NOISE

in Electronic Devices and Circuits

- Noise is any unwanted excitation of a circuit, any input that is not an information-bearing signal.

- Noise comes from
  - External sources: Unintended coupling with other parts of the physical world. In principle, this kind of noise can be virtually eliminated by careful design.
  - Internal sources: Unpredictable microscopic events that happen in the devices that constitute the circuit. In principle, this kind of noise can be reduced, but never eliminated.

- Noise is especially important to consider when designing low-power systems because the signal levels (typically voltages or currents) are small.

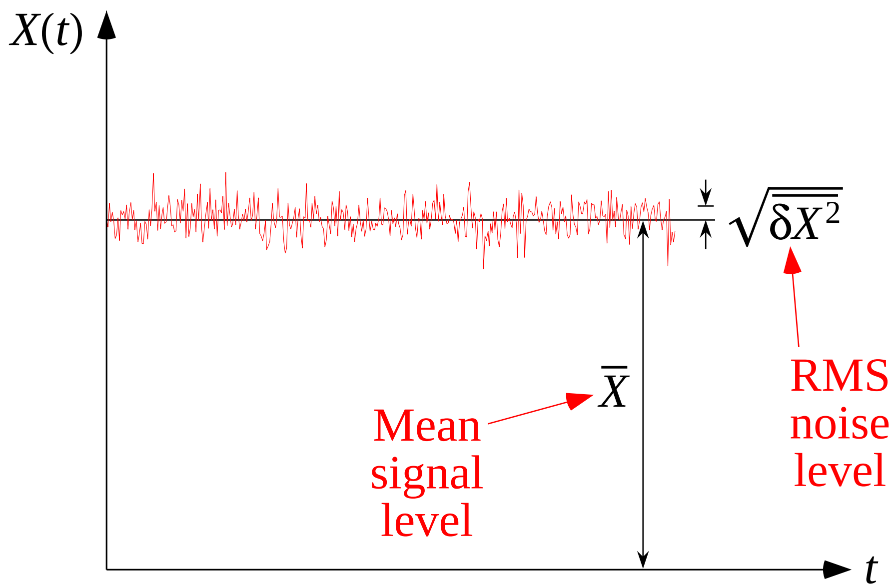
NOISE

- The amount of noise in a signal is characterized by its root mean square (RMS) value.

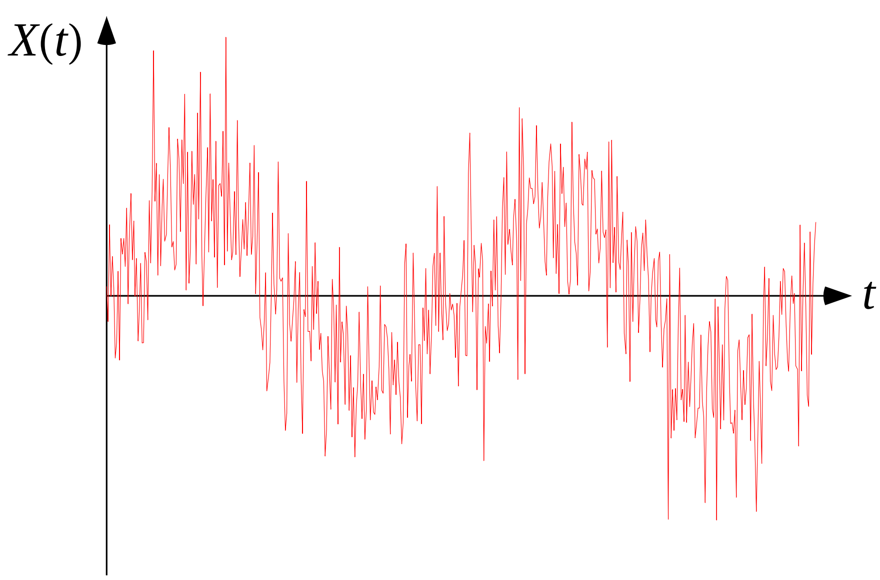
$$X(t) = \overline{X} + \delta X(t)$$

$$\overline{\delta X} = \frac{1}{T}\int_0^T \left(X(t) - \overline{X}\right) dt = 0$$

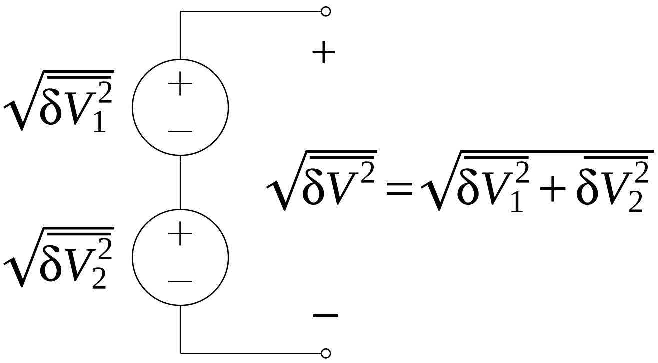
$$\overline{\delta X^2} = \frac{1}{T}\int_0^T \left(X(t) - \overline{X}\right)^2 dt$$

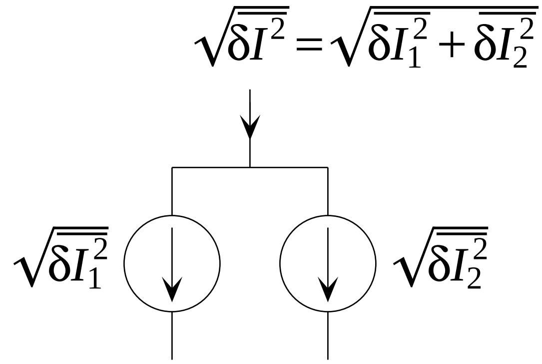


- The noise level sets the size of the smallest signal that can be processed meaningfully by a physical system.



NOISE





- The mean squared levels of (statistically) independent noise sources add:

$$\delta X(t) = \delta X_1(t) + \delta X_2(t)$$

$$\Rightarrow (\delta X(t))^2 = (\delta X_1(t))^2 + (\delta X_2(t))^2 + 2\delta X_1(t)\delta X_2(t)$$

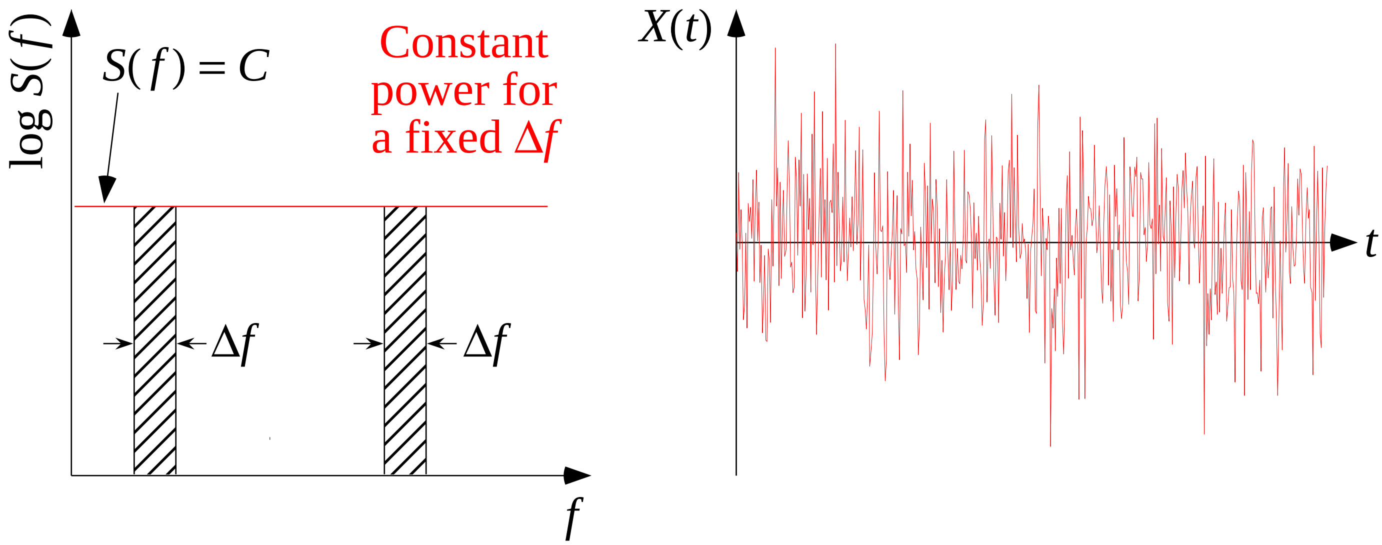
$$\Rightarrow \overline{\delta X^2} = \overline{\delta X_1^2} + \overline{\delta X_2^2} + 2\overline{\delta X_1 \delta X_2}$$

$$\Rightarrow \overline{\delta X^2} = \overline{\delta X_1^2} + \overline{\delta X_2^2} + 2\overline{\delta X_1}\,\overline{\delta X_2}$$

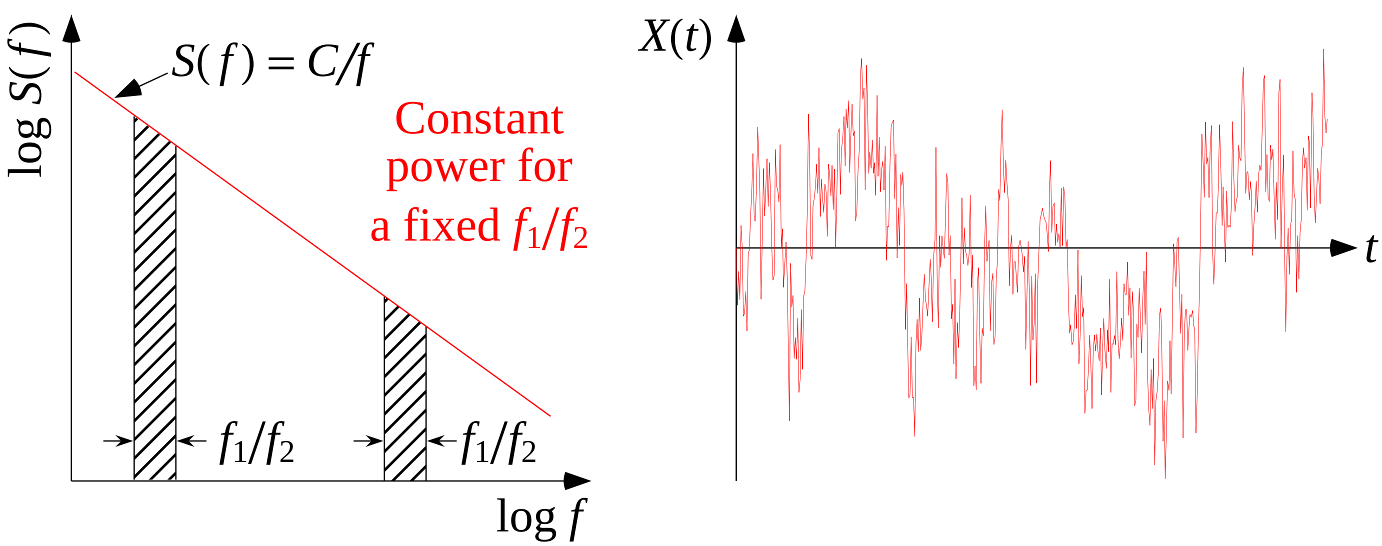
$$\Rightarrow \boxed{\overline{\delta X^2} = \overline{\delta X_1^2} + \overline{\delta X_2^2}}$$

NOISE

- The distribution of noise power over the spectrum is called the power spectral density (PSD) of the noise.
- White noise: Noise power is spread uniformly across the spectrum (cf. white light).



- Pink noise (a.k.a. flicker or $1/f$ noise): Noise power is concentrated at lower frequencies (cf. pink light).



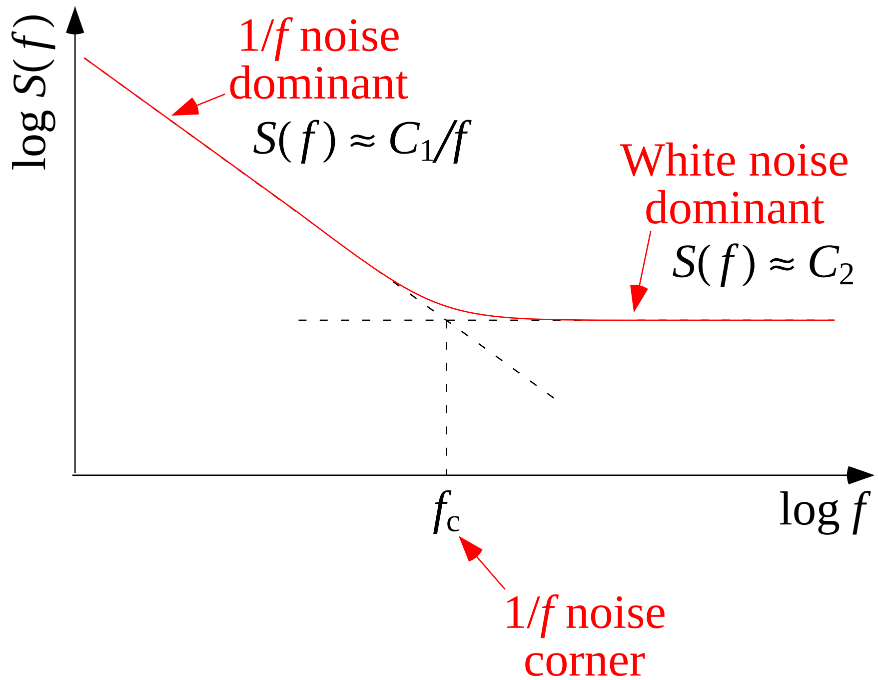
NOISE

- Usually, the noise in a device is a mixture of white noise and 1/*f* noise, where the two noise processes are independent.

PSD

$$S(f) = \frac{C_1}{f} + C_2$$

1/*f* noise component — white noise component



- In general, the 1/*f* noise corner frequency is highly process and bias dependent.

WHITE NOISE

- **Conventional view**: There are two distinct kinds.
- **Shot noise**: Variations in the arrival times of discrete charge carriers arising from the unpredictable time that each enters the device.

$$\overline{\delta I^2} = 2\, q\, \bar{I}\, \Delta f$$

Charge of each carrier (q) — Mean current level ($\bar{I}$) — System bandwidth (Δf)

  - Requires DC current flow.
  - Noise in diodes, BJTs, and vacuum tubes.

- **Thermal noise**: Variations in the arrival times of discrete charge carriers arising from their unpredictable thermal motions within in the device.

$$\overline{\delta I^2} = 4\, kT\, G\, \Delta f$$

Boltzmann constant (k) — Absolute temperature (T) — Conductance (G) — System bandwidth (Δf)

  - Requires no DC current flow.
  - Noise level is directly proportional to T.
  - Noise in resistors, MOSFETs, and JFETs.

- **Unconventional view**: Thermal noise is (two-sided) shot noise arising from diffusion currents.

SHOT NOISE

- We model the variation in the arrival times of discrete charge carriers at the terminal of a device as a Poisson process with a mean arrival rate, λ.
  - $\Pr\{\text{carrier arrives in } (t, t+dt)\} \approx \lambda\, dt$
  - Knowing how many carriers arrived in some past time interval tells us nothing about how many will arrive in a subsequent time interval.
- Let n be the number of carriers arriving in a given time interval T. For a Poisson process,

$$\bar{n} = \lambda T \text{ and } \overline{\delta n^2} = \lambda T$$

- Average current:

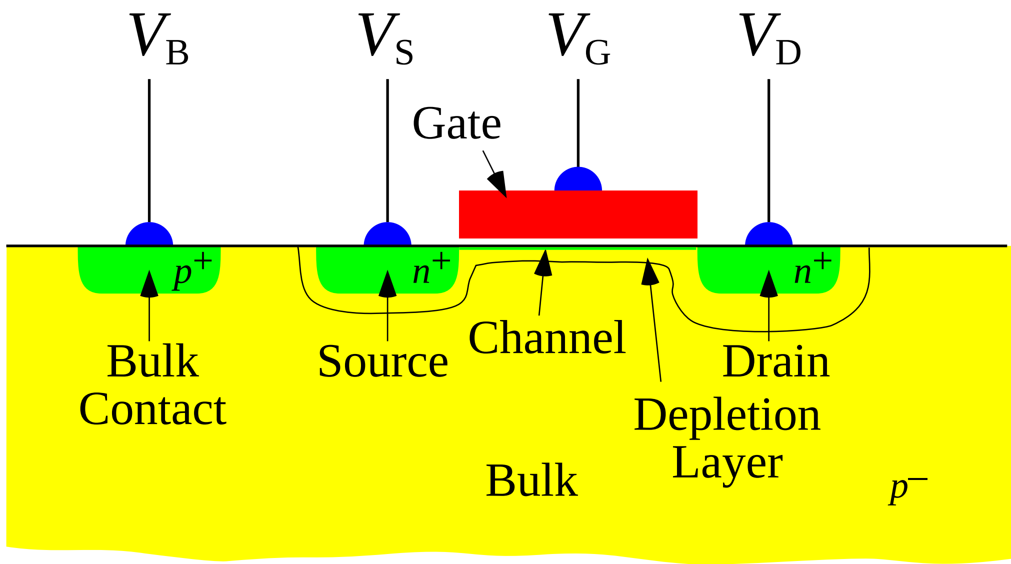
$$\bar{I} = \frac{q\bar{n}}{T} = \frac{q\lambda T}{T} = q\lambda$$

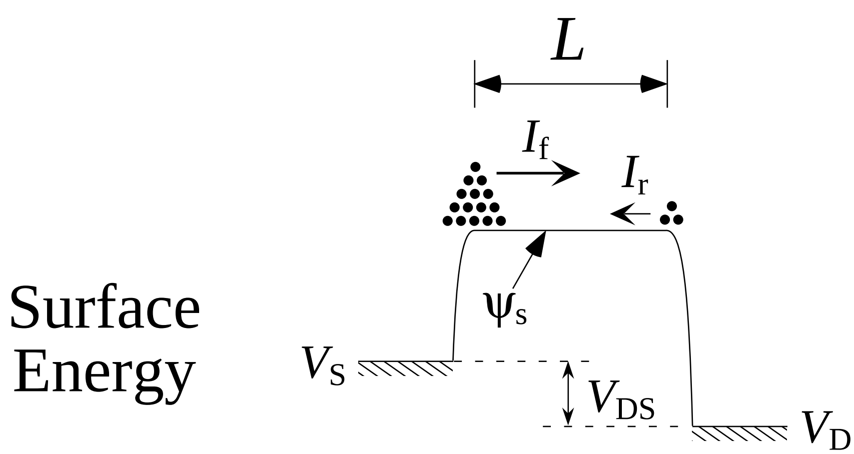
- Fluctuation in current over time scale T ($\Delta f \sim 1/T$):

$$\overline{\delta I^2} = \frac{q^2\,\overline{\delta n^2}}{T^2} = \frac{q^2 \lambda T}{T^2} = q\,\frac{q\lambda}{T} \sim q\,\bar{I}\Delta f$$

for every time scale T.

Subthreshold MOS Transistor





$$I_f = \frac{q\,D\,W\,Q_S}{L} \qquad I_r = \frac{q\,D\,W\,Q_D}{L}$$

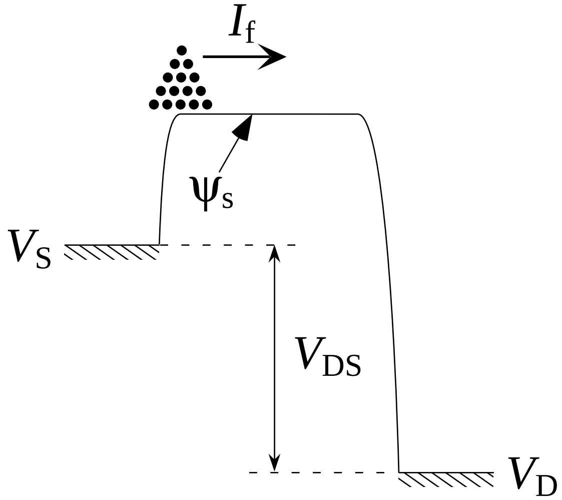
$$\overline{\delta I_f^2} = 2\,q\,I_f\,\Delta f \qquad \overline{\delta I_r^2} = 2\,q\,I_r\,\Delta f$$

$$I = I_f - I_r$$

$$\overline{\delta I^2} = \overline{\delta I_f^2} + \overline{\delta I_r^2} = 2\,q\,(I_f + I_r)\,\Delta f$$
$$= 2\,q\,I_{sat}\,(1 + e^{-V_{DS}/U_T})\,\Delta f$$

Subthreshold MOS Transistor

Saturation ($V_{DS} > 4\,U_T$)

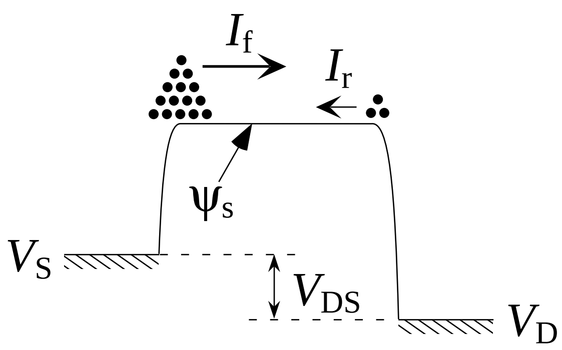


Noise is minimum because $I_r \approx 0$.

$$\overline{\delta I^2} = 2\,q\,I_{sat}\,\Delta f$$

$$I = I_{sat}$$

Ohmic ($0 < V_{DS} < 4\,U_T$)

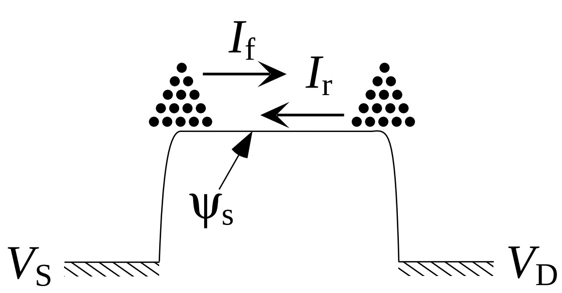


Noise is intermediate because $0 < I_r < I_{sat}$.

$$\overline{\delta I^2} = 2\,q\,I_{sat}\,(1 + e^{-V_{DS}/U_T})\,\Delta f$$

$$I = I_{sat}\,(1 - e^{-V_{DS}/U_T})$$

Linear ($V_{DS} = 0$)

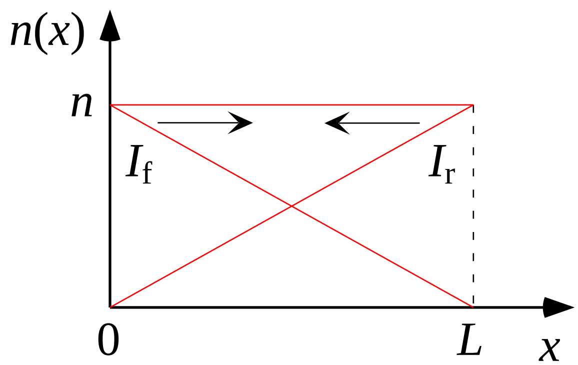


Noise is maximum because $I_r = I_{sat}$.

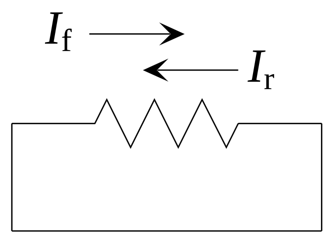
$$\overline{\delta I^2} = 4\,q\,I_{sat}\,\Delta f$$

$$I = 0$$

White Noise in a Resistor



$$I = I_f - I_r = 0$$



- Using the subthreshold MOS transistor with $V_{DS} = 0$ as a model, we can decompose the uniform concentration of electrons in a shorted resistor into equal forward and reverse components

$$I_f = \frac{qDnA}{L} \qquad I_r = \frac{qDnA}{L}$$

$$\overline{\delta I_f^2} = 2qI_f\Delta f \qquad \overline{\delta I_r^2} = 2qI_r\Delta f$$

$$\overline{\delta I^2} = \overline{\delta I_f^2} + \overline{\delta I_r^2} = 4q\frac{qDnA}{L}\Delta f$$

$$= 4kT\frac{q\mu nA}{L}\Delta f$$

$$= 4kTG\Delta f$$

Einstein relation

$$\frac{D}{\mu} = \frac{kT}{q}$$

- Nyquist's classic thermal noise result derived from a purely shot noise point of view!

WHITE NOISE
in Resistors and MOS Transistors

- One formula describes the white noise in all cases:

$$\overline{\delta I^2} = 4\,k\,T\,\mu\,\frac{W}{L}\,\overline{Q}\,\Delta f$$

Average charge per unit area

Subthreshold MOS Transistor:

$$\overline{\delta I^2} = 4\,k\,T\,\mu\,\frac{W}{L}\left(\frac{Q_S + Q_D}{2}\right)\Delta f$$

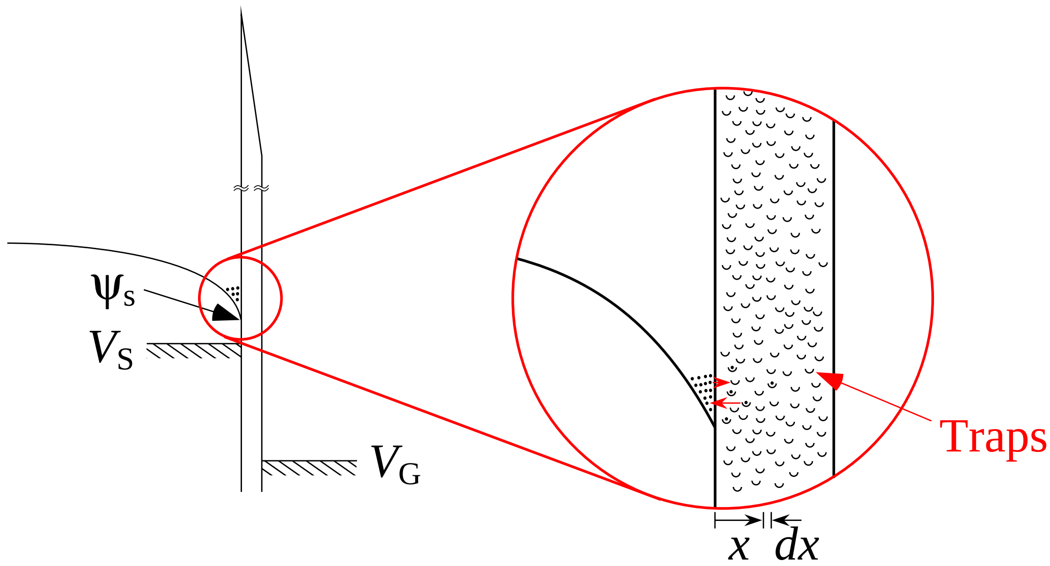
Above Threshold MOS Transistor:

$$\overline{\delta I^2} = 4\,k\,T\,\mu\,\frac{W}{L}\,\frac{2}{3}\left(\frac{Q_S^2 + Q_S Q_D + Q_D^2}{Q_S + Q_D}\right)\Delta f$$

Resistor:

$$\overline{\delta I^2} = 4\,k\,T\,G\,\Delta f$$

$1/f$ NOISE



- Assume that there is a uniform density, ρ, of traps in the gate oxide, independent of *x*.
- A charge, *q*, will enter or leave a trap at a time scale set by the tunneling probability:

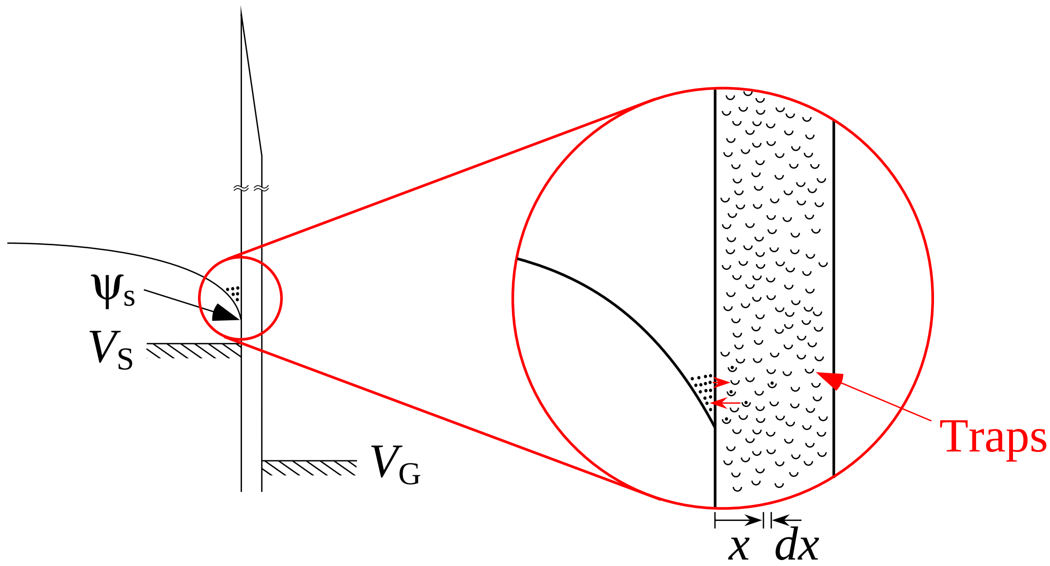
Fluctuation frequency → $f \sim e^{-\alpha x}$

$$\Rightarrow \log f \sim -\alpha x \Rightarrow \frac{df}{f} \sim -\alpha\, dx$$

- For a fixed frequency interval *df*, the total amount of trapped charge *Q* will fluctuate as $\sqrt{A \rho\, dx}$

$$\Rightarrow \overline{\delta Q^2} \sim A \rho\, dx \sim A \frac{df}{f}$$

$1/f$ NOISE



► We can think of the trapped charge as modulating the transistor's threshold voltage:

$$\overline{\delta Q^2} \sim A$$

$$\Rightarrow \overline{\delta V_T^2} \sim \frac{1}{C_G^2}\overline{\delta Q^2} \sim \frac{A}{A^2}\frac{df}{f} \sim \frac{1}{A}\frac{df}{f}$$

$$C_G \sim A$$

$$\Rightarrow \overline{\delta I^2} \sim g_m^2 \overline{\delta V_T^2} \sim \frac{g_m^2}{A}\frac{df}{f}$$