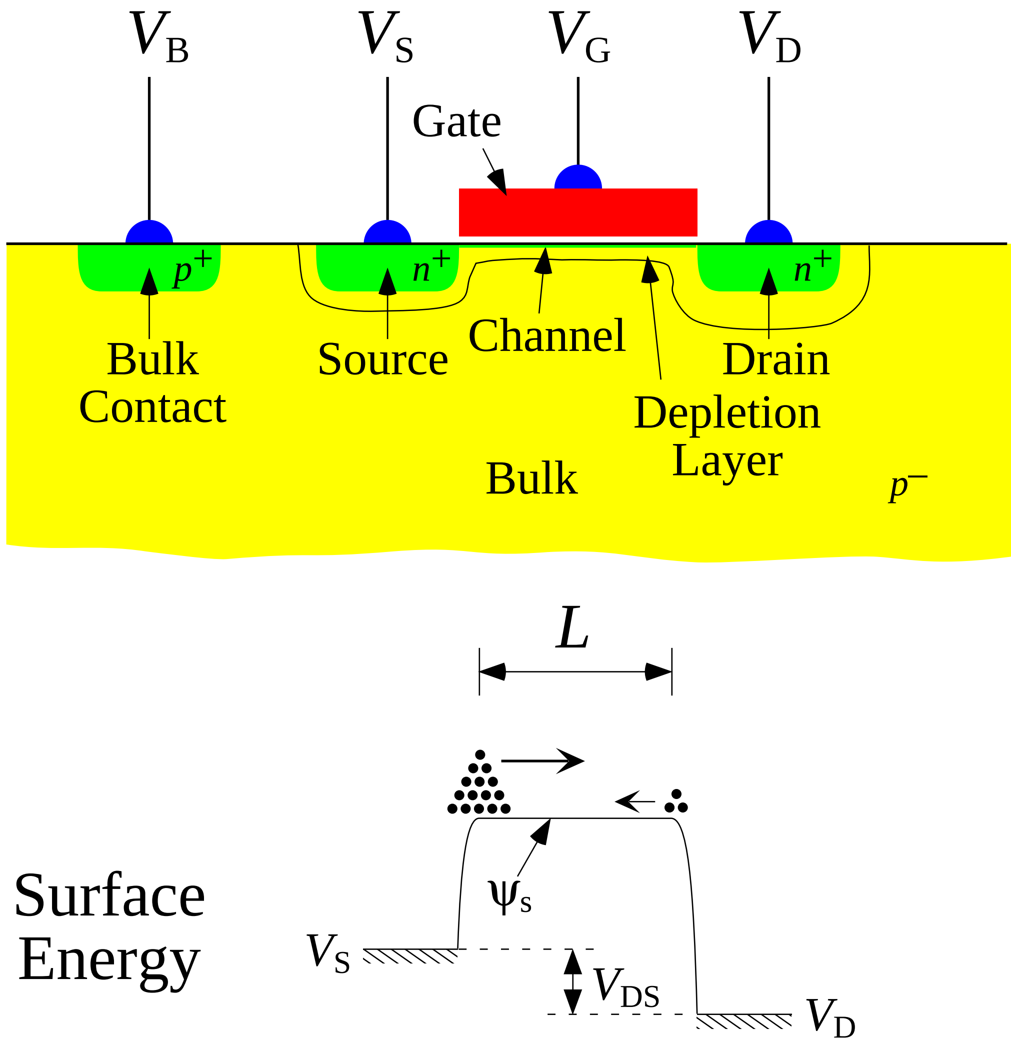
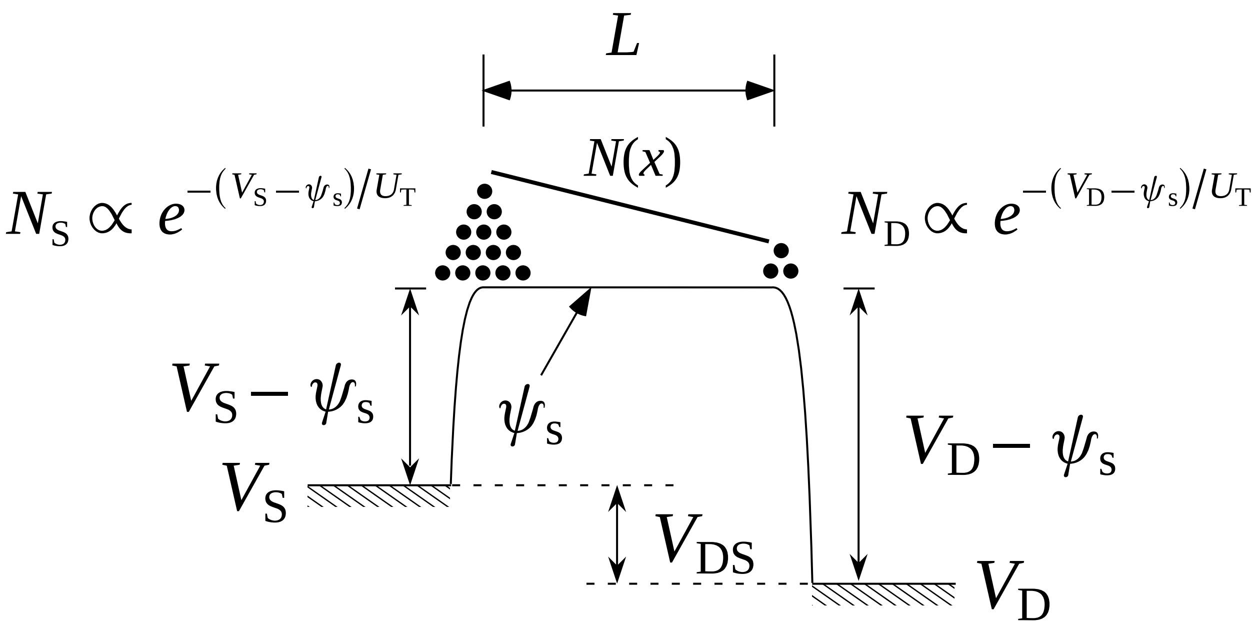
Surface Energy in Subthreshold



Surface Energy

Subthreshold Channel Current



- Current flow is by diffusion:

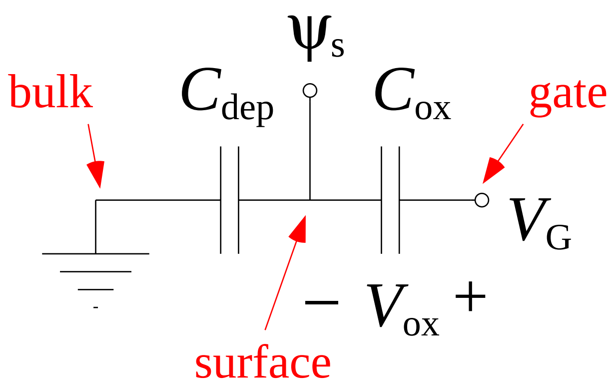
$$I = -WqD\frac{\partial N}{\partial x} = -WqD\frac{N_D - N_S}{L} = WqD\frac{N_S - N_D}{L}$$

$$= \frac{W}{L}qDN_0\left(e^{-(V_S-\psi_s)/U_T} - e^{-(V_D-\psi_s)/U_T}\right)$$

$$\psi_s \approx \psi_0 + \kappa V_G$$

$$I = \frac{W}{L}I_0 e^{\kappa V_G/U_T}\left(e^{-V_S/U_T} - e^{-V_D/U_T}\right)$$

Capacitive-Divider View of κ



$$V_{ox} = V_G - \psi_s$$

- Conservation of charge:

$$\Delta Q_s = 0 = \frac{\partial Q_s}{\partial \psi_s}\Delta\psi_s - \frac{\partial Q_s}{\partial V_{ox}}\Delta V_{ox}$$

($\frac{\partial Q_s}{\partial \psi_s}$: C_{dep}; $\frac{\partial Q_s}{\partial V_{ox}}$: C_{ox})

$$(C_{ox} + C_{dep})\,\Delta\psi_s = C_{ox}\,\Delta V_G$$

$$\frac{\partial\psi_s}{\partial V_G} \approx \frac{\Delta\psi_s}{\Delta V_G} = \frac{C_{ox}}{C_{ox} + C_{dep}} = \kappa$$

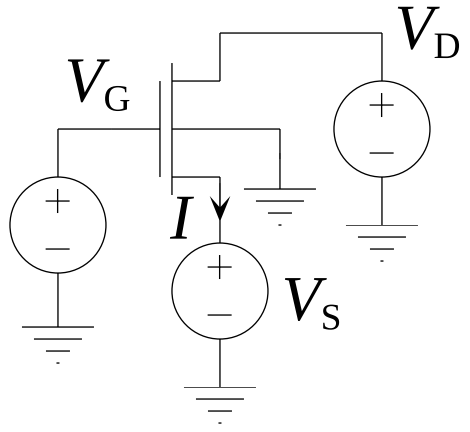
Subthreshold Channel Current

- *n*MOS:

$$I = \frac{W}{L} I_0 e^{\kappa V_G / U_T} \left(e^{-V_S / U_T} - e^{-V_D / U_T} \right)$$

$$= \frac{W}{L} I_0 e^{(\kappa V_G - V_S)/U_T} \left(1 - e^{-V_{DS} / U_T} \right)$$

$$\approx \frac{W}{L} I_0 e^{(\kappa V_G - V_S)/U_T} \quad (\text{for } V_{DS} \geq 4 U_T)$$

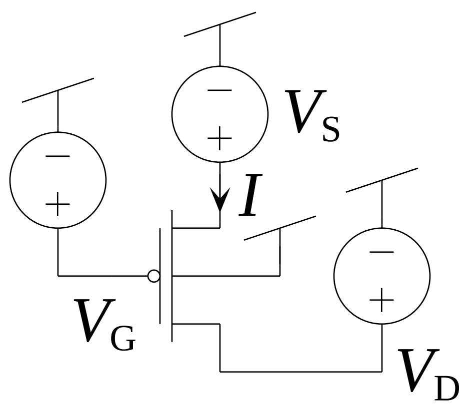


- *p*MOS:

$$I = \frac{W}{L} I_0 e^{-\kappa V_G / U_T} \left(e^{V_S / U_T} - e^{V_D / U_T} \right)$$

$$= \frac{W}{L} I_0 e^{(V_S - \kappa V_G)/U_T} \left(1 - e^{V_{DS} / U_T} \right)$$

$$\approx \frac{W}{L} I_0 e^{(V_S - \kappa V_G)/U_T} \quad (\text{for } V_{DS} \leq -4 U_T)$$



** Note here that $V_G < 0$, $V_S < 0$, and $V_D < 0$.

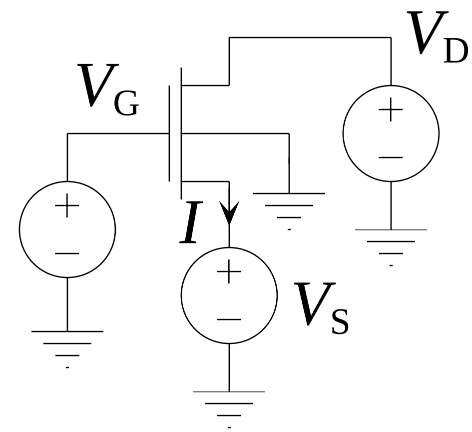
Subthreshold Channel Current

- *n*MOS:

$$I = \frac{W}{L} I_0 e^{\kappa V_G / U_T}\left(e^{-V_S/U_T} - e^{-V_D/U_T}\right)$$

$$= \frac{W}{L} I_0 e^{(\kappa V_G - V_S)/U_T}\left(1 - e^{-V_{DS}/U_T}\right)$$

$$\approx \frac{W}{L} I_0 e^{(\kappa V_G - V_S)/U_T} \quad (\text{for } V_{DS} \geq 4U_T)$$

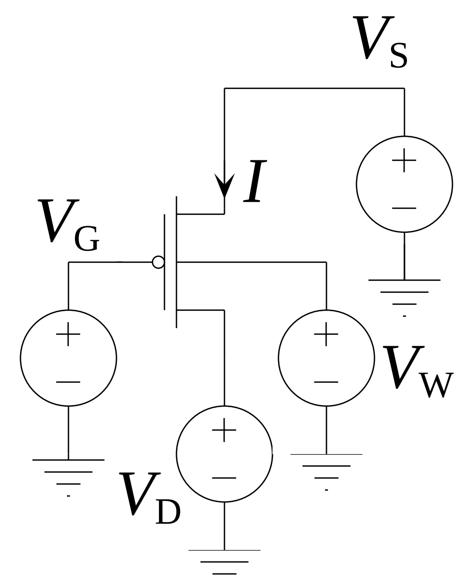


- *p*MOS:

$$I = \frac{W}{L} I_0 e^{\kappa (V_W - V_G)/U_T}\left(e^{-(V_W - V_S)/U_T} - e^{-(V_W - V_D)/U_T}\right)$$

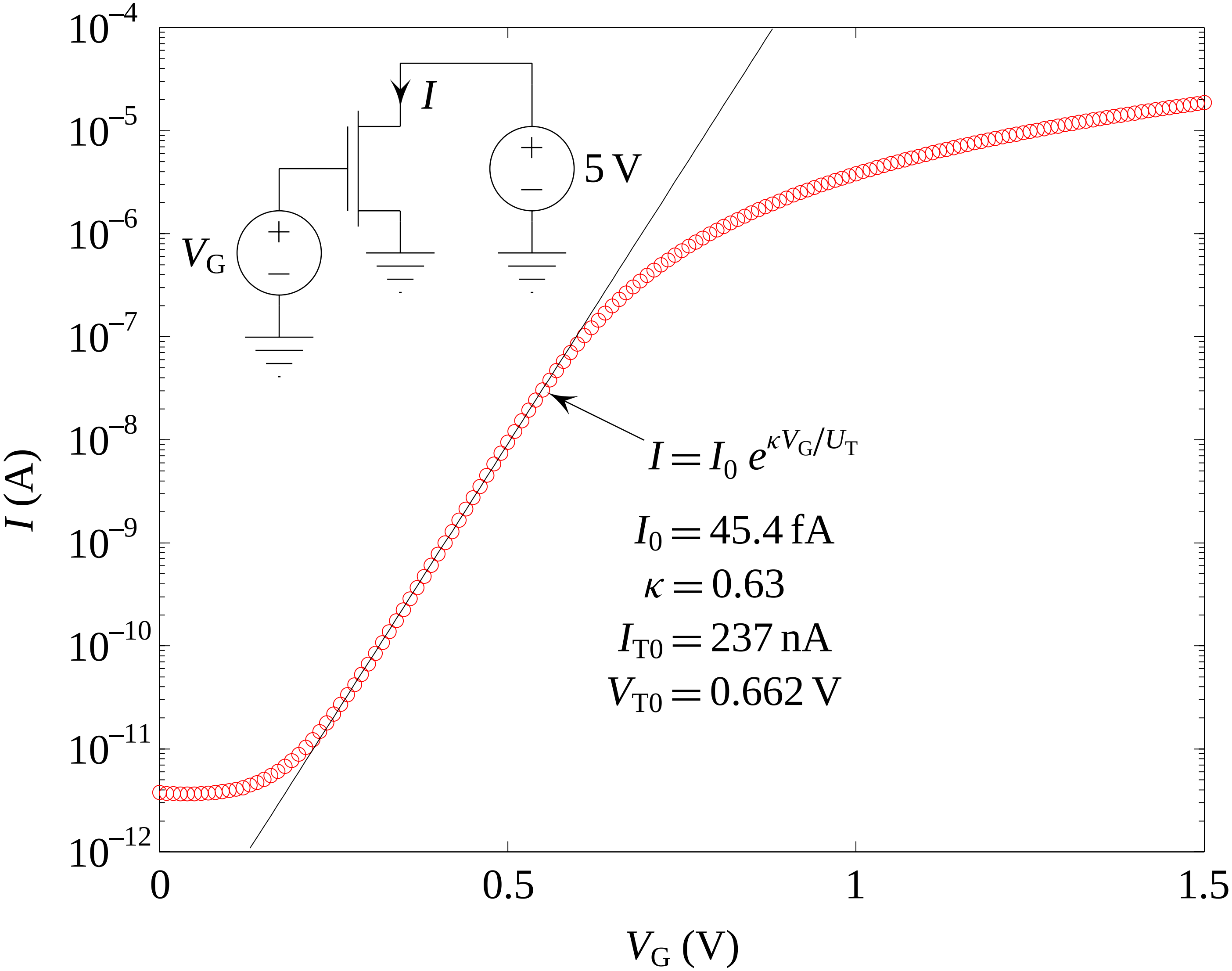
$$= \frac{W}{L} I_0 e^{(V_S - (1-\kappa)V_W - \kappa V_G)/U_T}\left(1 - e^{V_{DS}/U_T}\right)$$

$$\approx \frac{W}{L} I_0 e^{(V_S - (1-\kappa)V_W - \kappa V_G)/U_T} \quad (\text{for } V_{DS} \leq -4U_T)$$

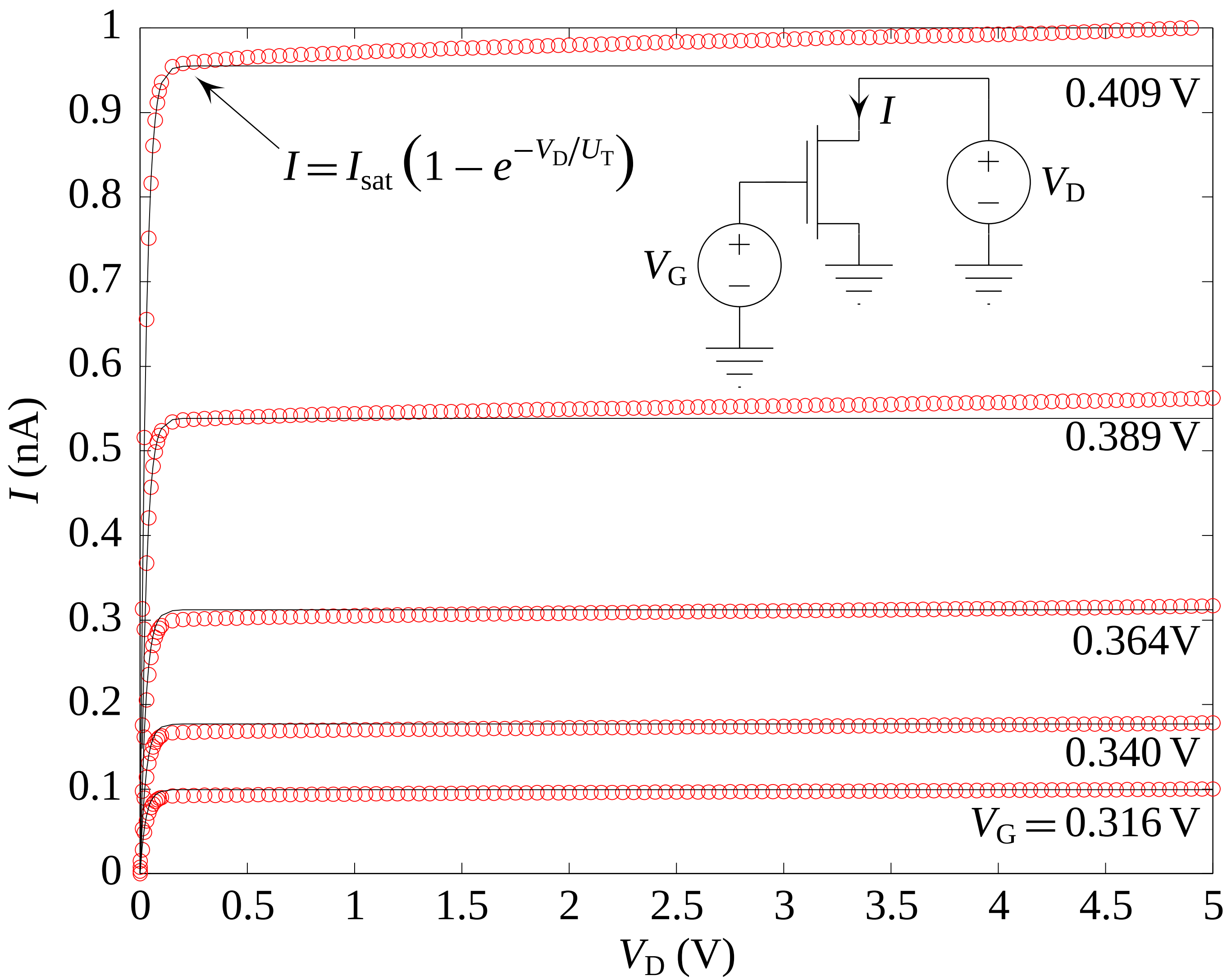


** Note here that $V_G > 0$, $V_S > 0$, $V_D > 0$, and $V_W > 0$.

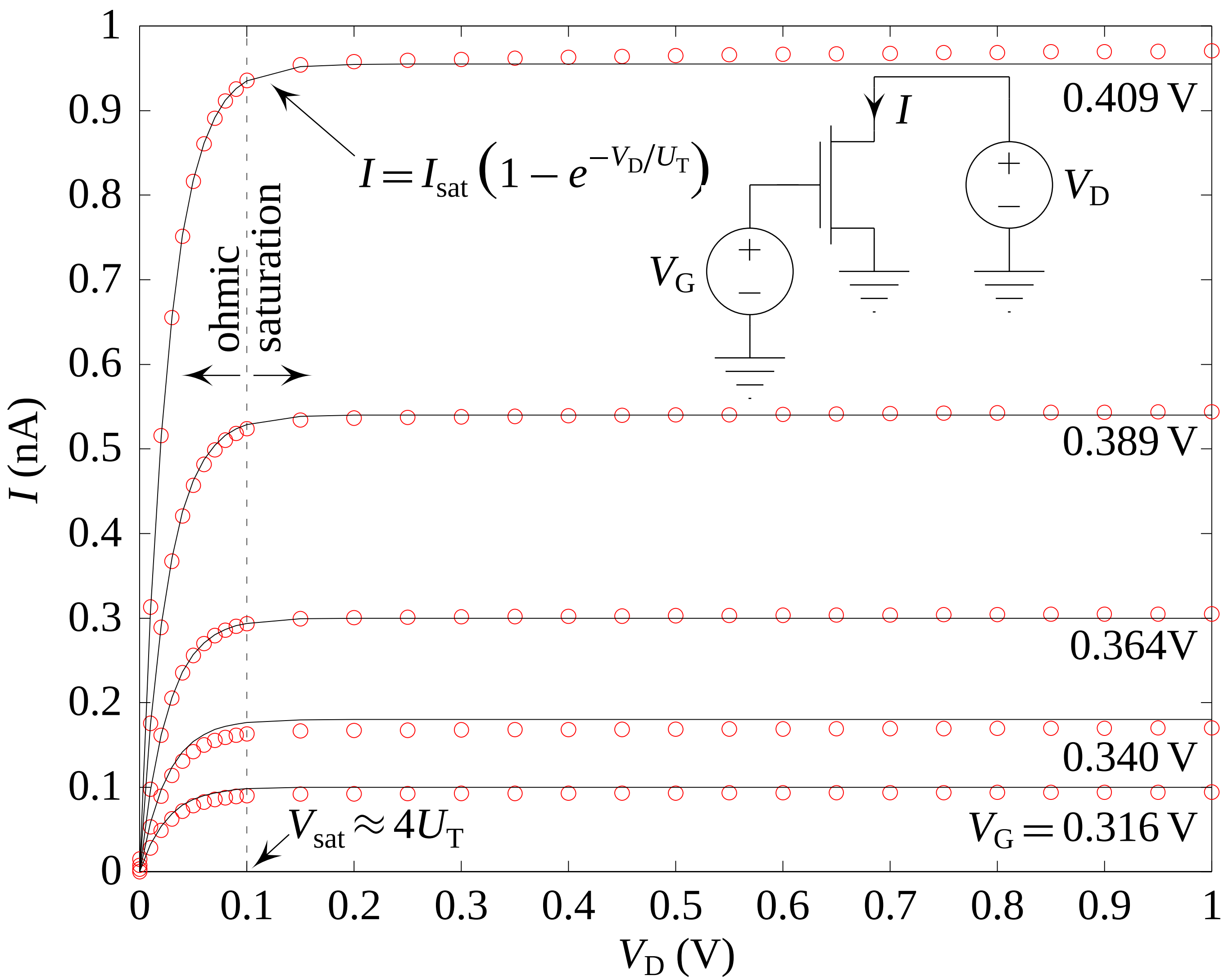
Subthreshold Channel Current in Saturation



Subthreshold Drain Characteristics



Subthreshold Drain Characteristics

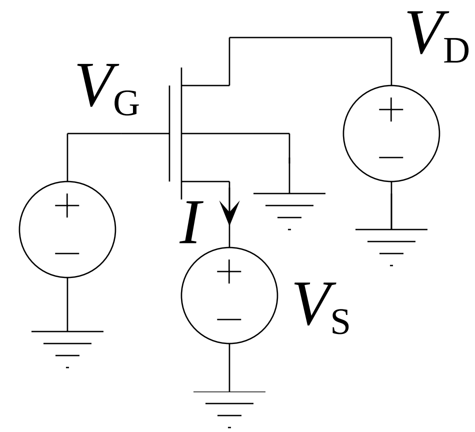


Above-Threshold Channel Current

- *n*MOS:

$$I = \frac{W}{L}\frac{\mu C_{ox}}{2\kappa}\left[\left(\kappa\left(V_G - V_{T0}\right) - V_S\right)^2 - \left(\kappa\left(V_G - V_{T0}\right) - V_D\right)^2\right]$$

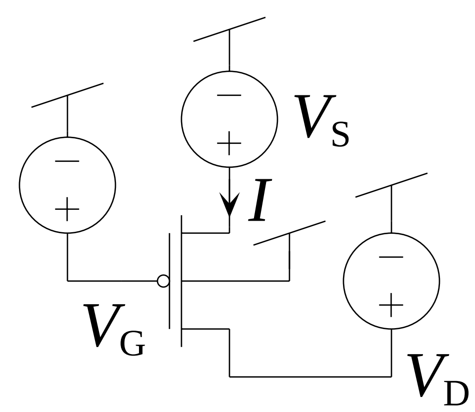
$$I = \frac{W}{L}\frac{\mu C_{ox}}{2\kappa}\left(\kappa\left(V_G - V_{T0}\right) - V_S\right)^2, \text{ for } V_D \geq \kappa\left(V_G - V_{T0}\right)$$



- *p*MOS:

$$I = \frac{W}{L}\frac{\mu C_{ox}}{2\kappa}\left[\left(V_S - \kappa\left(V_G - V_{T0}\right)\right)^2 - \left(V_D - \kappa\left(V_G - V_{T0}\right)\right)^2\right]$$

$$I = \frac{W}{L}\frac{\mu C_{ox}}{2\kappa}\left(V_S - \kappa\left(V_G - V_{T0}\right)\right)^2, \text{ for } V_D \leq \kappa\left(V_G - V_{T0}\right)$$



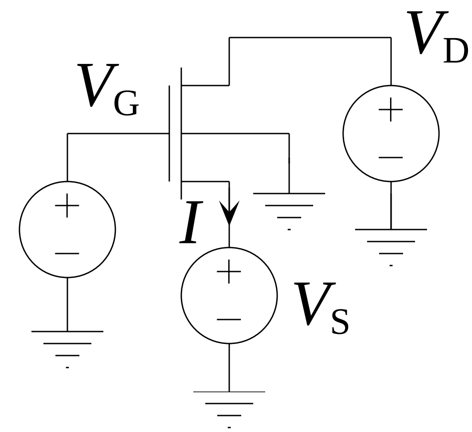
** Note here that $V_G < 0$, $V_S < 0$, $V_D < 0$, and $V_{T0} < 0$.

Above-Threshold Channel Current

- *n*MOS:

$$I = \frac{W}{L}\frac{\mu C_{ox}}{2\kappa}\left[\left(\kappa\left(V_G - V_{T0}\right) - V_S\right)^2 - \left(\kappa\left(V_G - V_{T0}\right) - V_D\right)^2\right]$$

$$I = \frac{W}{L}\frac{\mu C_{ox}}{2\kappa}\left(\kappa\left(V_G - V_{T0}\right) - V_S\right)^2, \text{ for } V_D \geq \kappa\left(V_G - V_{T0}\right)$$

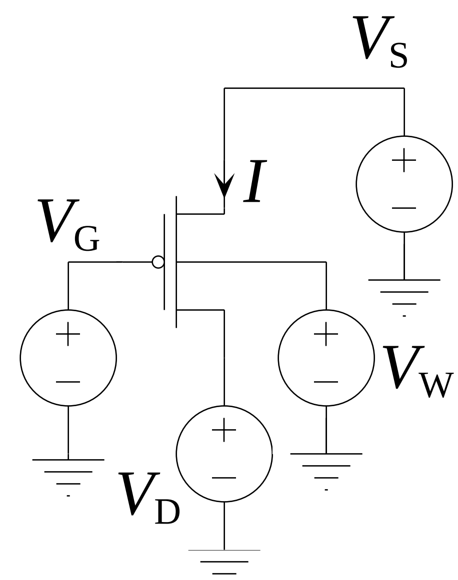


- *p*MOS:

$$I = \frac{W}{L}\frac{\mu C_{ox}}{2\kappa}\left[\left(\kappa\left(\left(V_W - V_G\right) - \left|V_{T0}\right|\right) - \left(V_W - V_S\right)\right)^2 - \left(\kappa\left(\left(V_W - V_G\right) - \left|V_{T0}\right|\right) - \left(V_W - V_D\right)\right)^2\right]$$

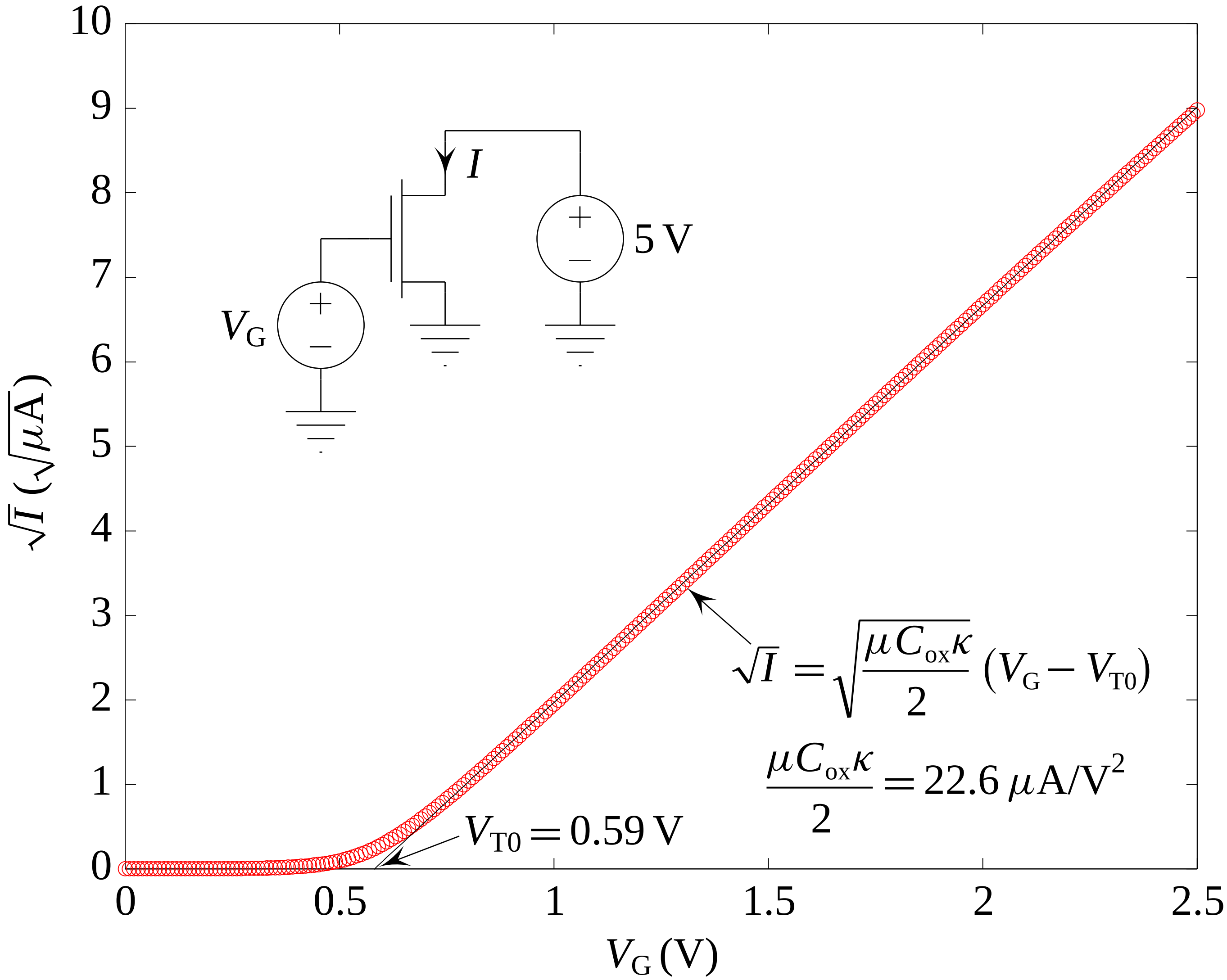
$$I = \frac{W}{L}\frac{\mu C_{ox}}{2\kappa}\left(\kappa\left(\left(V_W - V_G\right) - \left|V_{T0}\right|\right) - \left(V_W - V_S\right)\right)^2,$$

$$\text{for } V_D \leq V_W - \kappa\left(\left(V_W - V_G\right) - \left|V_{T0}\right|\right)$$

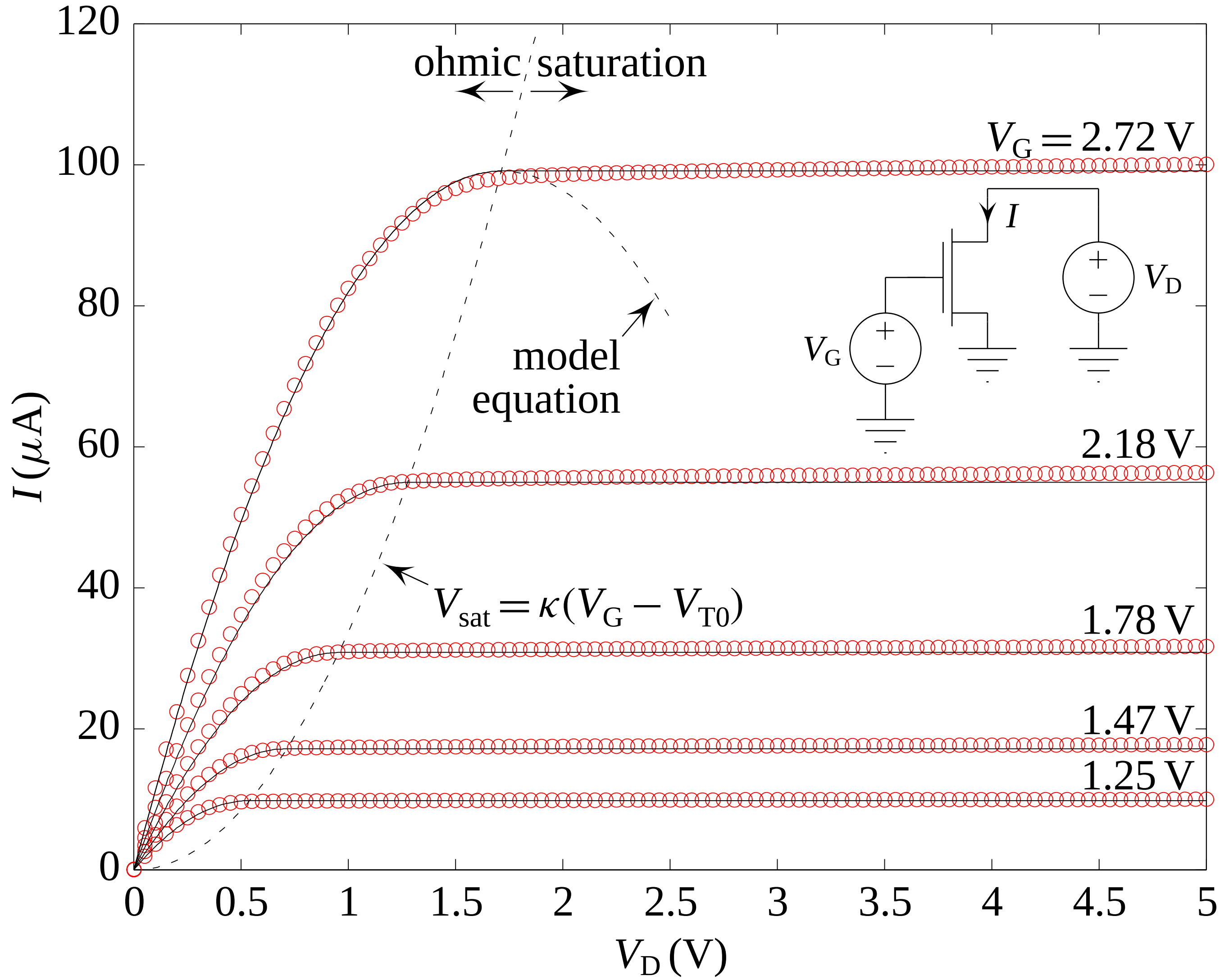


** Note here that $V_G > 0$, $V_S > 0$, $V_D > 0$, and $V_W > 0$.

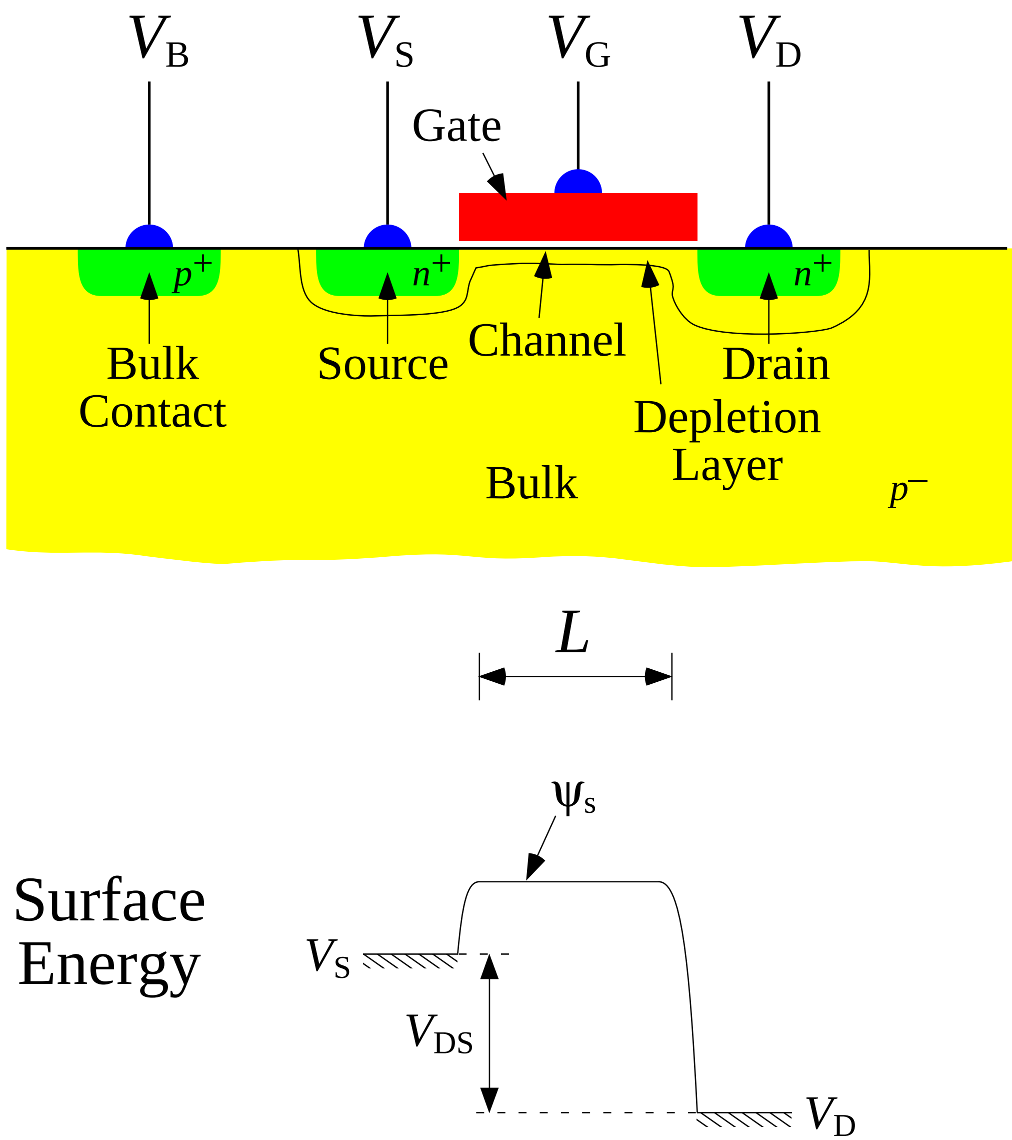
Above-Threshold Channel Current in Saturation



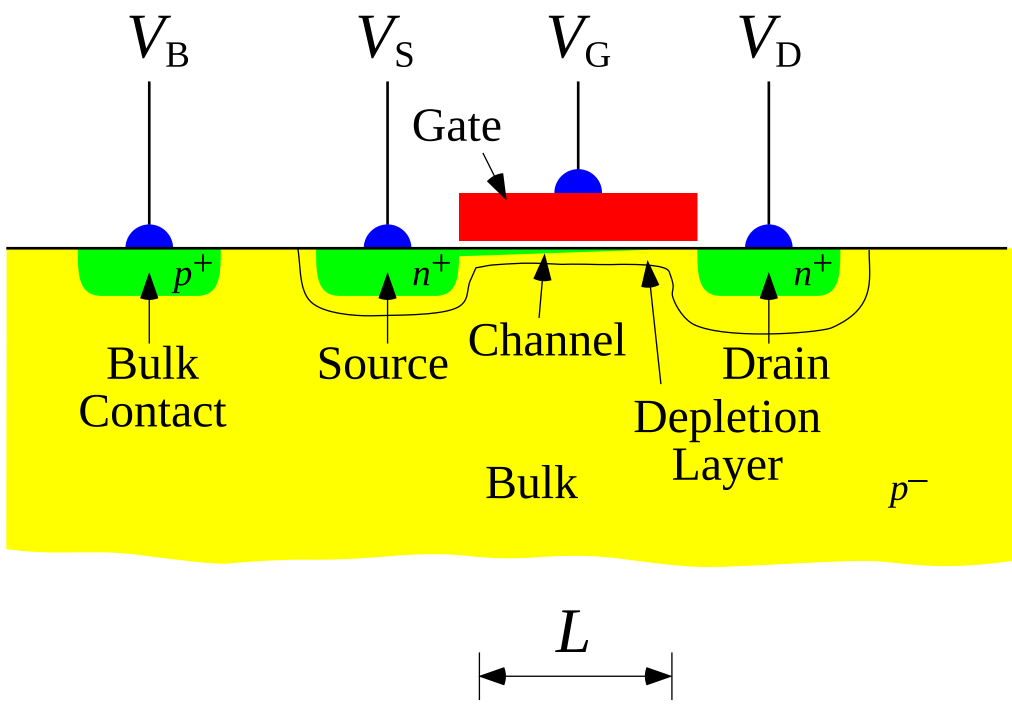
Above-Threshold Drain Characteristics

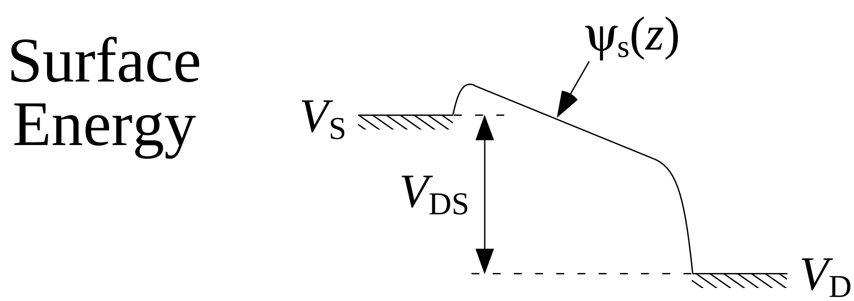


Surface Energy in Subthreshold

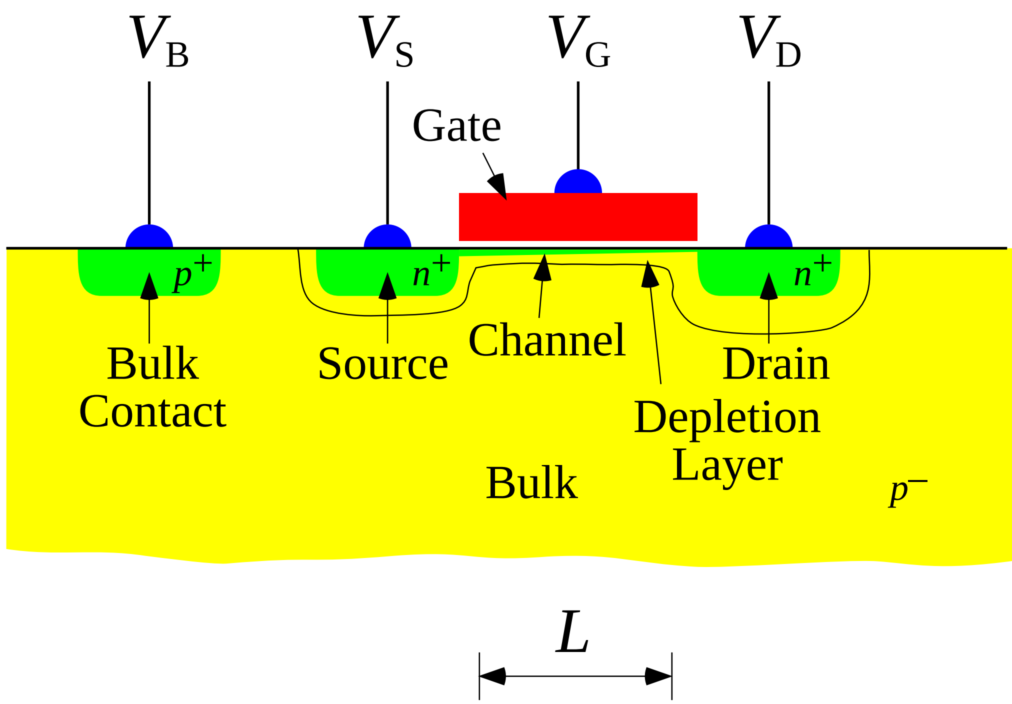


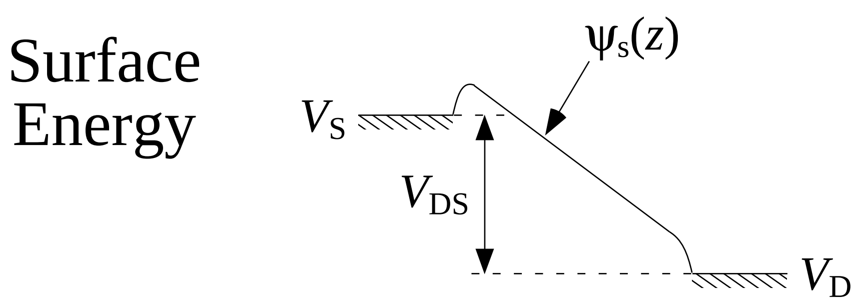
Surface Energy in Saturation Above Threshold



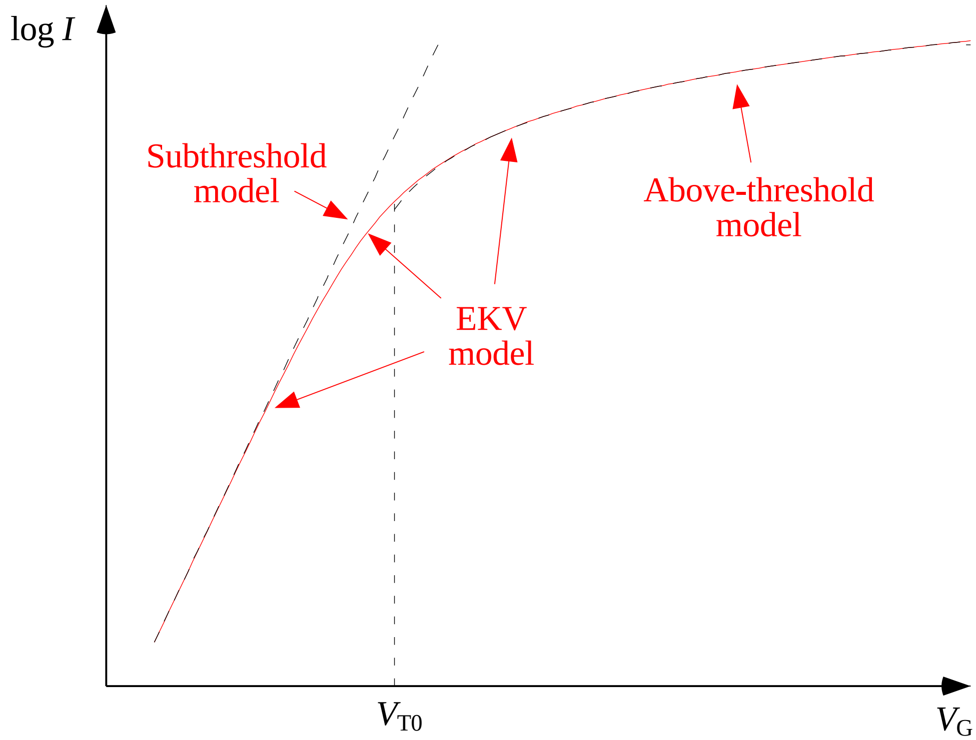


Surface Energy in Ohmic Region Above Threshold





Enz-Krummenacher-Vittoz Model



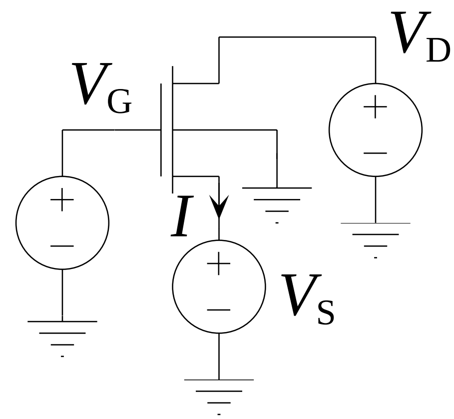
$$I = \frac{W}{L}\frac{\mu C_{ox}}{\kappa}2U_T^2\left[\log^2\left(1+e^{(\kappa(V_G-V_{T0})-V_S)/2U_T}\right) - \log^2\left(1+e^{(\kappa(V_G-V_{T0})-V_D)/2U_T}\right)\right]$$

- Model is continuous from subthreshold to above threshold and is valid in both the ohmic and saturation regions.
- $\log^2(1 + e^{x/2})$ function smoothly interpolates between e^x and x^2.
- Not based on first principles. (i.e., an elegant mathematical hack!)

EKV Model Channel Current

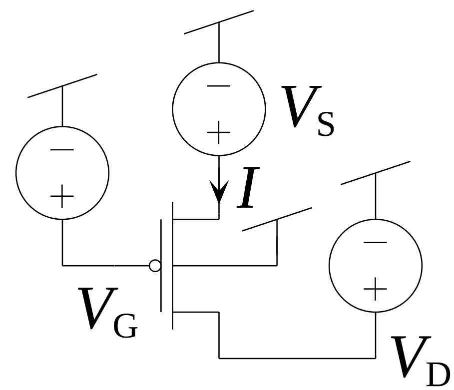
- *n*MOS:

$$I = \frac{W}{L}\frac{\mu C_{ox}}{\kappa} 2U_T^2 \left[\log^2\left(1+e^{(\kappa(V_G - V_{T0}) - V_S)/2U_T}\right) - \log^2\left(1+e^{(\kappa(V_G - V_{T0}) - V_D)/2U_T}\right)\right]$$



- *p*MOS:

$$I = \frac{W}{L}\frac{\mu C_{ox}}{\kappa} 2U_T^2 \left[\log^2\left(1+e^{(V_S - \kappa(V_G - V_{T0}))/2U_T}\right) - \log^2\left(1+e^{(V_D - \kappa(V_G - V_{T0}))/2U_T}\right)\right]$$

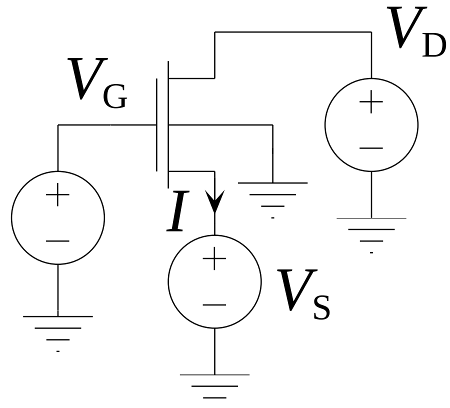


** Note here that $V_G < 0$, $V_S < 0$, $V_D < 0$, and $V_{T0} < 0$.

EKV Model Channel Current

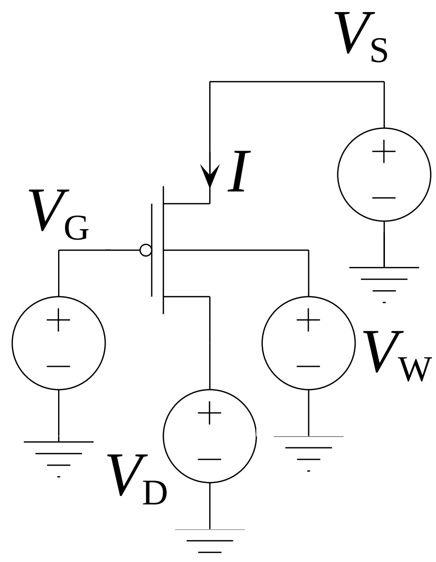
- *n*MOS:

$$I = \frac{W}{L}\frac{\mu C_{ox}}{\kappa} 2U_T^2 \left[\log^2\left(1 + e^{(\kappa(V_G - V_{T0}) - V_S)/2U_T}\right) - \log^2\left(1 + e^{(\kappa(V_G - V_{T0}) - V_D)/2U_T}\right) \right]$$



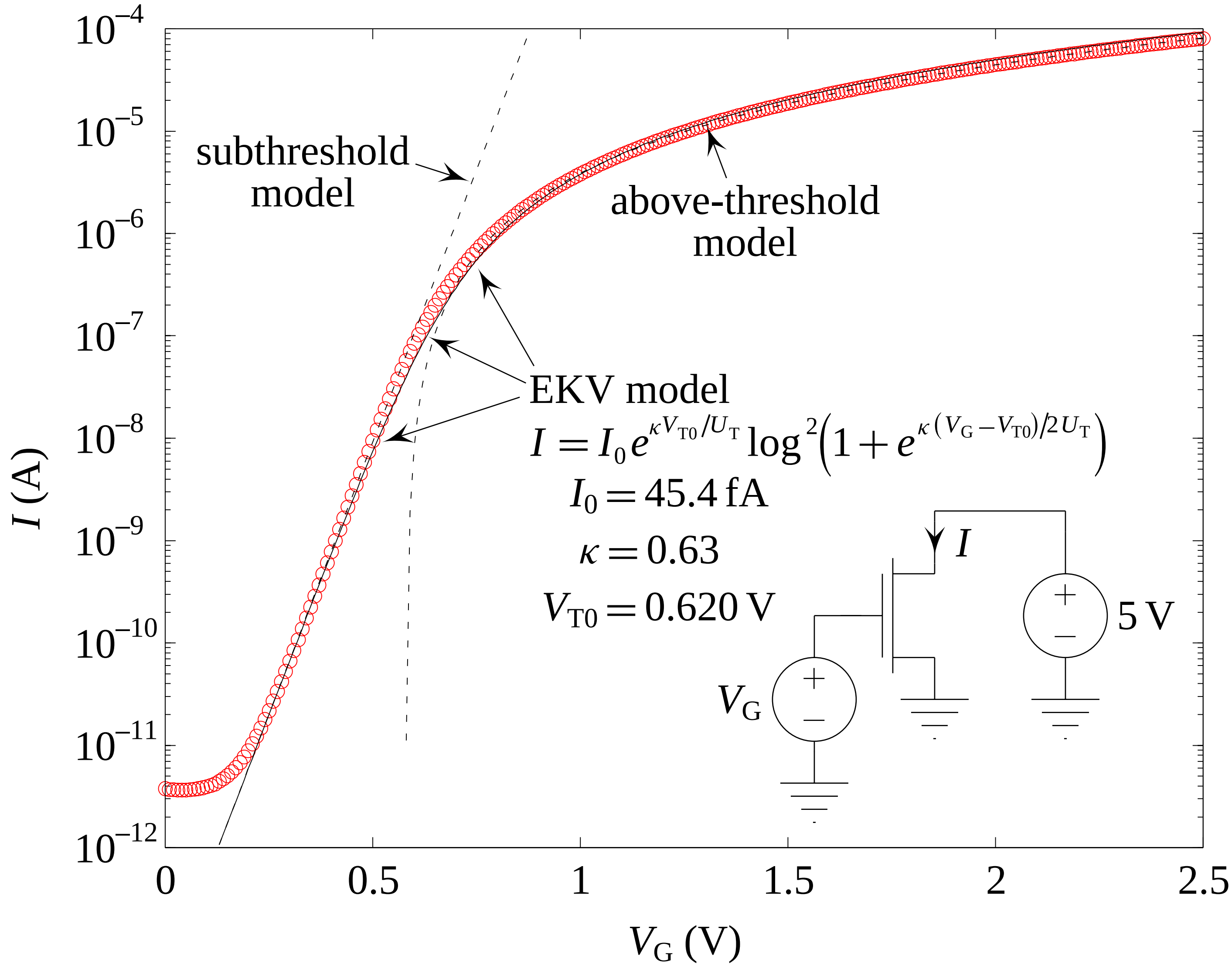
- *p*MOS:

$$I = \frac{W}{L}\frac{\mu C_{ox}}{\kappa} 2U_T^2 \left[\log^2\left(1 + e^{(\kappa((V_W - V_G) - |V_{T0}|) - (V_W - V_S))/2U_T}\right) - \log^2\left(1 + e^{(\kappa((V_W - V_G) - |V_{T0}|) - (V_W - V_D))/2U_T}\right) \right]$$

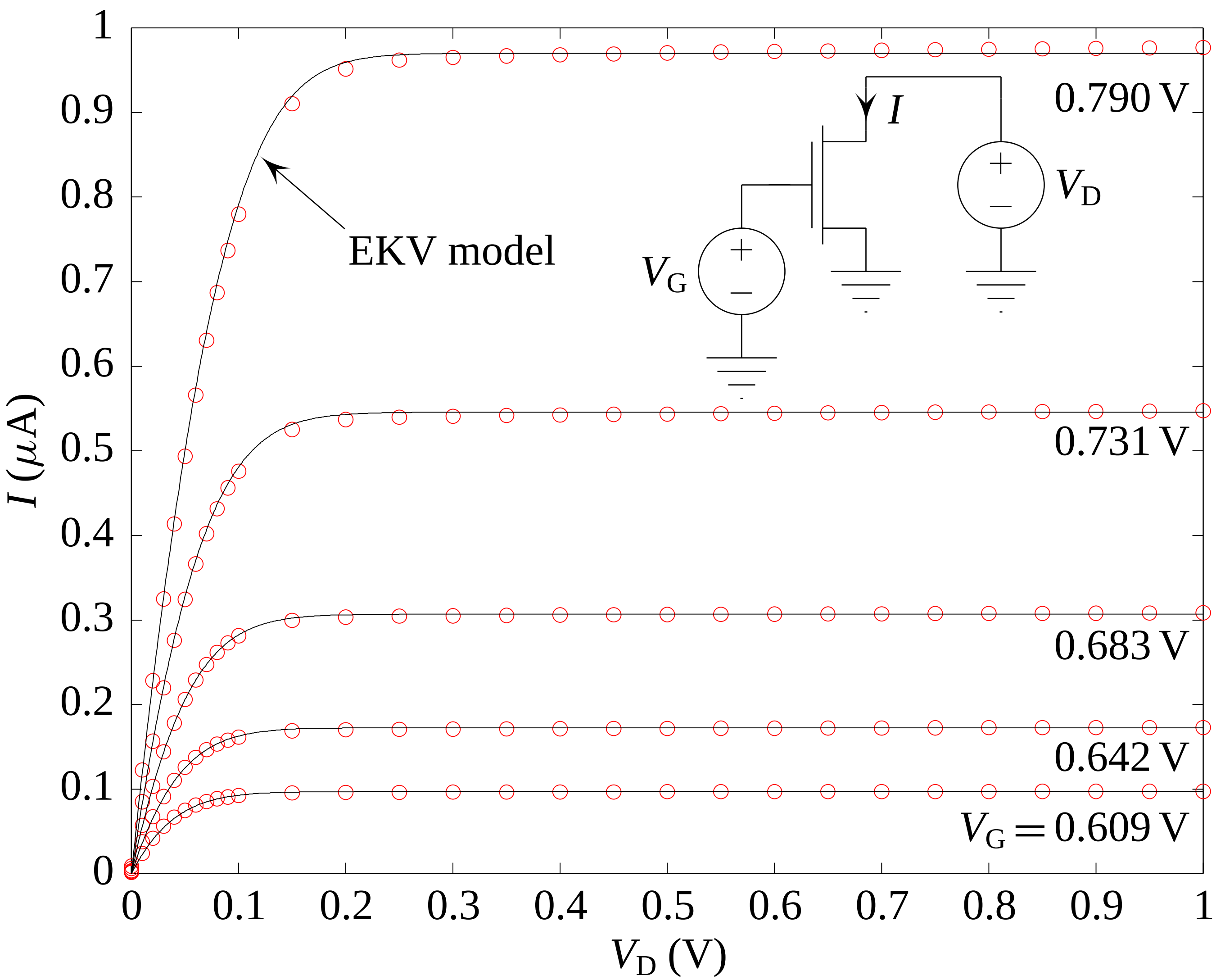


** Note here that $V_G > 0$, $V_S > 0$, $V_D > 0$, and $V_W > 0$.

EKV Model: Channel Current in Saturation



EKV Model: Drain Characteristics



Channel Length Modulation (a.k.a. The Early Effect)