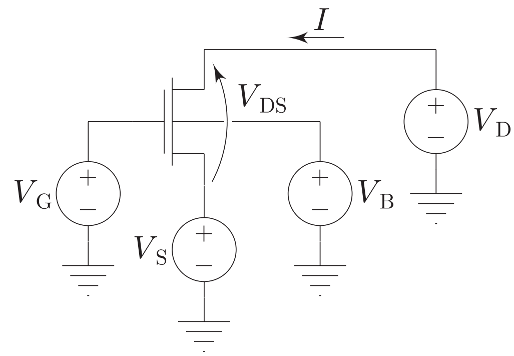
ENGR 2420: EKV MOS Transistor Model Summary

March 10, 2020

Table 1. Basic definitions and quantities used in the EKV model.

Qty	**Definition**	**Units**	**Description**
k	1.381×10^{-23}	J/K	Boltzmann constant
T		K	Absolute temperature
q	1.602×10^{-19}	C	Electronic charge
U_{T}	$\frac{kT}{q}$	V	Thermal voltage
W		m	Effective channel width
L		m	Effective channel length
S	$\frac{W}{L}$	-	Strength ratio
ϵ_{ox}	3.453×10^{-11}	F/m	Permittivity of silicon dioxide
ϵ_{si}	1.036×10^{-10}	F/m	Permittivity of silicon
t_{ox}		m	Gate-oxide thickness
t_{dep}		m	Depletion-layer thickness
C_{ox}	$\frac{\epsilon_{\mathrm{ox}}}{t_{\mathrm{ox}}}$	F/m^2	Gate-oxide capacitance per unit area
C_{dep}	$\frac{\epsilon_{\mathrm{si}}}{t_{\mathrm{dep}}}$	F/m^2	Incremental depletion-layer capacitance per unit area
κ	$\frac{C_{\mathrm{ox}}}{C_{\mathrm{ox}} + C_{\mathrm{dep}}}$	-	Reciprocal subthreshold slope factor
μ		m^2/V·s	Low-field carrier mobility
I_{sat}		A	Saturation current
I_{s}	$\frac{2S\mu C_{\mathrm{ox}} U_{\mathrm{T}}^2}{\kappa}$	A	Specific current (cf. $\approx 2I_{\mathrm{sat}}$ at threshold)
V_{T0}		V	Zero-bias threshold voltage
V_{A}		V	Early voltage
r_{o}	$\frac{V_{\mathrm{A}}}{I_{\mathrm{sat}}}$	Ω	Incremental output resistance in saturation
V_{XY}	$V_{\mathrm{X}} - V_{\mathrm{Y}}$	V	Potential difference between terminals X and Y

The relationships given below in Table 2 pertain to an nMOS transistor biased with $V_{\rm G} \geq V_{\rm B}$, $V_{\rm S} \geq V_{\rm B}$, and $V_{\rm D} \geq V_{\rm S}$, as shown to the right.



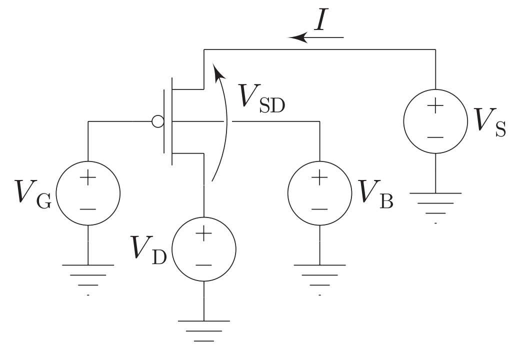
$$\text{Weak Inversion} \;\Rightarrow\; \kappa\left(V_{\rm GB} - V_{\rm T0}\right) < V_{\rm SB} \quad \text{and} \quad I_{\rm sat} \ll I_{\rm s}$$

$$\text{Strong Inversion} \;\Rightarrow\; \kappa\left(V_{\rm GB} - V_{\rm T0}\right) > V_{\rm SB} \quad \text{and} \quad I_{\rm sat} \gg I_{\rm s}$$

Table 2. Important EKV model relationships for an nMOS transistor.

Qty	**EKV Model**	**Weak Inversion**	**Strong Inversion**
$I_{\rm F}$	$I_{\rm s} \log^2\left(1 + e^{(\kappa(V_{\rm GB}-V_{\rm T0})-V_{\rm SB})/2U_{\rm T}}\right)$	$I_{\rm s} e^{(\kappa(V_{\rm GB}-V_{\rm T0})-V_{\rm SB})/U_{\rm T}}$	$I_{\rm s}\left(\frac{\kappa\left(V_{\rm GB}-V_{\rm T0}\right)-V_{\rm SB}}{2U_{\rm T}}\right)^2$
$I_{\rm R}$	$I_{\rm s} \log^2\left(1 + e^{(\kappa(V_{\rm GB}-V_{\rm T0})-V_{\rm DB})/2U_{\rm T}}\right)$	$I_{\rm s} e^{(\kappa(V_{\rm GB}-V_{\rm T0})-V_{\rm DB})/U_{\rm T}}$	$I_{\rm s}\left(\frac{\kappa\left(V_{\rm GB}-V_{\rm T0}\right)-V_{\rm DB}}{2U_{\rm T}}\right)^2$
$V_{\rm DS}$	$2U_{\rm T}\log\left(e^{\sqrt{I_{\rm F}/I_{\rm s}}}-1\right) - 2U_{\rm T}\log\left(e^{\sqrt{I_{\rm R}/I_{\rm s}}}-1\right)$	$U_{\rm T}\log\frac{I_{\rm F}}{I_{\rm R}}$	$2U_{\rm T}\left(\sqrt{\frac{I_{\rm F}}{I_{\rm s}}}-\sqrt{\frac{I_{\rm R}}{I_{\rm s}}}\right)$
I	$I_{\rm F}-I_{\rm R}$	$I_{\rm F}-I_{\rm R}$	$I_{\rm F}-I_{\rm R}$
$I_{\rm sat}$	$I_{\rm F}$	$I_{\rm F}$	$I_{\rm F}$
$V_{\rm DSsat}$	$2U_{\rm T}\log\left(e^{\sqrt{I_{\rm sat}/I_{\rm s}}}-1\right) - 2U_{\rm T}\log\left(e^{\sqrt{I_{\rm sat}/AI_{\rm s}}}-1\right)$	$U_{\rm T}\log A$	$2U_{\rm T}\sqrt{\frac{I_{\rm sat}}{I_{\rm s}}}\left(1-\frac{1}{\sqrt{A}}\right)$
$g_{\rm s}$	$\frac{\sqrt{I_{\rm s}I_{\rm sat}}}{U_{\rm T}}\left(1-e^{-\sqrt{I_{\rm sat}/I_{\rm s}}}\right)$	$\frac{I_{\rm sat}}{U_{\rm T}}$	$\frac{\sqrt{I_{\rm s}I_{\rm sat}}}{U_{\rm T}}$
$g_{\rm m}$	$\kappa g_{\rm s}$	$\kappa g_{\rm s}$	$\kappa g_{\rm s}$
$g_{\rm mb}$	$\left(1-\kappa\right)g_{\rm s}$	$\left(1-\kappa\right)g_{\rm s}$	$\left(1-\kappa\right)g_{\rm s}$
$\frac{g_{\rm m}}{I_{\rm sat}}$	$\frac{\kappa}{U_{\rm T}}\sqrt{\frac{I_{\rm s}}{I_{\rm sat}}}\left(1-e^{-\sqrt{I_{\rm sat}/I_{\rm s}}}\right)$	$\frac{\kappa}{U_{\rm T}}$	$\frac{\kappa}{U_{\rm T}}\sqrt{\frac{I_{\rm s}}{I_{\rm sat}}}$

The relationships given below in Table 3 pertain to a pMOS transistor biased with $V_{\mathrm{G}} \leq V_{\mathrm{B}}$, $V_{\mathrm{S}} \leq V_{\mathrm{B}}$, and $V_{\mathrm{D}} \leq V_{\mathrm{S}}$, as shown to the right.



$$\text{Weak Inversion} \Rightarrow \kappa\left(V_{\mathrm{BG}}-V_{\mathrm{T0}}\right) < V_{\mathrm{BS}} \quad \text{and} \quad I_{\mathrm{sat}} \ll I_{\mathrm{s}}$$

$$\text{Strong Inversion} \Rightarrow \kappa\left(V_{\mathrm{BG}}-V_{\mathrm{T0}}\right) > V_{\mathrm{BS}} \quad \text{and} \quad I_{\mathrm{sat}} \gg I_{\mathrm{s}}$$

Table 3. Important EKV model relationships for a pMOS transistor.

Qty	**EKV Model**	**Weak Inversion**	**Strong Inversion**
I_{F}	$I_{\mathrm{s}} \log^2\left(1+e^{(\kappa(V_{\mathrm{BG}}-V_{\mathrm{T0}})-V_{\mathrm{BS}})/2U_{\mathrm{T}}}\right)$	$I_{\mathrm{s}} e^{(\kappa(V_{\mathrm{BG}}-V_{\mathrm{T0}})-V_{\mathrm{BS}})/U_{\mathrm{T}}}$	$I_{\mathrm{s}}\left(\frac{\kappa\left(V_{\mathrm{BG}}-V_{\mathrm{T0}}\right)-V_{\mathrm{BS}}}{2U_{\mathrm{T}}}\right)^2$
I_{R}	$I_{\mathrm{s}} \log^2\left(1+e^{(\kappa(V_{\mathrm{BG}}-V_{\mathrm{T0}})-V_{\mathrm{BD}})/2U_{\mathrm{T}}}\right)$	$I_{\mathrm{s}} e^{(\kappa(V_{\mathrm{BG}}-V_{\mathrm{T0}})-V_{\mathrm{BD}})/U_{\mathrm{T}}}$	$I_{\mathrm{s}}\left(\frac{\kappa\left(V_{\mathrm{BG}}-V_{\mathrm{T0}}\right)-V_{\mathrm{BD}}}{2U_{\mathrm{T}}}\right)^2$
V_{SD}	$2U_{\mathrm{T}} \log\left(e^{\sqrt{I_{\mathrm{F}}/I_{\mathrm{s}}}}-1\right)$ $-2U_{\mathrm{T}} \log\left(e^{\sqrt{I_{\mathrm{R}}/I_{\mathrm{s}}}}-1\right)$	$U_{\mathrm{T}} \log \frac{I_{\mathrm{F}}}{I_{\mathrm{R}}}$	$2U_{\mathrm{T}}\left(\sqrt{\frac{I_{\mathrm{F}}}{I_{\mathrm{s}}}}-\sqrt{\frac{I_{\mathrm{R}}}{I_{\mathrm{s}}}}\right)$
I	$I_{\mathrm{F}}-I_{\mathrm{R}}$	$I_{\mathrm{F}}-I_{\mathrm{R}}$	$I_{\mathrm{F}}-I_{\mathrm{R}}$
I_{sat}	I_{F}	I_{F}	I_{F}
V_{SDsat}	$2U_{\mathrm{T}} \log\left(e^{\sqrt{I_{\mathrm{sat}}/I_{\mathrm{s}}}}-1\right)$ $-2U_{\mathrm{T}} \log\left(e^{\sqrt{I_{\mathrm{sat}}/AI_{\mathrm{s}}}}-1\right)$	$U_{\mathrm{T}} \log A$	$2U_{\mathrm{T}}\sqrt{\frac{I_{\mathrm{sat}}}{I_{\mathrm{s}}}}\left(1-\frac{1}{\sqrt{A}}\right)$
g_{s}	$\frac{\sqrt{I_{\mathrm{s}} I_{\mathrm{sat}}}}{U_{\mathrm{T}}}\left(1-e^{-\sqrt{I_{\mathrm{sat}}/I_{\mathrm{s}}}}\right)$	$\frac{I_{\mathrm{sat}}}{U_{\mathrm{T}}}$	$\frac{\sqrt{I_{\mathrm{s}} I_{\mathrm{sat}}}}{U_{\mathrm{T}}}$
g_{m}	κg_{s}	κg_{s}	κg_{s}
g_{mb}	$(1-\kappa) g_{\mathrm{s}}$	$(1-\kappa) g_{\mathrm{s}}$	$(1-\kappa) g_{\mathrm{s}}$
$\frac{g_{\mathrm{m}}}{I_{\mathrm{sat}}}$	$\frac{\kappa}{U_{\mathrm{T}}}\sqrt{\frac{I_{\mathrm{s}}}{I_{\mathrm{sat}}}}\left(1-e^{-\sqrt{I_{\mathrm{sat}}/I_{\mathrm{s}}}}\right)$	$\frac{\kappa}{U_{\mathrm{T}}}$	$\frac{\kappa}{U_{\mathrm{T}}}\sqrt{\frac{I_{\mathrm{s}}}{I_{\mathrm{sat}}}}$

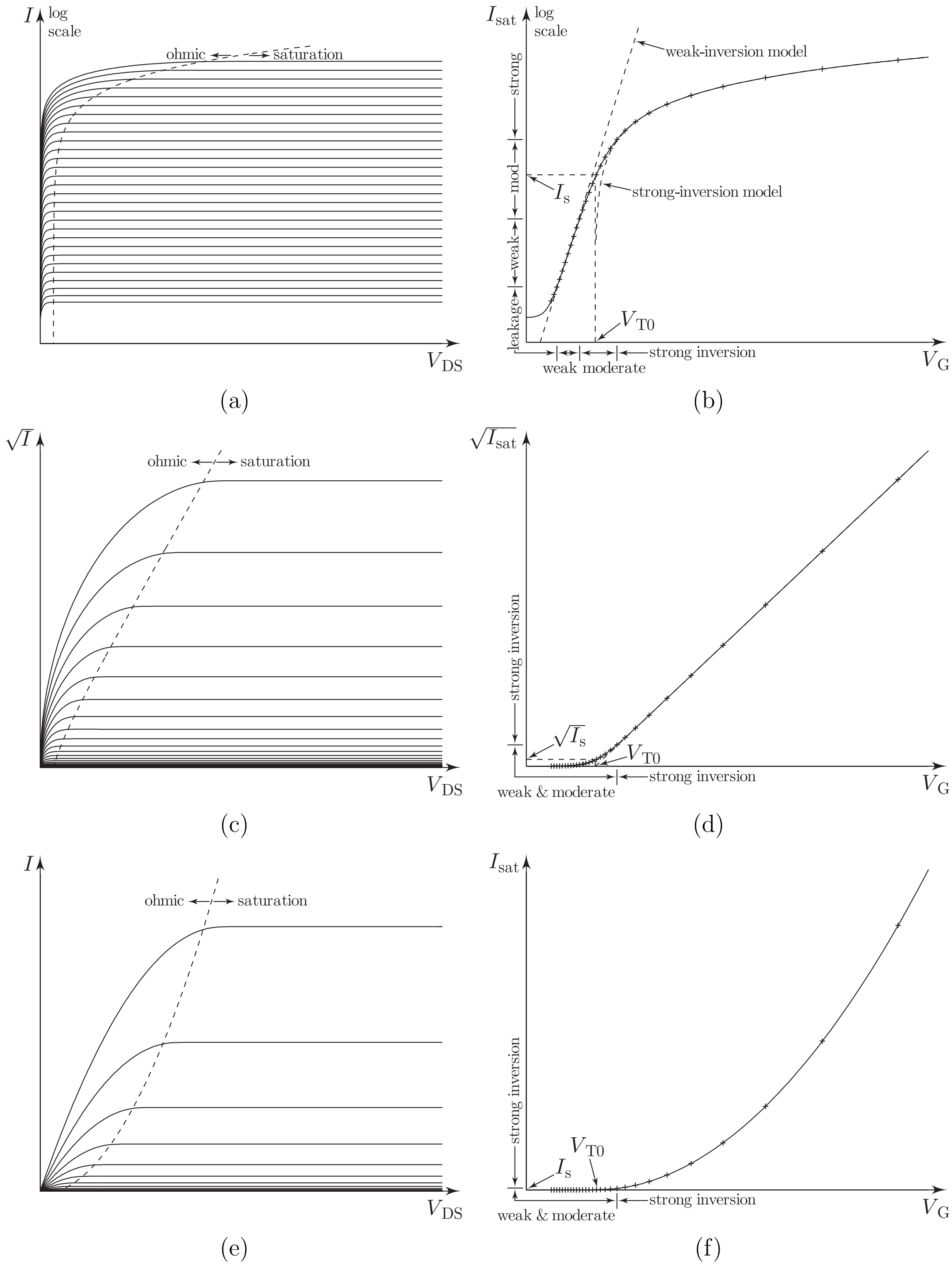


Figure 1. MOS-transistor current–voltage characteristics plotted on various scales highlighting different regions of operation.