ENGR 2420: Introduction to Microelectronic Circuits

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3 Characteristics of the Bipolar Transistor

The bipolar transistor is a functionally symmetric device—it will work in exactly the same manner if it is hooked up backwards instead of forwards. For most bipolar transistors, this symmetry is only qualitative, because most bipolar transistors are not physically symmetric devices (e.g., typically, the doping density of the emitter is much larger than that of the collector and the cross-sectional area of the emitter-base junction is smaller than that of the collector-base junction).

3.1 The Ebers-Moll Bipolar Transistor Model

The Ebers-Moll model [1] of the bipolar transistor provides a simple, closed-form expression for the collector, base, and emitter currents of a bipolar transistor in terms of its terminal voltages. The model is valid in all regions of operation of the bipolar transistor, transitioning between them smoothly. The Ebers-Moll bipolar transistor model expresses each of the terminal currents in terms of a forward component, I_{F}, which only depends on the base-to-emitter voltage and a reverse component, I_{R}, which only depends on the base-to-collector voltage. The terminal currents are expressed as

$$I_{\mathrm{C}} = I_{\mathrm{F}} - \frac{I_{\mathrm{R}}}{\alpha_{\mathrm{R}}}, \quad I_{\mathrm{E}} = \frac{I_{\mathrm{F}}}{\alpha_{\mathrm{F}}} - I_{\mathrm{R}}, \quad \text{and} \quad I_{\mathrm{B}} = \frac{I_{\mathrm{F}}}{\beta_{\mathrm{F}}} + \frac{I_{\mathrm{R}}}{\beta_{\mathrm{R}}}, \tag{1}$$

where α_{F} and α_{R} are, respectively, the forward-active and reverse-active common-base current gains of the bipolar transistor, and β_{F} and β_{R} are, respectively, the forward-active and reverse-active common-emitter current gains of the bipolar transistor. These current gains are related to one another by

$$\alpha_{\mathrm{F,R}} = \frac{\beta_{\mathrm{F,R}}}{\beta_{\mathrm{F,R}} + 1} \quad \text{and} \quad \beta_{\mathrm{F,R}} = \frac{\alpha_{\mathrm{F,R}}}{1 - \alpha_{\mathrm{F,R}}}.$$

By substituting these relationships into Eq. 1, we can show that the Ebers-Moll model satisfies Kirchhoff's current law, which implies that

$$I_{\mathrm{E}} = I_{\mathrm{C}} + I_{\mathrm{B}}. \tag{2}$$

If the emitter-base junction is forward biased and the collector-base junction is reverse biased, then $I_{\mathrm{F}} \gg I_{\mathrm{R}}$ and the bipolar transistor is said to be in the *forward-active mode* of operation. In this case, the terminal currents are given by

$$I_{\mathrm{C}} = I_{\mathrm{F}}, \quad I_{\mathrm{E}} = \frac{I_{\mathrm{F}}}{\alpha_{\mathrm{F}}}, \quad \text{and} \quad I_{\mathrm{B}} = \frac{I_{\mathrm{F}}}{\beta_{\mathrm{F}}}. \tag{3}$$

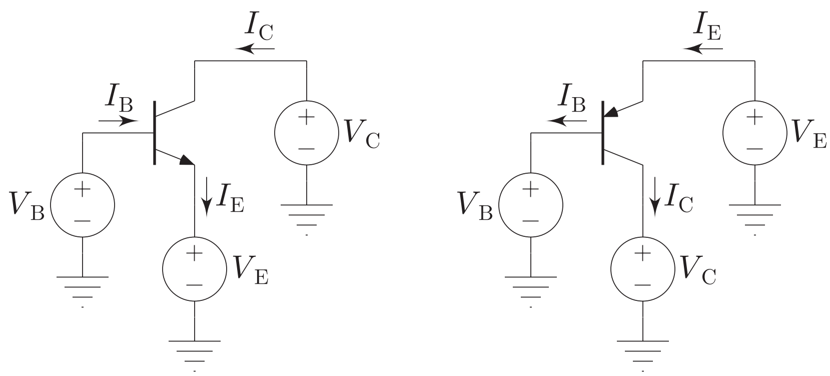

Figure 1: Biased bipolar transistors showing the direction of conventional current flow. (a) To bias an *npn* bipolar transistor into the forward-active region, we should ensure that $V_{BE} > 4U_T$ and $V_C \geq V_B$. In this case, I_C, I_E, and I_B are positive, flowing in the directions indicated. (b) To bias a *pnp* bipolar transistor into the forward-active region, we should ensure that $V_{EB} > 4U_T$ and $V_C \leq V_B$. In this case, I_C, I_E, and I_B are positive, flowing in the directions indicated.

By using Eqs. 2 and 3, we can express the terminal currents in terms of one another as

$$I_C = \beta_F I_B = \alpha_F I_E \quad \text{and} \quad I_E = (\beta_F + 1) I_B = \frac{I_C}{\alpha_F}.$$

If the collector-base junction is forward biased and the emitter-base junction is reverse biased, then $I_R \gg I_F$ and the bipolar transistor is said to be in the *reverse-active mode* of operation, in which the collector and emitters effectively reverse their roles. In this case, the terminal currents are given by

$$I_C = -\frac{I_R}{\alpha_R}, \quad I_E = -I_R, \quad \text{and} \quad I_B = \frac{I_R}{\beta_R}. \tag{4}$$

By using Eqs. 2 and 4, we can express the terminal currents in terms of one another as

$$I_E = -\beta_R I_B = \alpha_R I_C \quad \text{and} \quad I_C = -(\beta_R + 1) I_B = \frac{I_E}{\alpha_R}.$$

If both junctions are forward biased simultaneously, then the bipolar transistor is said to be in *saturation*, which is somewhat unfortunate, as we shall see later in the course, because this operating regime is analogous to the ohmic region of the MOS transistor. The MOS transistor has an operating regime called saturation that is analogous to the active region of the bipolar transistor.

For an *npn* bipolar transistor, biased as shown in Fig. 1a, the forward current component, I_F, is given by

$$I_F = I_s \left(e^{V_{BE}/U_T} - 1\right) = I_s \left(e^{(V_B - V_E)/U_T} - 1\right), \tag{5}$$

where I_s is the saturation current of the bipolar transistor and U_T is the thermal voltage, as before, V_B is the base voltage, and V_E is the emitter voltage. Likewise, the reverse current component, I_R, is given by

$$I_\mathrm{R} = I_\mathrm{s}\left(e^{V_\mathrm{BC}/U_\mathrm{T}} - 1\right) = I_\mathrm{s}\left(e^{(V_\mathrm{B}-V_\mathrm{C})/U_\mathrm{T}} - 1\right), \tag{6}$$

where V_C is the collector voltage and all of the other symbols are as they were defined previously. By substituting Eqs. 5 and 6 into Eq. 1, we can express the terminal currents in terms of the terminal voltages as

$$\left\{\begin{array}{lcl} I_\mathrm{C} & = & I_\mathrm{s}\left(e^{(V_\mathrm{B}-V_\mathrm{E})/U_\mathrm{T}} - 1\right) - \dfrac{I_\mathrm{s}}{\alpha_\mathrm{R}}\left(e^{(V_\mathrm{B}-V_\mathrm{C})/U_\mathrm{T}} - 1\right) \\ I_\mathrm{E} & = & \dfrac{I_\mathrm{s}}{\alpha_\mathrm{R}}\left(e^{(V_\mathrm{B}-V_\mathrm{E})/U_\mathrm{T}} - 1\right) - I_\mathrm{s}\left(e^{(V_\mathrm{B}-V_\mathrm{C})/U_\mathrm{T}} - 1\right) \\ I_\mathrm{B} & = & \dfrac{I_\mathrm{s}}{\beta_\mathrm{F}}\left(e^{(V_\mathrm{B}-V_\mathrm{E})/U_\mathrm{T}} - 1\right) + \dfrac{I_\mathrm{s}}{\beta_\mathrm{R}}\left(e^{(V_\mathrm{B}-V_\mathrm{C})/U_\mathrm{T}} - 1\right). \end{array}\right. \tag{7}$$

If $V_\mathrm{BE} > 4U_\mathrm{T}$ and $V_\mathrm{C} > V_\mathrm{B}$, then the bipolar transistor is biased into the forward-active region and Eq. 7 becomes

$$I_\mathrm{C} \approx I_\mathrm{s}e^{(V_\mathrm{B}-V_\mathrm{E})/U_\mathrm{T}}, \quad I_\mathrm{E} \approx \frac{I_\mathrm{s}}{\alpha_\mathrm{F}}e^{(V_\mathrm{B}-V_\mathrm{E})/U_\mathrm{T}}, \quad \text{and} \quad I_\mathrm{B} \approx \frac{I_\mathrm{s}}{\beta_\mathrm{F}}e^{(V_\mathrm{B}-V_\mathrm{E})/U_\mathrm{T}}. \tag{8}$$

If $V_\mathrm{BC} > 4U_\mathrm{T}$ and $V_\mathrm{E} > V_\mathrm{B}$, then the bipolar transistor is biased into the reverse-active region and Eq. 7 becomes

$$I_\mathrm{C} \approx -\frac{I_\mathrm{s}}{\alpha_\mathrm{R}}e^{(V_\mathrm{B}-V_\mathrm{C})/U_\mathrm{T}}, \quad I_\mathrm{E} \approx -I_\mathrm{s}e^{(V_\mathrm{B}-V_\mathrm{C})/U_\mathrm{T}}, \quad \text{and} \quad I_\mathrm{B} \approx \frac{I_\mathrm{s}}{\beta_\mathrm{R}}e^{(V_\mathrm{B}-V_\mathrm{C})/U_\mathrm{T}}. \tag{9}$$

If $V_\mathrm{BE} > 4U_\mathrm{T}$ and $V_\mathrm{BC} > 4U_\mathrm{T}$, then the bipolar transistor is biased into the saturation region (or is saturated) and Eq. 7 becomes

$$\left\{\begin{array}{lclcl} I_\mathrm{C} & \approx & I_\mathrm{s}e^{(V_\mathrm{B}-V_\mathrm{E})/U_\mathrm{T}} - \dfrac{I_\mathrm{s}}{\alpha_\mathrm{R}}e^{(V_\mathrm{B}-V_\mathrm{C})/U_\mathrm{T}} & = & I_\mathrm{s}e^{(V_\mathrm{B}-V_\mathrm{E})/U_\mathrm{T}}\left(1 - \dfrac{1}{\alpha_\mathrm{R}}e^{-V_\mathrm{CE}/U_\mathrm{T}}\right) \\ I_\mathrm{E} & \approx & \dfrac{I_\mathrm{s}}{\alpha_\mathrm{F}}e^{(V_\mathrm{B}-V_\mathrm{E})/U_\mathrm{T}} - I_\mathrm{s}e^{(V_\mathrm{B}-V_\mathrm{C})/U_\mathrm{T}} & = & \dfrac{I_\mathrm{s}}{\alpha_\mathrm{F}}e^{(V_\mathrm{B}-V_\mathrm{E})/U_\mathrm{T}}\left(1 - \alpha_\mathrm{F}e^{-V_\mathrm{CE}/U_\mathrm{T}}\right) \\ I_\mathrm{B} & \approx & \dfrac{I_\mathrm{s}}{\beta_\mathrm{F}}e^{(V_\mathrm{B}-V_\mathrm{E})/U_\mathrm{T}} + \dfrac{I_\mathrm{s}}{\beta_\mathrm{R}}e^{(V_\mathrm{B}-V_\mathrm{C})/U_\mathrm{T}} & = & \dfrac{I_\mathrm{s}}{\beta_\mathrm{F}}e^{(V_\mathrm{B}-V_\mathrm{E})/U_\mathrm{T}}\left(1 + \dfrac{\beta_\mathrm{F}}{\beta_\mathrm{R}}e^{-V_\mathrm{CE}/U_\mathrm{T}}\right). \end{array}\right. \tag{10}$$

If in addition V_CE is more than a few U_T, then each of the second terms in parentheses in Eq. 10 becomes negligible compared to one and Eq. 10 further reduces Eq. 8. In this region, which is sometimes called *soft saturation*, the bipolar transistor's behavior is nearly identical to that which it exhibits in the forward-active mode.

If $V_\mathrm{BE} < -4U_\mathrm{T}$ and $V_\mathrm{BC} < -4U_\mathrm{T}$, the bipolar transistor is in the cutoff region and Eq. 7 becomes

$$I_\mathrm{C} \approx \frac{I_\mathrm{s}}{\beta_\mathrm{R}}, \quad I_\mathrm{E} \approx \frac{I_\mathrm{s}}{\beta_\mathrm{F}}, \quad \text{and} \quad I_\mathrm{B} \approx -\frac{I_\mathrm{s}}{\beta_\mathrm{F}} - \frac{I_\mathrm{s}}{\beta_\mathrm{R}}. \tag{11}$$

For a *pnp* bipolar transistor, biased as shown in Fig. 1b, the forward current component, I_{F}, is given by

$$I_{\mathrm{F}} = I_{\mathrm{s}}\left(e^{V_{\mathrm{EB}}/U_{\mathrm{T}}} - 1\right) = I_{\mathrm{s}}\left(e^{(V_{\mathrm{E}}-V_{\mathrm{B}})/U_{\mathrm{T}}} - 1\right),$$

where all of the symbols have their previously defined meanings. Likewise, the reverse current component, I_{R}, is given by

$$I_{\mathrm{R}} = I_{\mathrm{s}}\left(e^{V_{\mathrm{CB}}/U_{\mathrm{T}}} - 1\right) = I_{\mathrm{s}}\left(e^{(V_{\mathrm{C}}-V_{\mathrm{B}})/U_{\mathrm{T}}} - 1\right),$$

where all symbols are as defined previously.

3.2 Incremental Bipolar Transistor Characteristics

In this section, we shall develop expressions for and relationships among several small-signal parameters for the bipolar transistor biased in the forward-active region by differentiating the Ebers-Moll model with respect to various terminal voltages. We shall find that restricting our attention to the forward-active region is not really a limitation, because the vast majority of all bipolar integrated circuits make use of these devices in this mode of operation. Moreover, because the bipolar transistor is functionally symmetric with respect to the interchange of its emitter and collector terminals, by using source splitting and superposition, we shall be able to use the incremental relationships that we shall derive in this section for the forward-active bipolar transistor and analogous ones for the reverse-active bipolar transistor to analyze circuits containing bipolar transistors biased into the saturation region.

First, we shall compute the incremental resistance seen looking into the base terminal of an *npn* bipolar transistor with the emitter voltage held fixed. Recalling that the base current in the forward-active region is given approximately by

$$I_{\mathrm{B}} \approx \frac{I_{\mathrm{s}}}{\beta_{\mathrm{F}}} \cdot e^{(V_{\mathrm{B}}-V_{\mathrm{E}})/U_{\mathrm{T}}},$$

we can obtain this incremental base resistance by differentiating I_{B} with respect to V_{B}, which yields

$$\begin{aligned} r_{\mathrm{b}} &= \frac{\partial V_{\mathrm{B}}}{\partial I_{\mathrm{B}}} \\ &= \left(\frac{\partial I_{\mathrm{B}}}{\partial V_{\mathrm{B}}}\right)^{-1} \\ &= \left(\underbrace{\frac{I_{\mathrm{s}}}{\beta_{\mathrm{F}}} \cdot e^{(V_{\mathrm{B}}-V_{\mathrm{E}})/U_{\mathrm{T}}}}_{I_{\mathrm{B}}} \cdot \frac{1}{U_{\mathrm{T}}}\right)^{-1} \\ &= \frac{U_{\mathrm{T}}}{I_{\mathrm{B}}}. \end{aligned}$$

Next, we shall compute the incremental resistance seen looking into the emitter terminal with the base voltage held constant. Recalling that incremental terminal resistances are defined on the current flowing into the terminal and that the emitter current in the forward-active region is given approximately by

$$I_{\mathrm{E}} \approx \frac{I_{\mathrm{s}}}{\alpha_{\mathrm{F}}} \cdot e^{(V_{\mathrm{B}}-V_{\mathrm{E}})/U_{\mathrm{T}}},$$

we can obtain this incremental emitter resistance by differentiating $-I_{\mathrm{E}}$ with respect to V_{E}, which yields

$$\begin{aligned} r_{\mathrm{e}} &= \frac{\partial V_{\mathrm{E}}}{\partial\left(-I_{\mathrm{E}}\right)} \\ &= -\left(\frac{\partial I_{\mathrm{E}}}{\partial V_{\mathrm{E}}}\right)^{-1} \\ &= -\left(\underbrace{\frac{I_{\mathrm{s}}}{\alpha_{\mathrm{F}}} \cdot e^{(V_{\mathrm{B}}-V_{\mathrm{E}})/U_{\mathrm{T}}}}_{I_{\mathrm{E}}} \cdot \left(-\frac{1}{U_{\mathrm{T}}}\right)\right)^{-1} \\ &= \frac{U_{\mathrm{T}}}{I_{\mathrm{E}}}. \end{aligned}$$

Next, we shall define the incremental transconductance gain of the *npn* bipolar transistor as the partial derivative of the collector current with respect to the base voltage. Recalling that the collector current in the forward-active mode of operation is given approximately by

$$I_{\mathrm{C}} \approx I_{\mathrm{s}} e^{(V_{\mathrm{B}}-V_{\mathrm{E}})/U_{\mathrm{T}}},$$

we can obtain this incremental transconductance gain from

$$\begin{aligned} g_{\mathrm{m}} &= \frac{\partial I_{\mathrm{C}}}{\partial V_{\mathrm{B}}} \\ &= \underbrace{I_{\mathrm{s}} e^{(V_{\mathrm{B}}-V_{\mathrm{E}})/U_{\mathrm{T}}}}_{I_{\mathrm{C}}} \cdot \frac{1}{U_{\mathrm{T}}} \\ &= \frac{I_{\mathrm{C}}}{U_{\mathrm{T}}}. \end{aligned}$$

Finally, we observe that, because we can express the terminal currents in the forward-active mode in terms of one another according to

$$I_{\mathrm{C}} = \beta_{\mathrm{F}} I_{\mathrm{B}} = \alpha_{\mathrm{F}} I_{\mathrm{E}}, \quad I_{\mathrm{E}} = \left(\beta_{\mathrm{F}}+1\right) I_{\mathrm{B}} = \frac{I_{\mathrm{C}}}{\alpha_{\mathrm{F}}}, \quad \text{and} \quad I_{\mathrm{B}} = \frac{I_{\mathrm{C}}}{\beta_{\mathrm{F}}} = \frac{I_{\mathrm{E}}}{\beta_{\mathrm{F}}+1},$$

we can also express these small-signal parameters in terms of one another as

$$r_{\mathrm{b}} = \frac{\beta_{\mathrm{F}}}{g_{\mathrm{m}}} = \left(\beta_{\mathrm{F}}+1\right) r_{\mathrm{e}}, \quad r_{\mathrm{e}} = \frac{\alpha_{\mathrm{F}}}{g_{\mathrm{m}}} = \frac{r_{\mathrm{b}}}{\beta_{\mathrm{F}}+1}, \quad \text{and} \quad g_{\mathrm{m}} = \frac{\beta_{\mathrm{F}}}{r_{\mathrm{b}}} = \frac{\alpha_{\mathrm{F}}}{r_{\mathrm{e}}}.$$

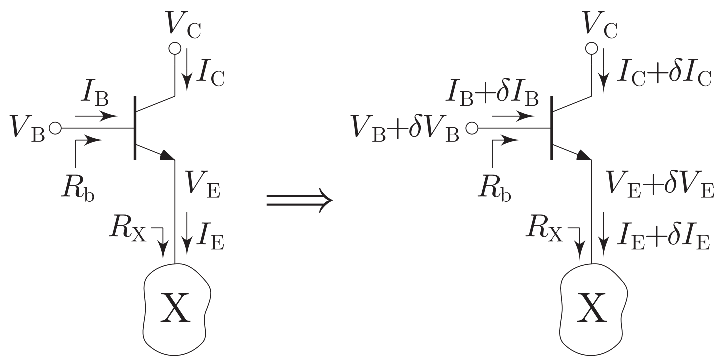

Figure 2: Incremental base resistance of an *npn* bipolar transistor with emitter degeneration.

3.3 Incremental Base Resistance and Transconductance Gain of an Emitter-Degenerated Bipolar Transistor

In this section, we shall determine what the incremental resistance of the base terminal is when a finite incremental resistance, R_X, is connected to the emitter terminal, as shown in Fig. 2. Under these circumstances, if we were to increase the base voltage by a small amount, we would increase the base current, the collector current, and the emitter current. However, the increased emitter current must also flow into the circuitry represented by X, which because of its finite incremental resistance, R_X, the emitter voltage must rise. This increase in the emitter voltage, in turn, reduces the increase in the base, emitter, and collector currents. If less current flows into the base for a given change in the base voltage, we should expect the incremental base resistance under these circumstances should be larger than r_b. From the incremental base resistance, we shall also determine what effect the emitter degeneration has on the incremental transconductance gain.

For a sufficiently small change in the emitter voltage and current, we have that

$$R_X = \frac{\delta V_E}{\delta I_E},$$

which implies that

$$\delta V_E = \delta I_E \cdot R_X.$$

Because $I_E = (\beta_F + 1)\, I_B$, we can express the change in V_E in terms of the change in I_B as

$$\delta V_E = \delta I_B\,(\beta_F + 1)\, R_X. \tag{12}$$

Before the small change in the base voltage, we have that the base current is given by

$$I_B = \frac{I_s}{\beta_F} \cdot e^{(V_B - V_E)/U_T}.$$

After the small change in the base voltage and the resulting change in the emitter voltage, we have that

$$I_B + \delta I_B \;\; = \;\; \frac{I_s}{\beta_F} \cdot e^{((V_B+\delta V_B)-(V_E+\delta V_E))/U_T}$$

$$
\begin{aligned}
&= \underbrace{\frac{I_s}{\beta_F} \cdot e^{(V_B - V_E)/U_T}}_{I_B} e^{(\delta V_B - \delta V_E)/U_T} \\
&= I_B \left(1 + \frac{\delta V_B - \delta V_E}{U_T} + \frac{1}{2}\left(\frac{\delta V_B - \delta V_E}{U_T}\right)^2 + \cdots\right) \\
&\approx I_B \left(1 + \frac{\delta V_B - \delta V_E}{U_T}\right) \\
&= I_B + \frac{\delta V_B - \delta V_E}{r_b},
\end{aligned}
$$

which we can rearrange to find that

$$\delta V_B = \delta I_B \cdot r_b + \delta V_E.$$

By substituting Eq. 12 into this equation, we can eliminate δV_E, thereby expressing the change in the base voltage as

$$\delta V_B = \delta I_B \cdot r_b + \delta I_B \left(\beta_F + 1\right) R_X,$$

which implies that

$$R_b = \frac{\delta V_B}{\delta I_B} = r_b + \left(\beta_F + 1\right) R_X.$$

Thus, the incremental base resistance in the presence of emitter degeneration is the undegenerated incremental base resistance in series with the degenerating incremental resistance magnified by $\beta_F + 1$.

From this result, we can also obtain the incremental transconductance gain of the bipolar transistor with emitter degeneration. Given a change in I_B, we can obtain the change in I_C as

$$
\begin{aligned}
\delta I_C &= \beta_F \delta I_B \\
&= \beta_F \cdot \frac{\delta V_B}{R_b} \\
&= \frac{\beta_F \delta V_B}{r_b + \left(\beta_F + 1\right) R_X} \\
&= \frac{\beta_F / r_b}{1 + \dfrac{\beta_F + 1}{r_b} \cdot R_X} \cdot \delta V_B \\
&= \frac{g_m}{1 + R_X / r_e} \cdot \delta V_B,
\end{aligned}
$$

which we can rearrange to express the transconductance gain of the bipolar transistor with emitter degeneration as

$$G_m = \frac{\delta I_C}{\delta V_B} = \frac{g_m}{1 + R_X / r_e}.$$

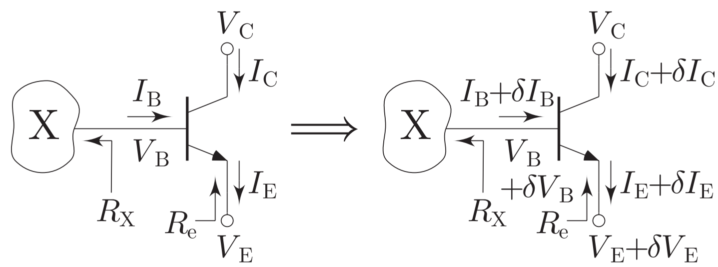

Figure 3: Incremental emitter resistance of an *npn* bipolar transistor with a finite incremental resistance in the base.

3.4 Incremental Emitter Resistance of an Incremental Base Resistance

In this section, we shall examine the effect on the incremental emitter resistance of a finite incremental resistance connected to the base of a bipolar transistor, as shown in Fig. 3. Given the reference directions for I_B shown in Fig. 3, for a sufficiently small change in the base voltage and current, we have that

$$R_X = \frac{\delta V_B}{\delta(-I_B)} = -\frac{\delta V_B}{\delta I_B},$$

which implies that

$$\delta V_B = -\delta I_B \cdot R_X.$$

Because $I_B = I_E/(\beta_F + 1)$, we can express the change in V_B in terms of the change in I_E as

$$\delta V_B = -\delta I_E \cdot \frac{R_X}{\beta_F + 1}. \quad (13)$$

Before the small change in the emitter voltage, we have that the emitter current is given by

$$I_E = \frac{I_s}{\alpha_F} \cdot e^{(V_B - V_E)/U_T}.$$

After the small change in the emitter voltage and the resulting change in the base voltage, we have that

$$\begin{aligned} I_E + \delta I_E &= \frac{I_s}{\alpha_F} \cdot e^{((V_B+\delta V_B)-(V_E+\delta V_E))/U_T} \\ &= \underbrace{\frac{I_s}{\alpha_F} \cdot e^{(V_B-V_E)/U_T}}_{I_E} e^{(\delta V_B - \delta V_E)/U_T} \\ &= I_E\left(1 + \frac{\delta V_B - \delta V_E}{U_T} + \frac{1}{2}\left(\frac{\delta V_B - \delta V_E}{U_T}\right)^2 + \cdots\right) \\ &\approx I_E\left(1 + \frac{\delta V_B - \delta V_E}{U_T}\right) \end{aligned}$$

$$= \quad I_{\mathrm{E}} + \frac{\delta V_{\mathrm{B}} - \delta V_{\mathrm{E}}}{r_{\mathrm{e}}},$$

which we can rearrange to find that

$$-\delta V_{\mathrm{E}} = \delta I_{\mathrm{E}} \cdot r_{\mathrm{e}} - \delta V_{\mathrm{B}}.$$

By substituting Eq. 13 into this equation, we can eliminate δV_{B}, thereby expressing the change in the emitter voltage as

$$-\delta V_{\mathrm{E}} = \delta I_{\mathrm{E}} \cdot r_{\mathrm{e}} + \delta I_{\mathrm{E}} \cdot \frac{R_{\mathrm{X}}}{\beta_{\mathrm{F}} + 1},$$

which implies that

$$R_{\mathrm{e}} = \frac{\delta V_{\mathrm{E}}}{\delta\left(-I_{\mathrm{E}}\right)} = -\frac{\delta V_{\mathrm{E}}}{\delta I_{\mathrm{E}}} = r_{\mathrm{e}} + \frac{R_{\mathrm{X}}}{\beta_{\mathrm{F}} + 1}.$$

Thus, the incremental emitter resistance in the presence of a finite incremental resistance connected to the base is given by r_{e} in series with the incremental resistance in the base reduced by a factor of $\beta_{\mathrm{F}} + 1$.

References

[1] J. J. Ebers and J. L. Moll, "Large-Signal Behavior of Junction Transistors," *Proceedings of the IRE*, vol. 42, no. ?, pp. 1761–1772, 1954.