ENGR 2420: Introduction to Microelectronic Circuits

February 4, 2020

1 Source Splitting

In this handout, we shall describe two source transformations, called *source splitting*, by which we can split a single voltage source or a single current source, respectively, into multiple voltage sources or current sources of the same value. These source transformations are not included in many elementary courses on circuit analysis. When they are described, the main motivation for applying them is almost invariably to create situations in which we can apply either Thévenin's theorem or Norton's theorem to a portion of some circuit [1–3]. In DPI circuit analysis, however, we shall primarily use these transformations to create situations in which we can apply superposition where none existed [4]. After describing the two types of source splitting, we shall illustrate each one in this context with worked examples.

The first of the two transformations, called *voltage-source splitting*, is illustrated in Fig. 1. Consider a single voltage source, V, connected to an arbitrary circuit, as shown in Fig. 1a. We can add another voltage source with value V in parallel with the first one, as shown in Fig. 1b. In many elementary circuits texts, such a parallel connection of voltage sources is termed a "forbidden" configuration, because if the voltage sources happened to have different values, Kirchhoff's voltage law (KVL) would be violated around the loop containing the two voltage sources. However, in this instance, there is no such violation of KVL, because we have made the values of the two sources equal to each other. Finally, we can divide the branches connected to the positive terminal of the voltage sources, connecting these other branches to the individual voltage sources, as shown in Fig. 1c, without altering any of the KVL equations for the circuit. All of the loops in the circuit that contained the voltage source, such as loop A in Fig. 1, still contain one such voltage source. Loops that go through the node that was split, such as loop B in Fig. 1, now contain a voltage drop of V followed by a voltage rise of V, which leaves the KVL equations around such loops unaltered. In effect, we

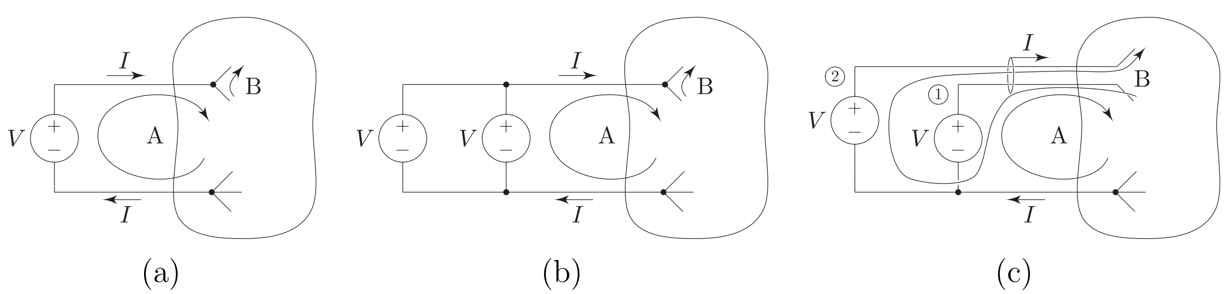

Figure 1: Voltage-source splitting. (a) A single voltage source, V, connected to an arbitrary circuit. (b) We can add another voltage source in parallel with the original one as long as it has the same value as the first one. (c) We can divide the branches connected to the positive terminals of the voltage sources, connecting them to the separate voltage sources, as shown, without affecting the behavior of the circuit.

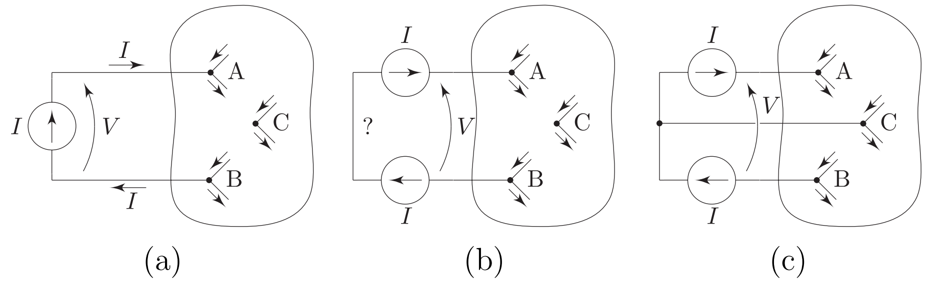

Figure 2: Current-source splitting. (a) A single current source, I, connected to an arbitrary circuit. (b) We can add another current source in series with the original one as long as it has the same value as the first one. (c) We can connect the node between the two current sources to any other node in the circuit, as shown, without altering the behavior of the circuit.

have added and subtracted V from each such equation. The Kirchhoff's current law (KCL) equation that held at the node connected to the common negative terminal of the voltage sources also remains unaltered. Thus, the behavior of a circuit remains unaltered under this transformation. Moreover, it is easy to see that we can add as many copies of the voltage source as there are branches connected to the positive terminal. This transformation can also be visualized as shifting voltage source V through one of the nodes to which it was connected; consequently, voltage-source splitting is also sometimes referred to as *voltage-source shifting*.

The second transformation, called *current-source splitting*, is illustrated in Fig. 2. Here, a single current source, I, connected to an arbitrary circuit, as shown in Fig. 2a. We can add another current source with value I in series with the first one, as shown in Fig. 2b. Like the parallel connection of voltage sources, such a series connection of current sources is termed a "forbidden" configuration in many elementary texts, because if the current sources happened to have different values, KCL would be violated at the node between the current sources. As before, there is no such violation of KCL in this situation, because we have made the values of the two sources equal to each other. The voltage on the intermediate node is indeterminate, because the current sources will continue to pass the same current regardless of the voltage across them. This node voltage can take on any value and the circuit will still function as it did before; a current of I will still be sourced to node A and a current of I will still be sunk from node B, as was the original case. We can, in fact, connect this intermediate node to any other node in the circuit, such as node C, as shown in Fig. 2c, without altering any of the KCL equations in the circuit. The KCL equations of the nodes to which the original current source was connected, such as nodes A and B, remain unaltered. The KCL equation of a node, such as node C, connected to a pair of current sources will have I added and subtracted from it, leaving it unaffected. Because the current sources inject no net current into such nodes, their voltages will remain unaltered. Thus, the behavior of a circuit remains unchanged by this transformation. Moreover, it is easy to see that we can add as many copies of the current source as we would like in series with the original, connecting the intermediate nodes to any other nodes in the circuit. We shall often find it convenient to connect these intermediate nodes to ground.

It is interesting to note that neither of these source-splitting transformations rely on

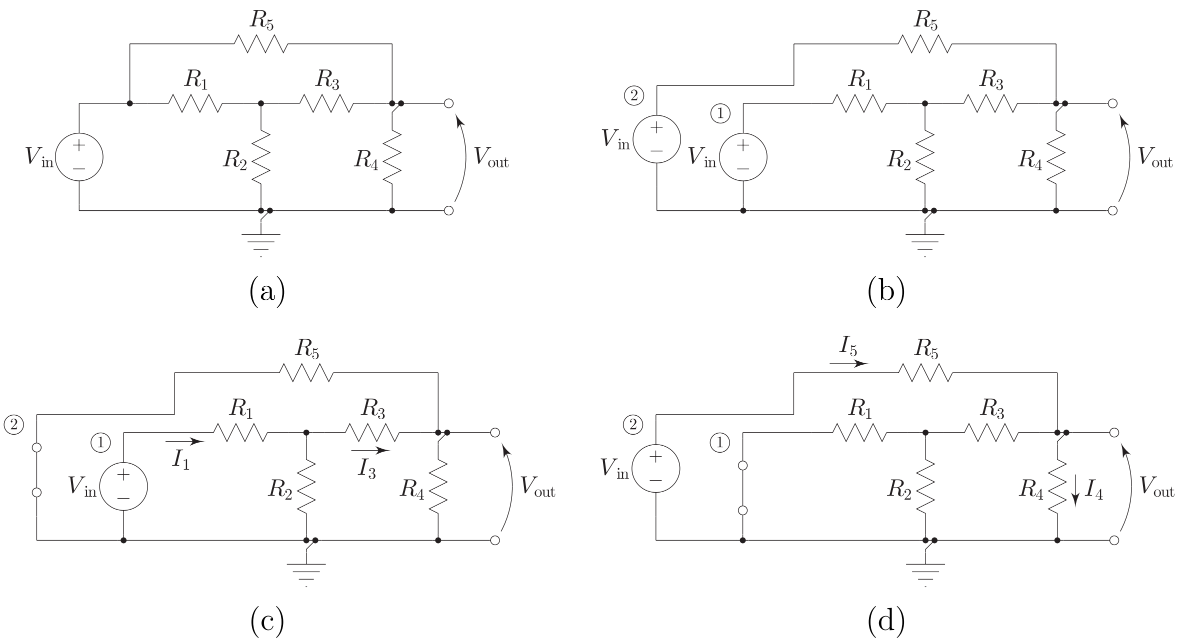

Figure 3: Example illustrating the use of voltage-source splitting and superposition to calculate the voltage gain of a bridged ladder network.

assumptions of linearity; rather, they are based on the notions of ideal voltage and current sources, KVL, and KCL. Thus, we can apply them both to linear circuits and to nonlinear circuits. We shall now illustrate the use of each of these source-splitting transformations in conjunction with superposition to analyze simple circuits. Superposition is a property of linear circuits, so such applications of source splitting apply only to linear circuits or to nonlinear circuits for excursions about a quiescent operating point that are sufficiently small that the circuit is incrementally linear.

Example 1.1

Consider the bridged ladder circuit shown in Fig. 3a. We would like to calculate the voltage gain of this circuit, $V_{\text{out}}/V_{\text{in}}$. If the resistor connecting V_{in} directly to V_{out}, R_5, were absent, this circuit would be a ladder network, and the required voltage gain could be written out by inspection using voltage dividers or current dividers and Ohm's law. Resistor R_5 provides feedback from the output back to the input, preventing us from applying such techniques directly. There are many ways of solving this problem. For example, we could apply a delta-wye transformation to the delta comprising resistors R_1, R_3, and R_5 to obtain an equivalent purely series-parallel network that we could solve by inspection using resistive dividers and Ohm's law. Alternatively, we could apply a wye-delta transformation to the wye comprising resistors R_1, R_2, and R_3 to obtain a different equivalent network that we could also solve by inspection. However, such transformations are difficult to remember, the resistor values in the equivalent circuit are complicated combinations of the resistor values in the original circuit. Moreover, the equivalent circuit bears little resemblance to the original circuit and provides little design-oriented insight into the functioning of the original circuit.

We could also apply node-voltage analysis or mesh-current analysis, solving for the output voltage by Cramer's rule or Gaussian elimination. Whereas there are three mesh-current equations, there are only two node-voltage equations, so nodal analysis seems easier in this case. Calling the intermediate node voltage V_{A}, we can write the nodal equations by inspection as

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_3} \\ -\frac{1}{R_3} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} \end{bmatrix} \begin{bmatrix} V_{\mathrm{A}} \\ V_{\mathrm{out}} \end{bmatrix} = \begin{bmatrix} \frac{V_{\mathrm{in}}}{R_1} \\ \frac{V_{\mathrm{in}}}{R_5} \end{bmatrix}.$$

We can write an equation for V_{out}, using Cramer's rule on the preceding equation, as the following ratio of two determinants

$$\begin{aligned} V_{\mathrm{out}} &= \frac{\begin{vmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{V_{\mathrm{in}}}{R_1} \\ -\frac{1}{R_3} & \frac{V_{\mathrm{in}}}{R_5} \end{vmatrix}}{\begin{vmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_3} \\ -\frac{1}{R_3} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} \end{vmatrix}} \\ &= V_{\mathrm{in}} \frac{\frac{1}{R_1R_3}+\frac{1}{R_1R_5}+\frac{1}{R_2R_5}+\frac{1}{R_3R_5}}{\frac{1}{R_1R_3}+\frac{1}{R_1R_4}+\frac{1}{R_1R_5}+\frac{1}{R_2R_3}+\frac{1}{R_2R_4}+\frac{1}{R_2R_5}+\frac{1}{R_3R_4}+\frac{1}{R_3R_5}}. \end{aligned}$$

Clearing out all of the nested fractions and dividing both sides of this equation by V_{in}, we obtain an expression for the required voltage gain as

$$\frac{V_{\mathrm{out}}}{V_{\mathrm{in}}} = \frac{R_2R_4R_5+R_2R_3R_4+R_1R_3R_4+R_1R_2R_4}{R_2R_4R_5+R_2R_3R_5+R_2R_3R_4+R_1R_4R_5+R_1R_3R_5+R_1R_3R_4+R_1R_2R_5+R_1R_2R_4}.$$

All of the component values in this formidable expression for the voltage gain of the circuit of Fig. 3a are multiplied together, almost completely obscuring their relative significance in determining the final answer. In many cases, design-oriented insight into a circuit is confounded by matrix algebra when we try to apply node-voltage analysis or mesh-current analysis, especially when the circuits become more complex than this five-resistor circuit.

We can also analyze the circuit of Fig. 3a by splitting V_{in} into two replica voltage sources, as shown in Fig. 3b. By doing so, we can calculate V_{out} by superposition as

$$V_{\mathrm{out}} = V_{\mathrm{out1}} + V_{\mathrm{out2}}, \tag{1}$$

where V_{out1} is the component of V_{out} due to the first copy of V_{in} and V_{out2} is the component of V_{out} due to the second copy of V_{in}. Each of the two terms in this equation can be written by

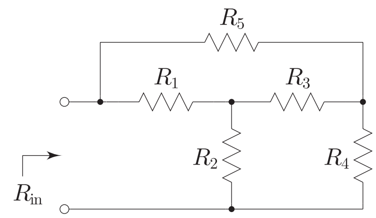

Figure 4: Bridged ladder network circuit.

inspection using resistive dividers and Ohm's law. With the copy of $V_{\rm in}$ on branch 2 shorted, as shown in Fig. 3c, we can write the output voltage as

$$V_{\rm out1} = (R_4 \| R_5)\, I_3 = (R_4 \| R_5) \cdot \underbrace{\frac{V_{\rm in}}{R_1 + (R_2 \| (R_3 + (R_4 \| R_5)))}}_{I_1} \cdot \underbrace{\frac{R_2}{R_2 + R_3 + (R_4 \| R_5)}}_{I_3/I_1}. \qquad (2)$$

Similarly, with the copy of $V_{\rm in}$ on branch 1 shorted, as shown in Fig. 3d, we can write the output voltage as

$$V_{\rm out2} = R_4 I_4 = R_4 \cdot \underbrace{\frac{V_{\rm in}}{R_5 + (R_4 \| (R_3 + (R_1 \| R_2)))}}_{I_5} \cdot \underbrace{\frac{R_3 + (R_1 \| R_2)}{R_4 + R_3 + (R_1 \| R_2)}}_{I_4/I_5}. \qquad (3)$$

By substituting Eqs. 2 and 3 into Eq. 1, we can write the voltage gain of the circuit of Fig. 3a as

$$\frac{V_{\rm out}}{V_{\rm in}} = \frac{R_4 \| R_5}{R_1 + (R_2 \| (R_3 + (R_4 \| R_5)))} \cdot \frac{R_2}{R_2 + R_3 + (R_4 \| R_5)} + \frac{R_4 \| (R_3 + (R_1 \| R_2))}{R_5 + (R_4 \| (R_3 + (R_1 \| R_2)))}.$$

While this expression for the gain of the circuit of Fig. 3a is by no means simple, the terms are combined in natural groupings of series and parallel combinations of resistors, allowing us to compare them easily. Moreover, this style of analysis requires us to think about how currents propagate through the circuit and divide at each point. Such a mental exercise yields direct insight into *how* the circuit actually works, not just merely the required answer. ∎

Example 1.2

Consider the bridged resistive ladder circuit shown in Fig. 4. We shall use source splitting and superposition to calculate the equivalent input resistance of the bridge, $R_{\rm in}$, as indicated in Fig. 4. To determine $R_{\rm in}$, we shall apply a test voltage source of value $V_{\rm in}$ to the circuit and determine the resulting current flowing into the circuit, $I_{\rm in}$, as shown in Fig. 5a. First, we split $V_{\rm in}$ into two replica sources, as shown. We are interested computing $I_{\rm in}$, the total current flowing through the loop indicated. There are four components of $I_{\rm in}$, two excited by the input source on branch 1 and two excited by the input source on branch 2. To facilitate the calculation of these four currents, we shall denote by $I_{{\rm in}ij}$ the component of $I_{\rm in}$ excited in the ith branch due to the replica source on the jth branch. We consider each input source

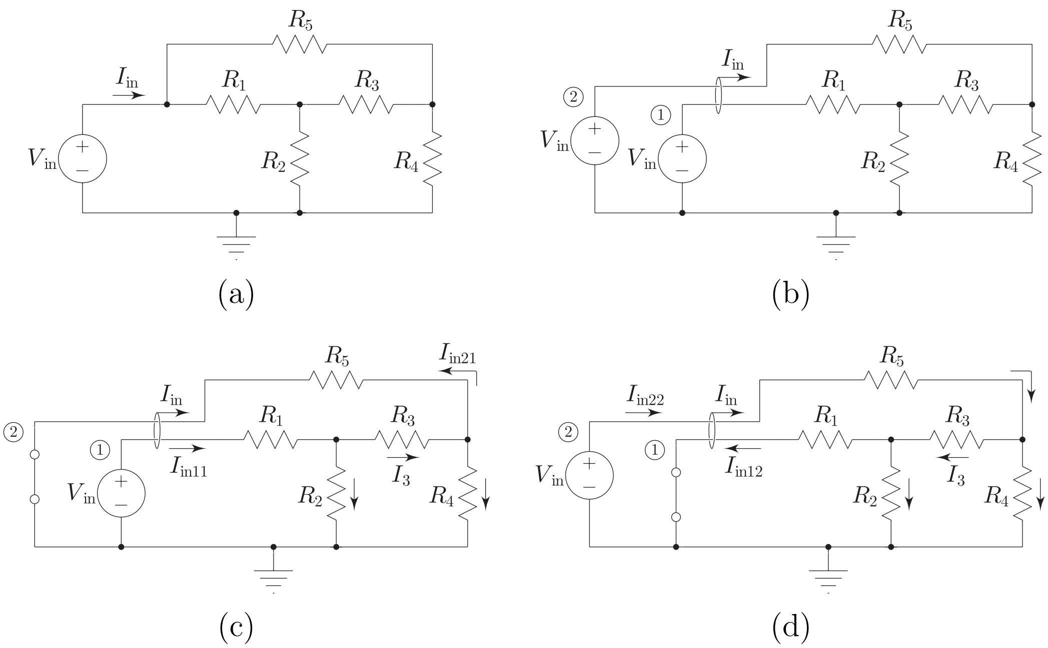

Figure 5: Calculation of the input resistance of the bridged ladder network using voltage-source splitting and superposition.

in turn, computing the components of I_{in} that it excites, and use superposition to put the results together. These two situations are depicted in Fig. 5b and Fig. 5c, along with the components of I_{in} relevant to each.

We write I_{in} as

$$\begin{aligned} I_{\text{in}} &= \underbrace{\frac{V_{\text{in}}}{R_1 + (R_2 \| (R_3 + (R_4 \| R_5)))}}_{I_{\text{in11}}} \left(1 - \underbrace{\frac{R_2}{R_2 + R_3 + (R_4 \| R_5)}}_{I_3/I_{\text{in11}}} \cdot \underbrace{\frac{R_4}{R_4 + R_5}}_{I_{\text{in21}}/I_3}\right) \\ &\quad + \underbrace{\frac{V_{\text{in}}}{R_5 + (R_4 \| (R_3 + (R_1 \| R_2)))}}_{I_{\text{in22}}} \left(1 - \underbrace{\frac{R_4}{R_4 + R_3 + (R_1 \| R_2)}}_{I_3/I_{\text{in22}}} \cdot \underbrace{\frac{R_2}{R_1 + R_2}}_{I_{\text{in12}}/I_3}\right). \end{aligned}$$

From this expression, we get an expression for $R_{\text{in}} = V_{\text{in}}/I_{\text{in}}$ as

$$R_{\text{in}} = \frac{V_{\text{in}}}{I_{\text{in}}} = \frac{R_1 + (R_2 \| (R_3 + (R_4 \| R_5)))}{1 - \dfrac{R_4}{R_4 + R_3 + (R_1 \| R_2)} \cdot \dfrac{R_4}{R_4 + R_5}} \left\| \frac{R_5 + (R_4 \| (R_3 + (R_1 \| R_2)))}{1 - \dfrac{R_2}{R_2 + R_3 + (R_4 \| R_5)} \cdot \dfrac{R_2}{R_1 + R_2}} \right. .$$

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Example 1.3

Consider the unbalanced resistive Wheatstone bridge circuit shown in Fig. 6. We shall use source splitting and superposition to calculate the equivalent input resistance of the bridge,

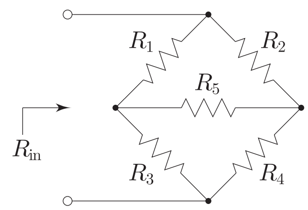

Figure 6: Unbalanced Wheatstone bridge circuit.

R_{in}, as indicated in Fig. 6. To determine R_{in}, we can use one of two methods. We can apply a test voltage source of value V_{in} and determine the resulting current flowing into the circuit, I_{in}. Alternatively, we can apply a test current source of value I_{in} and determine the voltage, V_{in}, that results across the port. In either case, we can apply source splitting and superposition to facilitate the calculation of R_{in}. For this example, we shall consider each method in turn.

First, we shall consider the voltage-source method. We apply a test voltage source, V_{in}, across the input port, as shown in Fig. 7a. For convenience, we choose the bottom node to be ground and we split V_{in} into two replica sources, as shown. We are interested computing I_{in}, the total current flowing through the loop indicated. There are four components of I_{in}, two excited by the input source on branch 1 and two excited by the input source on branch 2. To facilitate the calculation of these four currents, we shall consider each input source in turn, computing the components of I_{in} that it excites, and use superposition to put the results together. These two situations are depicted in Fig. 7b and Fig. 7c, along with the components of I_{in} relevant to each.

We write I_{in} as

$$\begin{aligned} I_{\text{in}} \quad &= \quad \underbrace{\frac{V_{\text{in}}}{R_1 + (R_3 \| (R_5 + (R_2 \| R_4)))}}_{I_{\text{in}11}} \left(1 - \underbrace{\frac{R_3}{R_3 + R_5 + (R_2 \| R_4)}}_{I_5/I_{\text{in}11}} \cdot \underbrace{\frac{R_4}{R_2 + R_4}}_{I_{\text{in}21}/I_5} \right) \\ &\qquad + \underbrace{\frac{V_{\text{in}}}{R_2 + (R_4 \| (R_5 + (R_1 \| R_3)))}}_{I_{\text{in}22}} \left(1 - \underbrace{\frac{R_4}{R_4 + R_5 + (R_1 \| R_3)}}_{I_5/I_{\text{in}22}} \cdot \underbrace{\frac{R_3}{R_1 + R_3}}_{I_{\text{in}12}/I_5} \right). \end{aligned}$$

Now, consider the first quantity in parenthesis in the above expression for I_{in}. We can express it as

$$\begin{aligned} 1 - \frac{R_3}{R_3 + R_5 + (R_2 \| R_4)} \cdot \frac{R_4}{R_2 + R_4} \quad &= \quad \frac{R_3 + R_5 + (R_2 \| R_4) - R_3 \cdot \dfrac{R_4}{R_2 + R_4}}{R_3 + R_5 + (R_2 \| R_4)} \\ &= \quad \frac{R_3 \left(1 - \dfrac{R_4}{R_2 + R_4} \right) + R_5 + (R_2 \| R_4)}{R_3 + R_5 + (R_2 \| R_4)} \end{aligned}$$

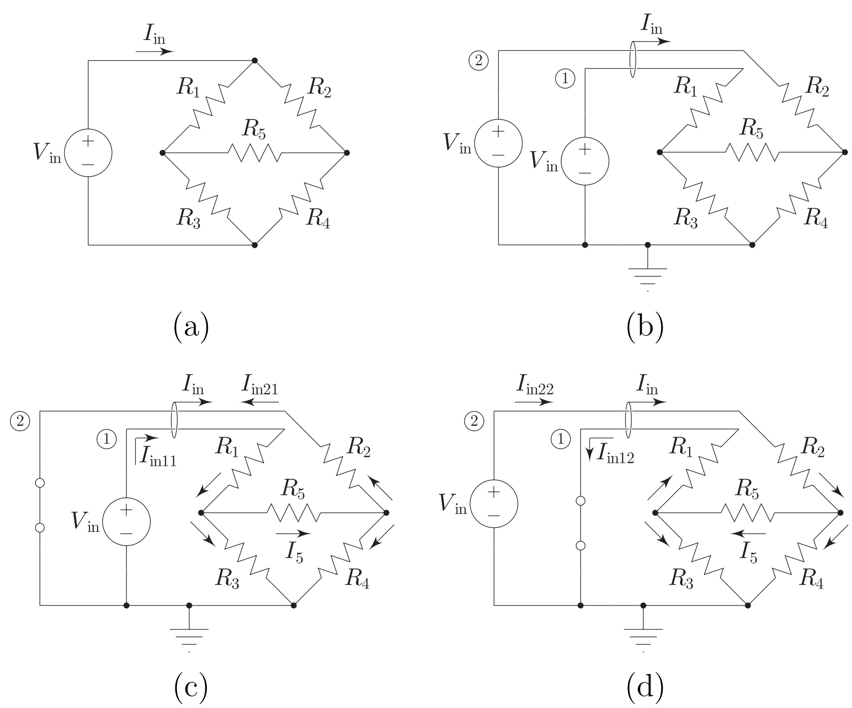

Figure 7: Calculation of the input resistance of the unbalanced Wheatstone Bridge using voltage-source splitting and superposition.

$$\begin{aligned}
&= \frac{R_3 \cdot \dfrac{R_2}{R_2 + R_4} + R_5 + (R_2 \| R_4)}{R_3 + R_5 + (R_2 \| R_4)} \\
&= \frac{R_3 \,(R_2 \| R_4)\,/R_4 + R_5 + (R_2 \| R_4)}{R_3 + R_5 + (R_2 \| R_4)} \\
&= \frac{(1 + R_3/R_4)\,(R_2 \| R_4) + R_5}{R_3 + R_5 + (R_2 \| R_4)}.
\end{aligned}$$

By a similar sequence of steps, we can transform the second expression in parenthesis as

$$1 - \frac{R_4}{R_4 + R_5 + (R_1 \| R_3)} \cdot \frac{R_3}{R_1 + R_3} = \frac{(1 + R_4/R_3)\,(R_1 \| R_3) + R_5}{R_4 + R_5 + (R_1 \| R_3)}.$$

Thus, we can express $R_{\text{in}} = V_{\text{in}}/I_{\text{in}}$ as

$$\begin{aligned}
R_{\text{in}} \;=\;& \frac{(R_1 + (R_3 \| \,(R_5 + (R_2 \| R_4))))\,(R_3 + R_5 + (R_2 \| R_4))}{(1 + R_3/R_4)\,(R_2 \| R_4) + R_5} \\
& \Bigg\| \frac{(R_2 + (R_4 \| \,(R_5 + (R_1 \| R_3))))\,(R_4 + R_5 + (R_1 \| R_3))}{(1 + R_4/R_3)\,(R_1 \| R_3) + R_5}.
\end{aligned}$$

Next, we shall consider the current-source method. We apply a test current source, I_{in}, across the input port, as shown in Fig. 8a. For convenience, we choose the left-most node of

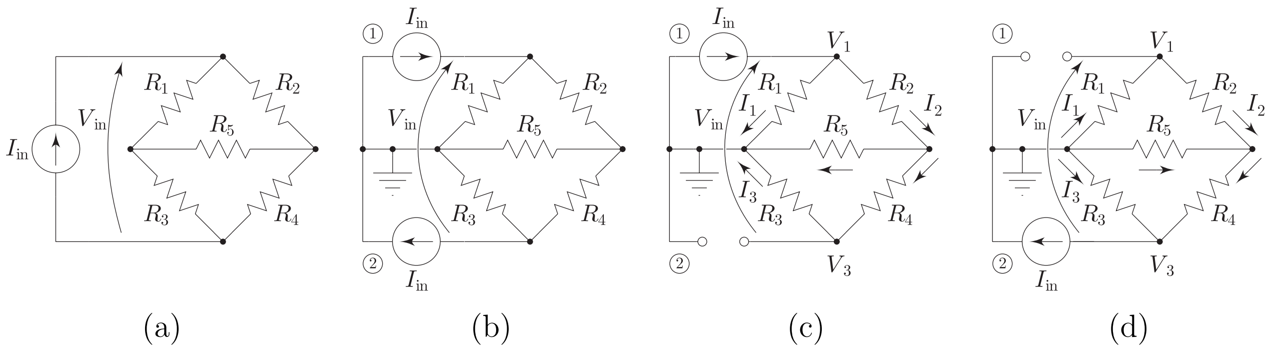

Figure 8: Calculation of the input resistance of the unbalanced Wheatstone Bridge using current-source splitting and superposition.

the bridge to be ground and we split I_{in} into two replica sources, as shown. We are interested computing V_{in}, the total voltage that develops across the input port. The input voltage is given by the difference between node voltages V_1 and V_3. To facilitate the calculation of these voltages, we consider each input source in turn, computing the value of V_{in} that results, and use superposition to put them together. These two situations are depicted in Fig. 8b and Fig. 8c.

For the first source, we can compute V_1 as R_1 times I_1, which is given by

$$V_1 = R_1 \cdot \underbrace{\frac{R_2 + (R_5 \| (R_4 + R_3))}{R_1 + R_2 + (R_5 \| (R_4 + R_3))}}_{I_1/I_{\text{in}}} \cdot I_{\text{in}}.$$

Likewise, we can compute V_3 as R_3 times I_3, which is given by

$$V_3 = R_3 \cdot \underbrace{\frac{R_1}{R_1 + R_2 + (R_5 \| (R_4 + R_3))}}_{I_2/I_{\text{in}}} \cdot \underbrace{\frac{R_5}{R_5 + R_4 + R_3}}_{I_3/I_2} \cdot I_{\text{in}}.$$

Thus, the component of V_{in} due to the first input current source is given by

$$\begin{aligned}
V_{\text{in}} &= V_1 - V_3 \\
&= I_{\text{in}} \cdot \frac{R_1}{R_1 + R_2 + (R_5 \| (R_4 + R_3))} \left(R_2 + (R_5 \| (R_4 + R_3)) - \frac{R_3 R_5}{R_5 + R_4 + R_3} \right) \\
&= I_{\text{in}} \cdot \frac{R_1}{R_1 + R_2 + (R_5 \| (R_4 + R_3))} \left(R_2 + \frac{R_5 R_4}{R_5 + R_4 + R_3} + \frac{R_5 R_3}{R_5 + R_4 + R_3} \right. \\
&\qquad \left. - \frac{R_3 R_5}{R_5 + R_4 + R_3} \right) \\
&= I_{\text{in}} \cdot \frac{R_1}{R_1 + R_2 + (R_5 \| (R_4 + R_3))} \left(R_2 + \frac{R_4 R_5}{R_5 + R_4 + R_3} \right).
\end{aligned}$$

For the second source, we can again compute V_1 as R_1 times I_1, which is given by

$$V_1 = -R_1 \cdot \underbrace{\frac{R_3}{R_3 + R_4 + (R_5 \| (R_1 + R_2))}}_{I_4/I_{\text{in}}} \cdot \underbrace{\frac{R_5}{R_5 + R_1 + R_2}}_{I_1/I_4} \cdot I_{\text{in}}.$$

Likewise, we can compute V_3 as R_3 times I_3, which is given by

$$V_3 = -R_3 \cdot \underbrace{\frac{R_4 + (R_5 \| (R_1 + R_2))}{R_3 + R_4 + (R_5 \| (R_1 + R_2))}}_{I_3/I_{\text{in}}} \cdot I_{\text{in}}.$$

Thus, the component of V_{in} due to the second input current source is given by

$$\begin{aligned}
V_{\text{in}} &= V_1 - V_3 \\
&= I_{\text{in}} \cdot \frac{R_3}{R_3 + R_4 + (R_5 \| (R_1 + R_2))} \left(-\frac{R_1 R_5}{R_5 + R_1 + R_2} + (R_4 + (R_5 \| (R_1 + R_2))) \right) \\
&= I_{\text{in}} \cdot \frac{R_3}{R_3 + R_4 + (R_5 \| (R_1 + R_2))} \left(R_4 + \frac{R_5 R_1}{R_5 + R_1 + R_2} + \frac{R_5 R_2}{R_5 + R_1 + R_2} \right. \\
&\qquad \left. - \frac{R_5 R_1}{R_5 + R_1 + R_2} \right) \\
&= I_{\text{in}} \cdot \frac{R_3}{R_3 + R_4 + (R_5 \| (R_1 + R_2))} \left(R_4 + \frac{R_2 R_5}{R_5 + R_1 + R_2} \right).
\end{aligned}$$

By superposing both components of V_{in} and dividing both sides of the equation by I_{in}, we find $R_{\text{in}} = V_{\text{in}}/I_{\text{in}}$ to be

$$\begin{aligned}
R_{\text{in}} &= \frac{R_1}{R_1 + R_2 + (R_5 \| (R_4 + R_3))} \left(R_2 + \frac{R_4 R_5}{R_5 + R_4 + R_3} \right) \\
&\qquad + \frac{R_3}{R_3 + R_4 + (R_5 \| (R_1 + R_2))} \left(R_4 + \frac{R_2 R_5}{R_5 + R_1 + R_2} \right).
\end{aligned}$$

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Example 1.4

Consider the infinite hexagonal mesh of resistors, each of which has value R, shown in Fig. 9a. We would like to determine the driving-point resistance, R_{in}, looking into the port shown (i.e., between two adjacent nodes). We could apply a voltage source to the port and see how much current is drawn by the circuit. Alternatively, we can apply a current source to the port and see how much voltage develops across the current source. In either case, the required resistance is given by the ratio of the voltage across the port to the current flowing into the port. In this case, it is convenient to chose the latter method, as shown in Fig. 9b. At this point, our task may seem somewhat daunting. It is unclear whether this problem can be solved using node-voltage analysis or using mesh-current analysis. At the very least,

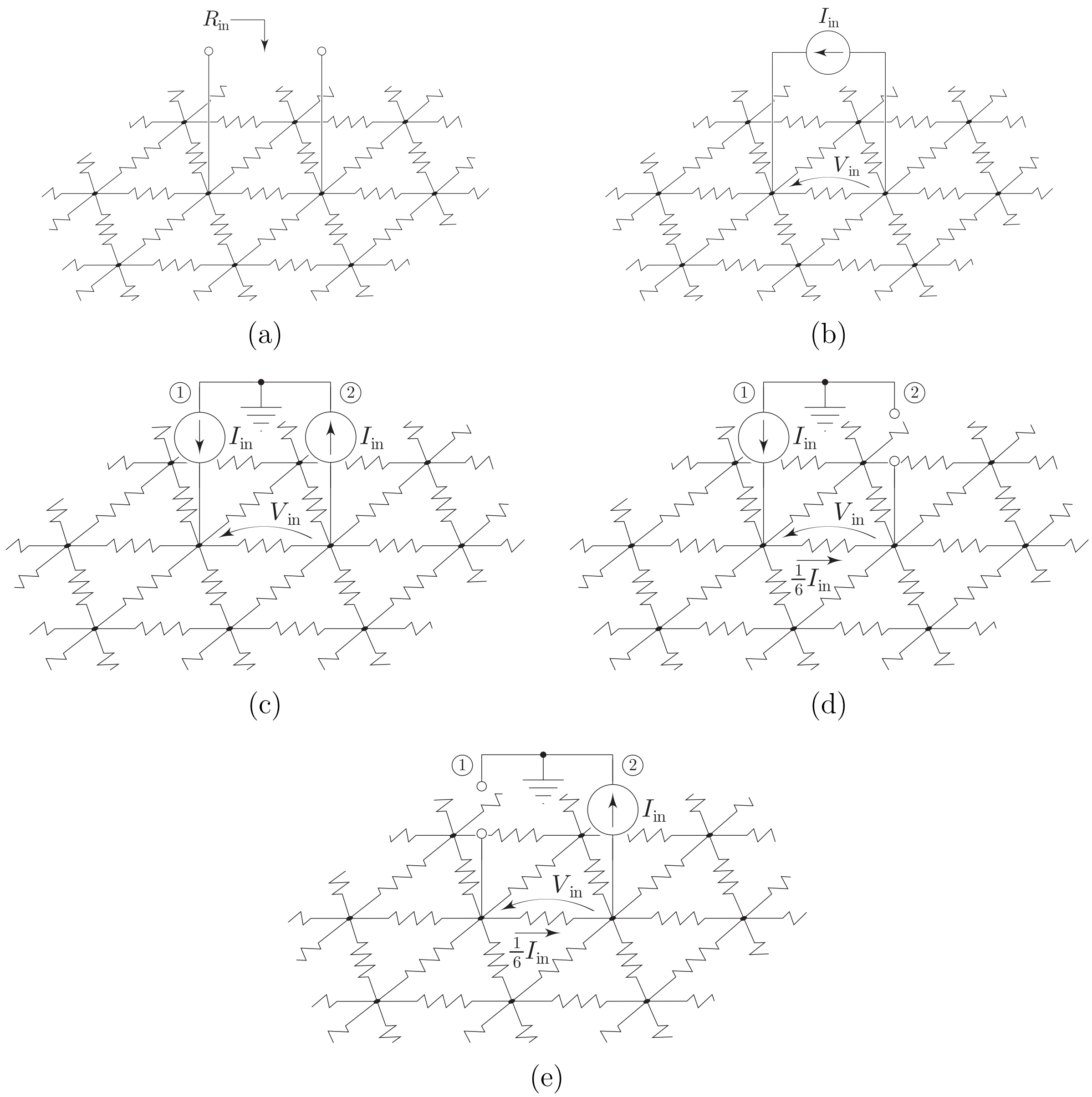


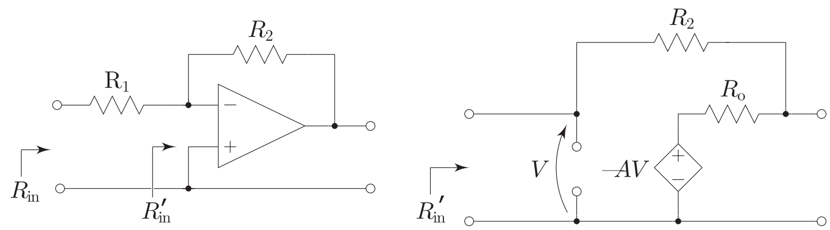
Figure 9: Example illustrating the use of current-source splitting and superposition.

we would have to invent some clever scheme for indexing the nodes on this hexagonal grid, which is itself nontrivial.

Instead, we shall split the test current source and then apply superposition to solve the problem. First, we split I_{in} into two replica sources, as shown in Fig. 9c. We can use superposition to write V_{in} as

$$V_{\text{in}} = V_{\text{in1}} + V_{\text{in2}},$$

where V_{in1} is the component of V_{in} due to the replica current source attached to port 1 and V_{in2} is the component of V_{in} due to the test source attached to port 2. With the current source on port 2 open circuited, we are injecting a current I_{in} into one location in this infinite



(a) (b)

Figure 10: (a) Inverting amplifier circuit. (b) Linearized model for calculating the input resistance R'_{in}, assuming the op-amp has a finite gain, A, and output resistance, R_{o}.

network, as shown in Fig. 9d. By symmetry, the current I_{in} must split evenly into six parts. We can calculate V_{in} using Ohm's law as $RI_{\text{in}}/6 = RI_{\text{in}}/6$. With the current source on port 1 open circuited, we are sinking a current I_{in} from one location in the infinite network, as shown in Fig. 9e. Again, by symmetry, the current I_{in} must be pulled together from the network in six equal parts. We can again calculate V_{in} using Ohm's law as $RI_{\text{in}}/6 = RI_{\text{in}}/6$. Superposing these results, we have that

$$V_{\text{in}} = R \cdot \frac{I_{\text{in}}}{6} + R \cdot \frac{I_{\text{in}}}{6} = R \cdot \frac{I_{\text{in}}}{3},$$

which implies that

$$R_{\text{in}} = \frac{V_{\text{in}}}{I_{\text{in}}} = \frac{R}{3}.$$

∎

Example 1.5

Consider the inverting amplifier circuit comprising an op-amp and two resistors, R_1 and R_2, shown in Fig. 10. We shall assume that the op-amp has a finite voltage gain, A, and a finite output resistance, R_{o}. We would like to determine the equivalent input resistance, R_{in}, of this circuit. Because the input port is only connected to a single branch, it does not seem like we will be able to apply source splitting to facilitate the calculation. However, if we recognize that we can compute R_{in} as

$$R_{\text{in}} = R_1 + R'_{\text{in}},$$

where R'_{in} is the equivalent input resistance of the circuit looking directly into the virtual ground node of the op-amp, then we can apply source splitting to perform the calculation of R'_{in}.

Figure 10b shows the equivalent circuit model that we shall use for the op-amp in order to compute R'_{in}. We shall apply a test voltage source, V'_{in}, to the circuit, as shown in Fig. 11a, and determine the amount of current I'_{in} drawn into the circuit as a result. First, we split V'_{in} into two replica sources, as shown. We are interested computing I'_{in}, the total current flowing through the loop indicated. There are four components of I'_{in}, two excited by the input source on branch 1 and two excited by the input source on branch 2. We consider each

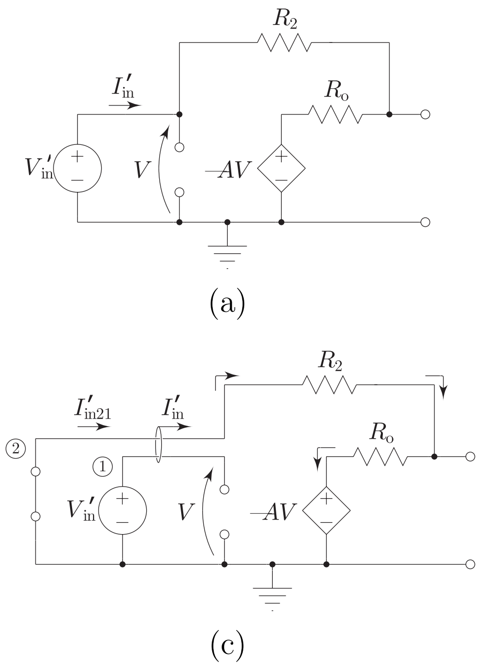


Figure 11: Calculation of the input resistance, R'_{in}, of the inverting amplifier using voltage-source splitting and superposition.

input source in turn, computing the components of I'_{in} that it excites, and use superposition to put the results together. First, consider the situation shown in Fig. 11b, in which the test source on branch 2 is disabled. In this case, even though we apply V'_{in} on branch 1, no current flows into this branch, so $I'_{\text{in}11}$ is zero. However, the dependent source will cause a current of $AV'_{\text{in}}/\left(R_{\text{o}}+R_2\right)$ to flow into the second branch of the circuit, which we denote by $I'_{\text{in}21}$. Next, consider the situation depicted in Fig. 11c, in which the test source on branch 1 is disabled. In this case, the dependent source has a value of zero, because V is zero, and a current of $V'_{\text{in}}/\left(R_{\text{o}}+R_2\right)$ flows into branch 2, because of the test source that we have applied on that branch. As in the first case, there is no current on branch 1, so $I'_{\text{in}12}$ is zero as well.

Thus, we have that

$$I'_{\text{in}} = \underbrace{\frac{AV'_{\text{in}}}{R_{\text{o}}+R_2}}_{I'_{\text{in}21}} + \underbrace{\frac{V'_{\text{in}}}{R_{\text{o}}+R_2}}_{I'_{\text{in}22}} = V'_{\text{in}} \cdot \frac{A+1}{R_{\text{o}}+R_2},$$

from which we can write R'_{in} as

$$R'_{\text{in}} = \frac{V'_{\text{in}}}{I'_{\text{in}}} = \frac{R_{\text{o}}+R_2}{A+1}.$$

Therefore, we can express the input resistance, R_{in}, as

$$R_{\text{in}} = R_1 + R'_{\text{in}} = R_1 + \frac{R_{\text{o}}+R_2}{A+1},$$

which approaches R_1 as A gets large. ∎

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