Synthesis of Dynamic Multiple-Input Translinear Element Networks

Bradley A. Minch

School of Electrical and Computer Engineering
Cornell University
Ithaca, NY 14853–5401

minch@ee.cornell.edu
http://www.ee.cornell.edu/~minch

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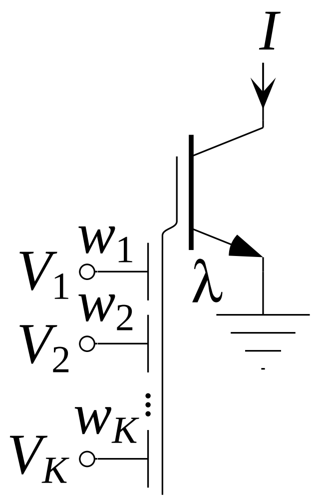
The Multiple-Input Translinear Element

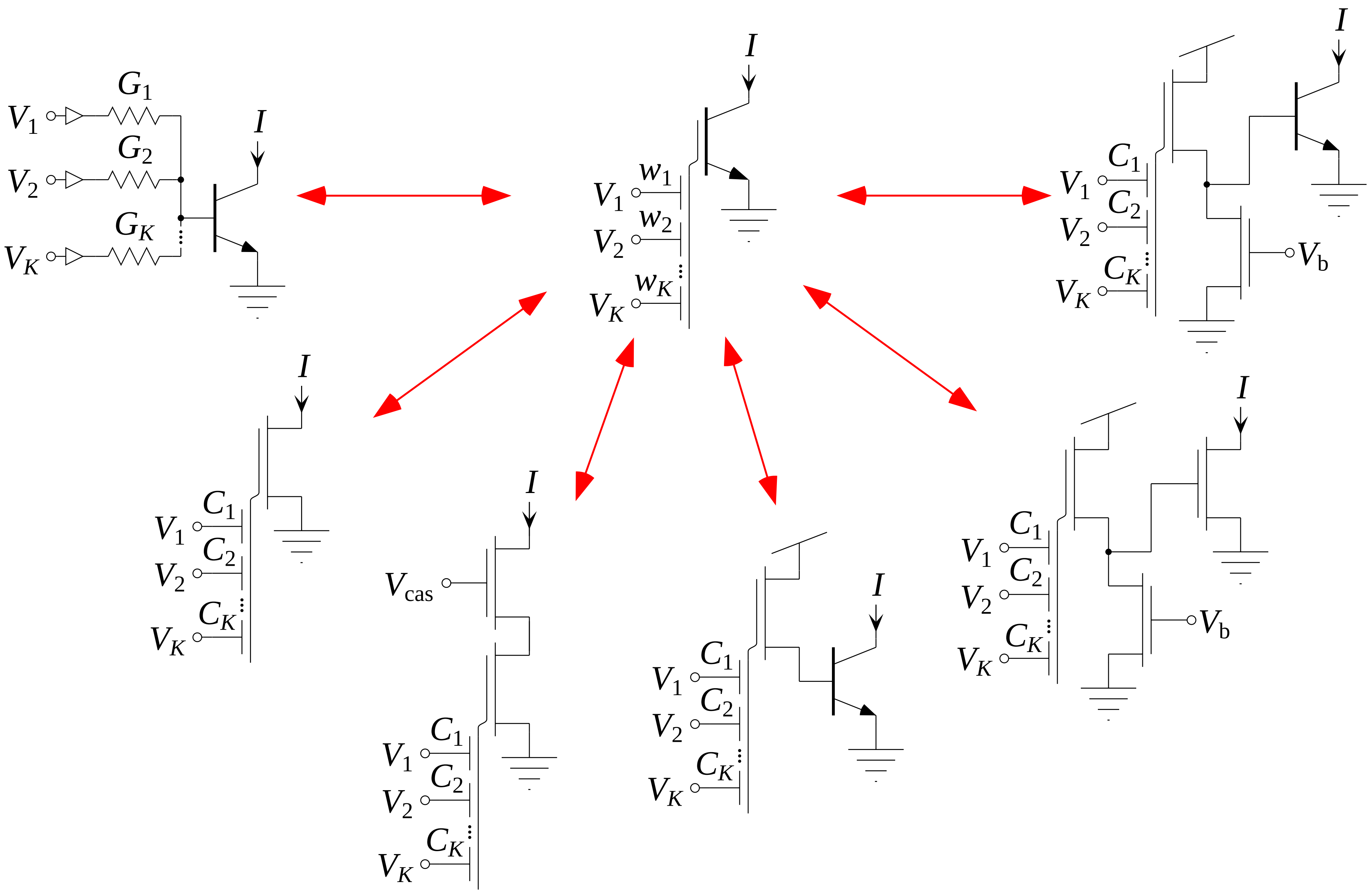
$$I = \lambda I_s \exp\left[\sum_{k=1}^{K} \frac{w_k V_k}{U_T}\right]$$



- The MITE has K *trans*conductances, each of which is *linear* in the output current, I:

$$g_k = \frac{\partial I}{\partial V_k} = \frac{w_k}{U_T}\lambda I_s \exp\left[\sum_{k=1}^{K} \frac{w_k V_k}{U_T}\right] = \frac{w_k}{U_T} I$$

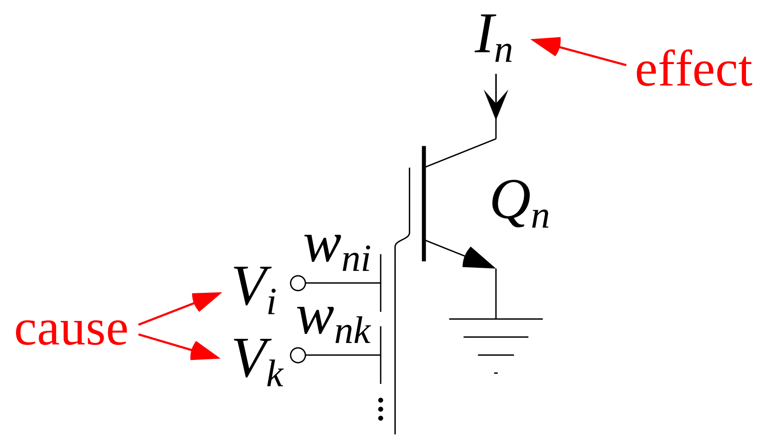
MITE Implementations



Basic MITE Configurations: Voltage-In, Current-Out

$$I_n \propto \exp\left[\frac{w_{ni}V_i + w_{nk}V_k + \ldots}{U_\mathrm{T}}\right]$$

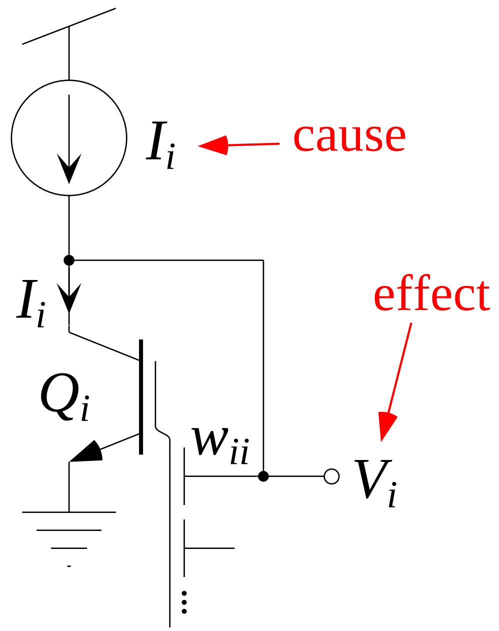
$$\Rightarrow \boxed{I_n \propto \exp\left[\frac{w_{ni}V_i}{U_\mathrm{T}}\right]\exp\left[\frac{w_{nk}V_k}{U_\mathrm{T}}\right]}$$



Basic MITE Configurations: Current-In, Voltage-Out

$$I_i \propto \exp\left[\frac{w_{ii} V_i + \ldots}{U_T}\right]$$

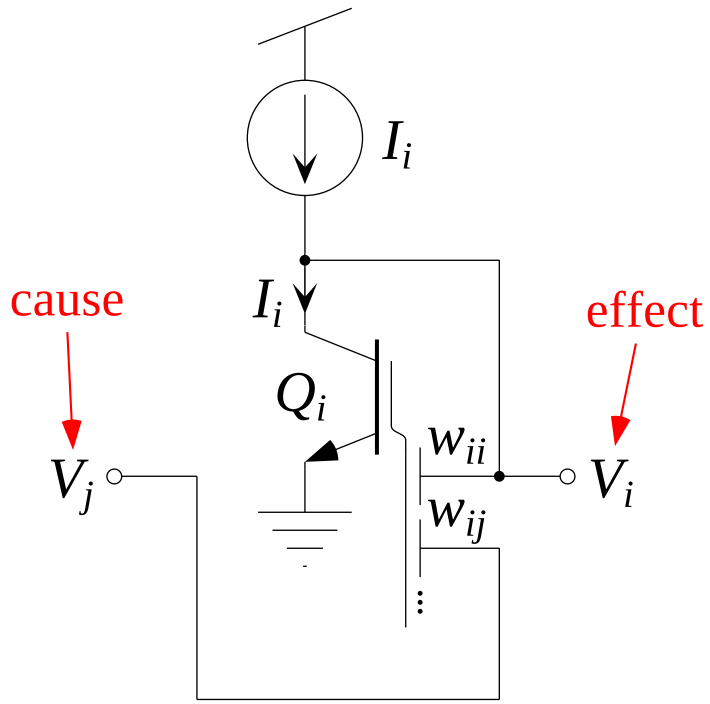
$$\Rightarrow \boxed{V_i = \frac{U_T}{w_{ii}} \log I_i - \ldots}$$



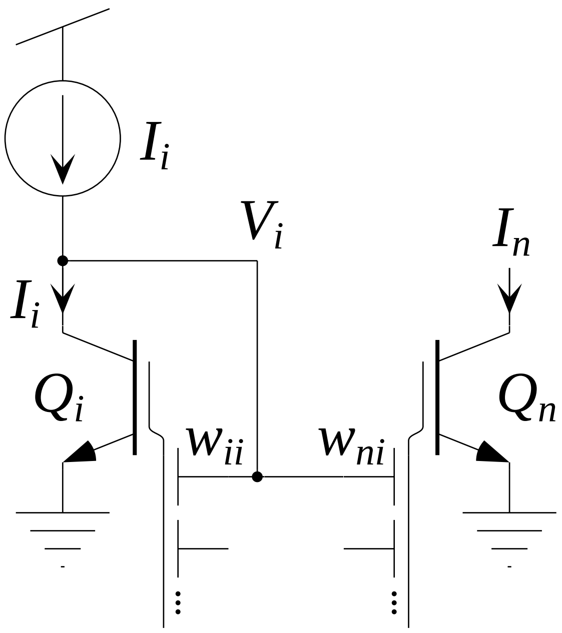
Basic MITE Configurations: Voltage-In, Voltage-Out

$$I_i \propto \exp\left[\frac{w_{ii} V_i + w_{ij} V_j + \ldots}{U_T}\right]$$

$$\Rightarrow \boxed{V_i = \frac{U_T}{w_{ii}} \log I_i - \frac{w_{ij}}{w_{ii}} V_j - \ldots}$$



Static MITE Networks: Power-Law Circuits



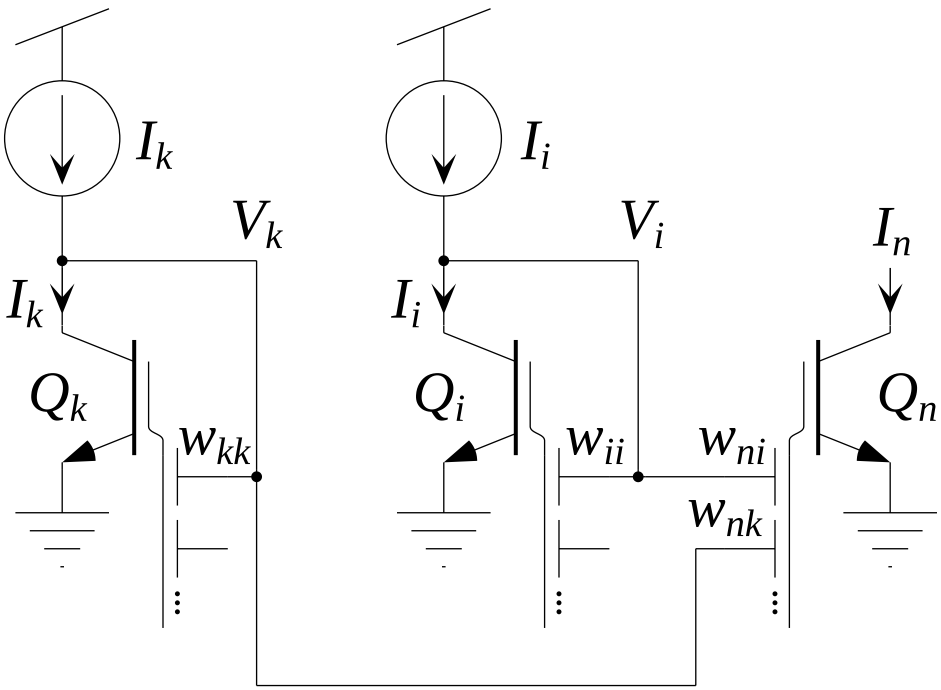
$$I_n \propto \exp\left[\frac{w_{ni} V_i + \ldots}{U_T}\right]$$

$$\Rightarrow I_n \propto \exp\left[\frac{w_{ni}}{U_T}\left(\frac{U_T}{w_{ii}} \log I_i - \ldots\right)\right]$$

$$\Rightarrow I_n \propto \exp\left[\frac{\cancel{U_T}}{\cancel{U_T}} \frac{w_{ni}}{w_{ii}} \log I_i\right]$$

$$\Rightarrow \boxed{I_n \propto I_i^{\frac{w_{ni}}{w_{ii}}}}$$

Static MITE Networks:
Product-of-Power-Law Circuits



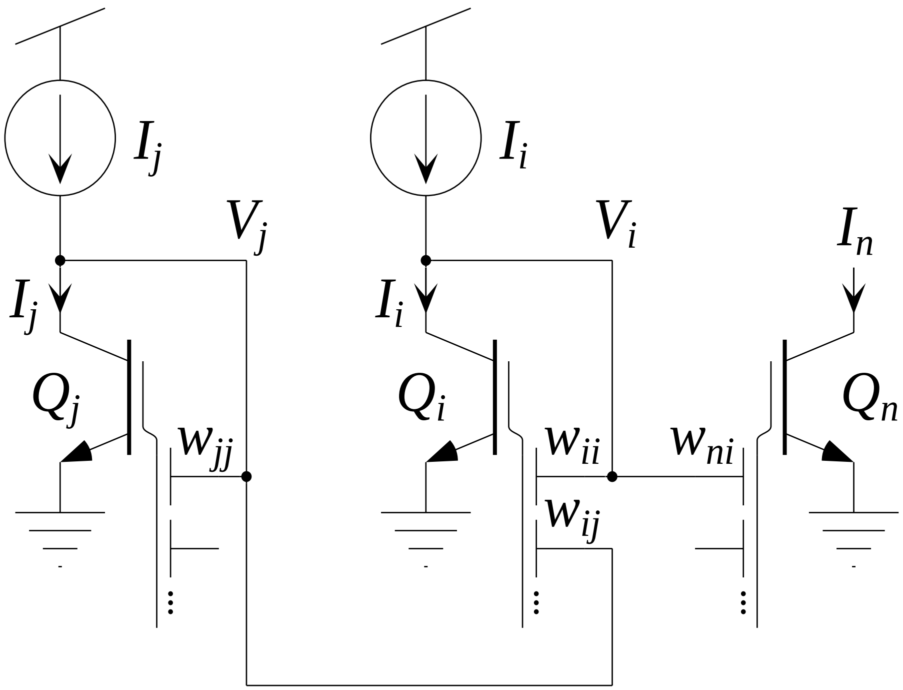
$$I_n \propto \exp\left[\frac{w_{ni}V_i}{U_T}\right]\exp\left[\frac{w_{nk}V_k}{U_T}\right]$$

$$\Rightarrow I_n \propto \exp\left[\frac{w_{ni}}{U_T}\left(\frac{U_T}{w_{ii}}\log I_i - \ldots\right)\right] \times \exp\left[\frac{w_{nk}}{U_T}\left(\frac{U_T}{w_{kk}}\log I_k - \ldots\right)\right]$$

$$\Rightarrow I_n \propto \exp\left[\frac{\cancel{U_T}}{\cancel{U_T}}\frac{w_{ni}}{w_{ii}}\log I_i\right] \times \exp\left[\frac{\cancel{U_T}}{\cancel{U_T}}\frac{w_{nk}}{w_{kk}}\log I_k\right]$$

$$\Rightarrow \boxed{I_n \propto I_i^{\frac{w_{ni}}{w_{ii}}} \times I_k^{\frac{w_{nk}}{w_{kk}}}}$$

Static MITE Networks: Quotient-of-Power-Law Circuits



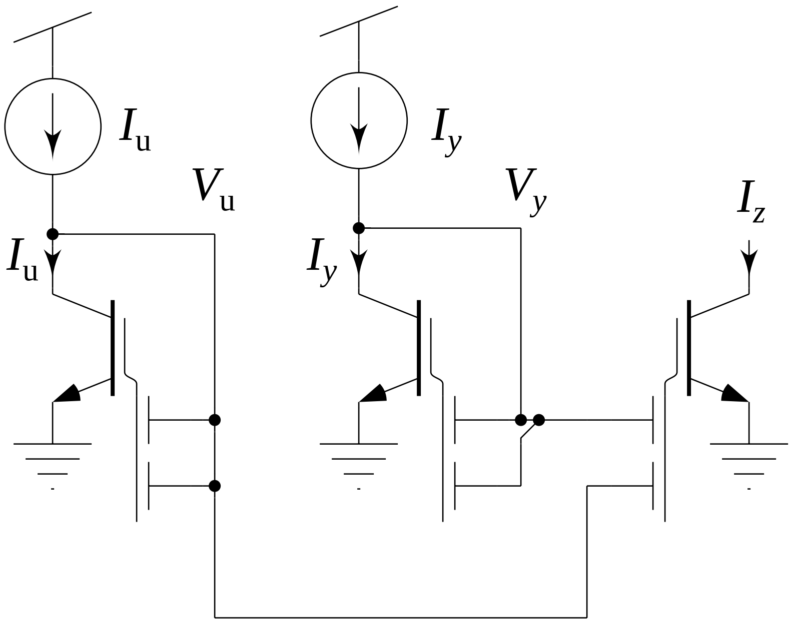
$$I_n \propto \exp\left[\frac{w_{ni}V_i + \ldots}{U_T}\right]$$

$$\Rightarrow I_n \propto \exp\left[\frac{w_{ni}}{U_T}\left(\frac{U_T}{w_{ii}}\log I_i - \frac{w_{ij}}{w_{ii}}\left(\frac{U_T}{w_{jj}}\log I_j - \ldots\right)\ldots\right)\right]$$

$$\Rightarrow I_n \propto \exp\left[\frac{\cancel{U_T}}{\cancel{U_T}}\frac{w_{ni}}{w_{ii}}\log I_i\right] \times \exp\left[-\frac{\cancel{U_T}}{\cancel{U_T}}\frac{w_{ni}}{w_{ii}}\frac{w_{ij}}{w_{jj}}\log I_j\right]$$

$$\Rightarrow I_n \propto I_i^{\frac{w_{ni}}{w_{ii}}} \times I_j^{-\frac{w_{ni}}{w_{ii}}\frac{w_{ij}}{w_{jj}}} \Rightarrow \boxed{I_n \propto I_i^{\frac{w_{ni}}{w_{ii}}} \div I_j^{\frac{w_{ni}}{w_{ii}}\frac{w_{ij}}{w_{jj}}}}$$

Static MITE Networks: Square-Root



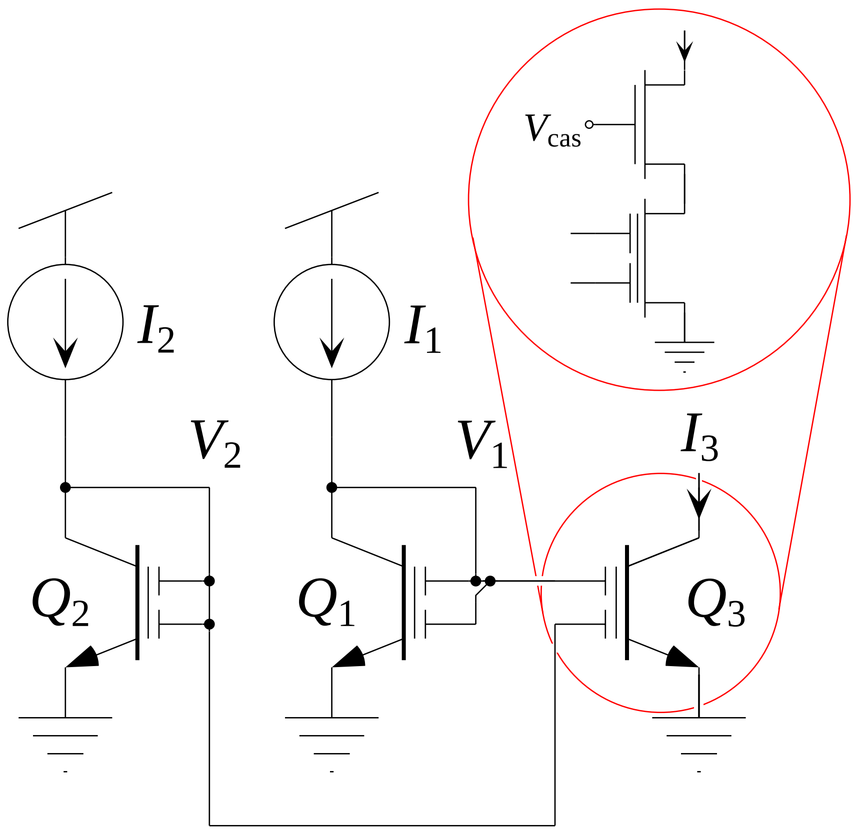
$$I_u = \lambda I_s e^{2wV_u/U_T} \Rightarrow \frac{wV_u}{U_T} = \log\sqrt{\frac{I_u}{\lambda I_s}}$$

$$I_y = \lambda I_s e^{2wV_y/U_T} \Rightarrow \frac{wV_y}{U_T} = \log\sqrt{\frac{I_y}{\lambda I_s}}$$

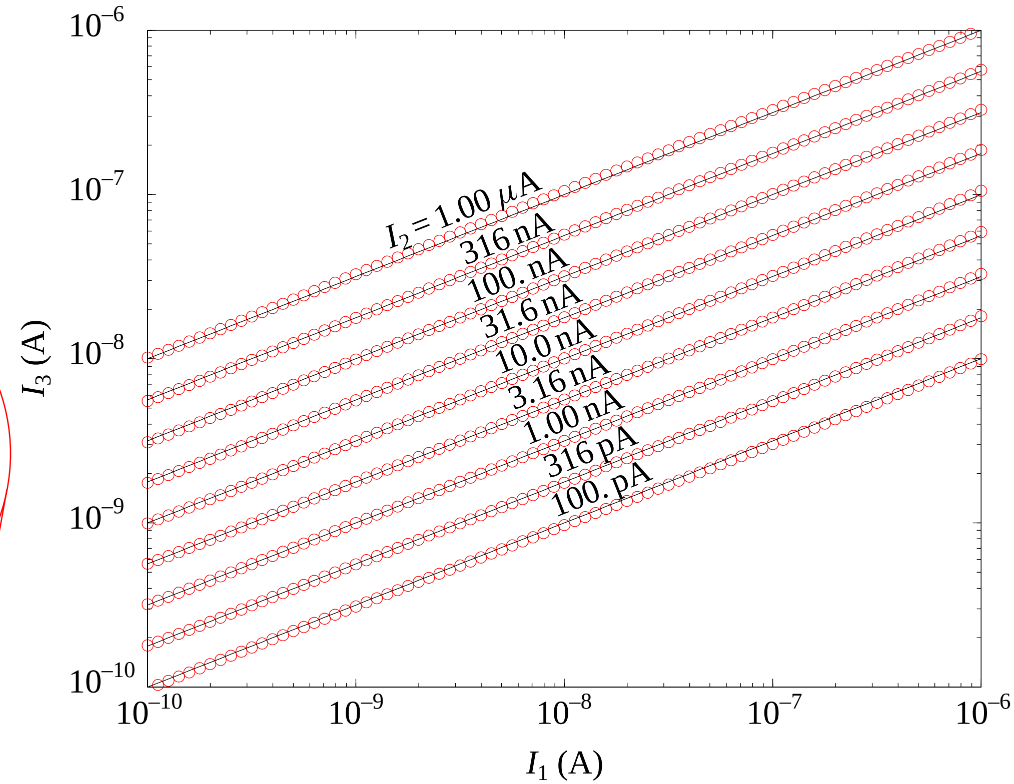
$$I_z = \lambda I_s e^{w\left(V_y + V_u\right)/U_T} = \lambda I_s e^{\log\sqrt{I_y/\lambda I_s} + \log\sqrt{I_u/\lambda I_s}}$$

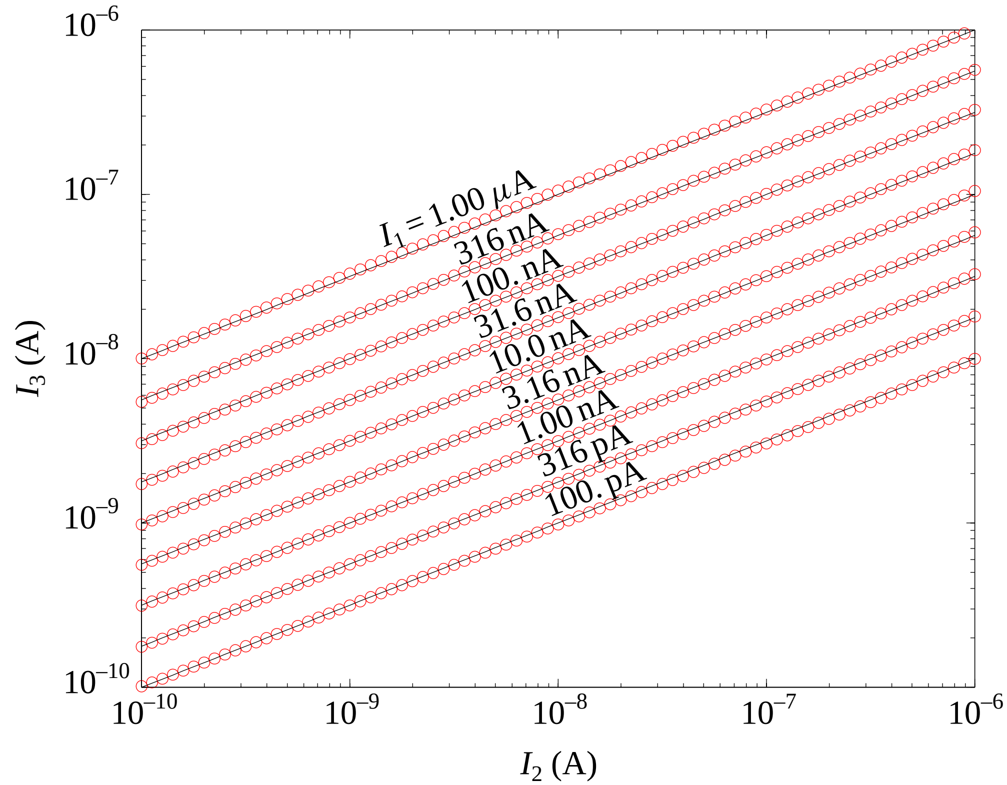
$$\Rightarrow I_z = \sqrt{I_y I_u} \Rightarrow \underbrace{\left(\frac{I_z}{I_u}\right)}_{z} = \sqrt{\underbrace{\frac{I_y}{I_u}}_{y}} \Rightarrow \boxed{z = \sqrt{y}}$$

MITE Networks: Geometric Mean

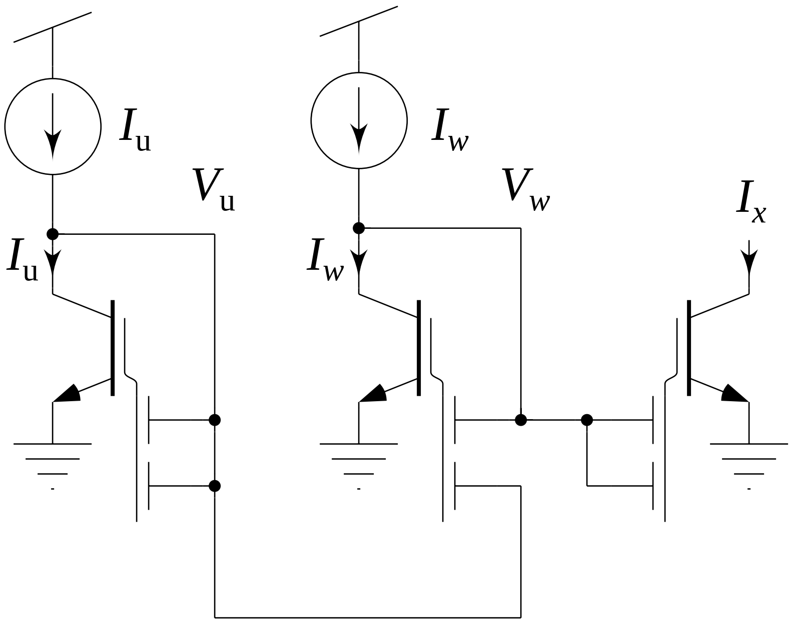


$$I_3 = \sqrt{I_1 I_2}$$





Static MITE Networks: Square



$$I_u = \lambda I_s e^{2wV_u/U_T} \Rightarrow \frac{wV_u}{U_T} = \log\sqrt{\frac{I_u}{\lambda I_s}}$$

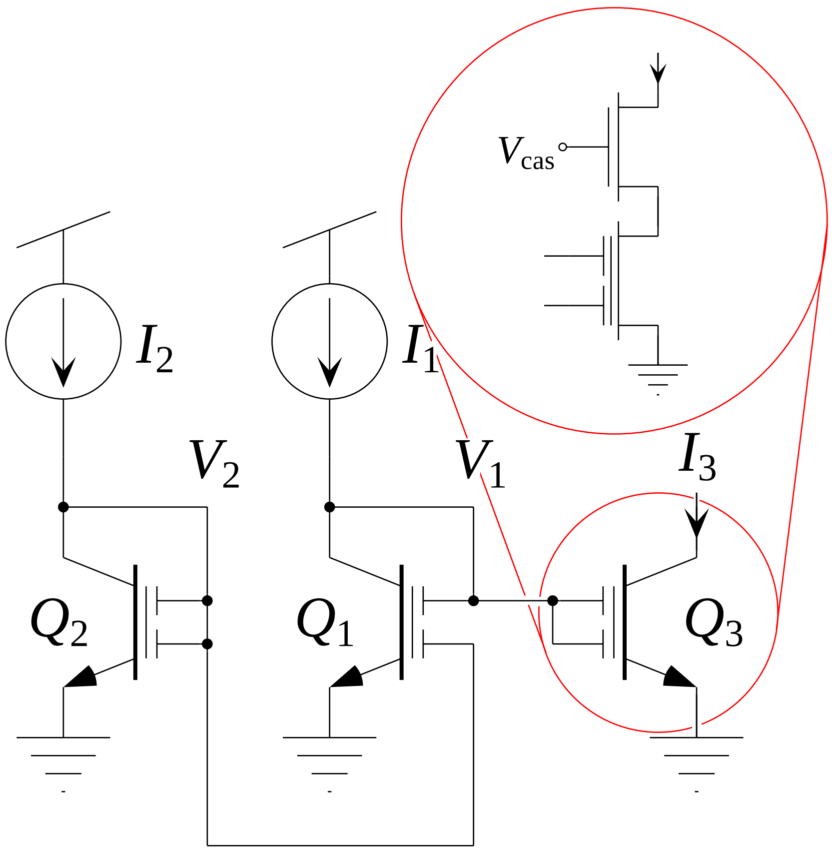
$$I_w = \lambda I_s e^{w(V_w + V_u)/U_T} = \lambda I_s e^{wV_w/U_T} e^{\log\sqrt{I_u/\lambda I_s}}$$

$$\Rightarrow \frac{wV_w}{U_T} = \log\frac{I_w}{\sqrt{\lambda I_s I_u}}$$

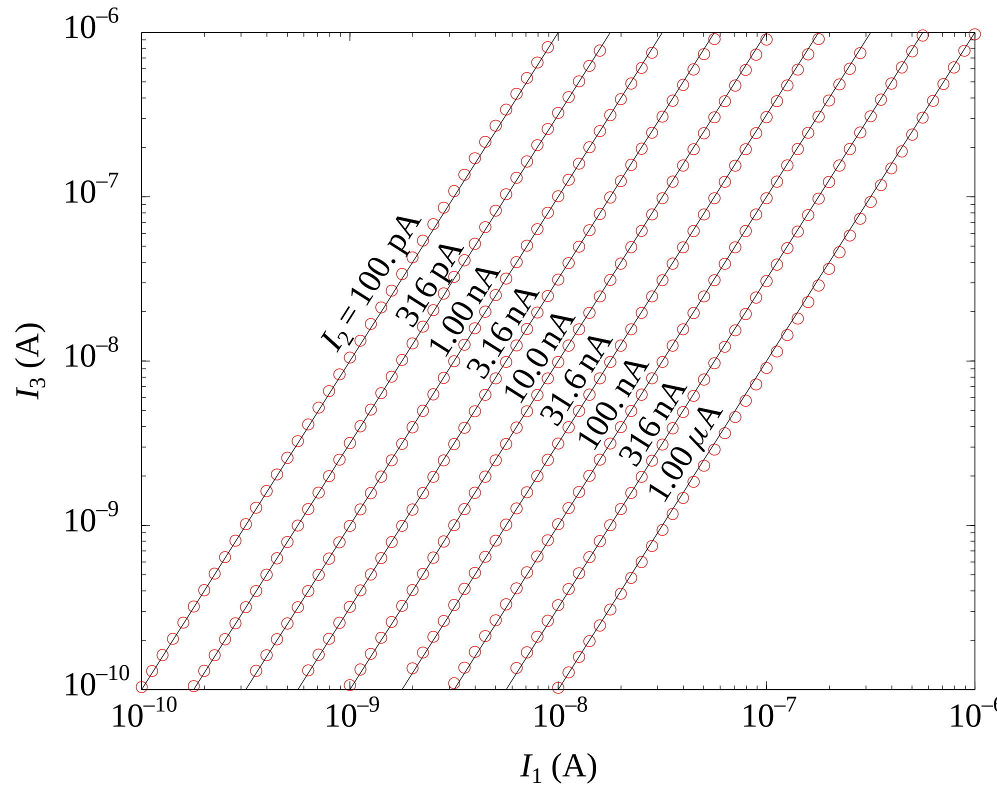
$$I_x = \lambda I_s e^{2wV_w/U_T} = \lambda I_s e^{2\log I_w/\sqrt{\lambda I_s I_u}} = \frac{I_w^2}{I_u}$$

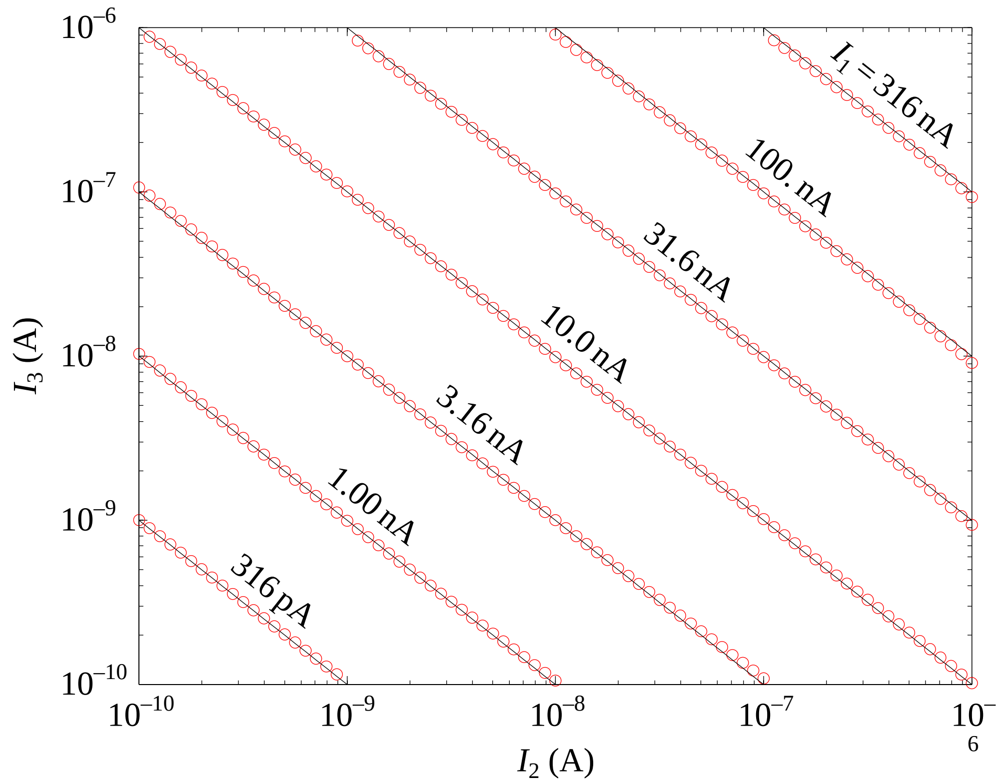
$$\Rightarrow \underbrace{\left(\frac{I_x}{I_u}\right)}_{x} = \underbrace{\left(\frac{I_w}{I_u}\right)}_{w}^2 \Rightarrow \boxed{x = w^2}$$

MITE Networks: Square/Reciprocal

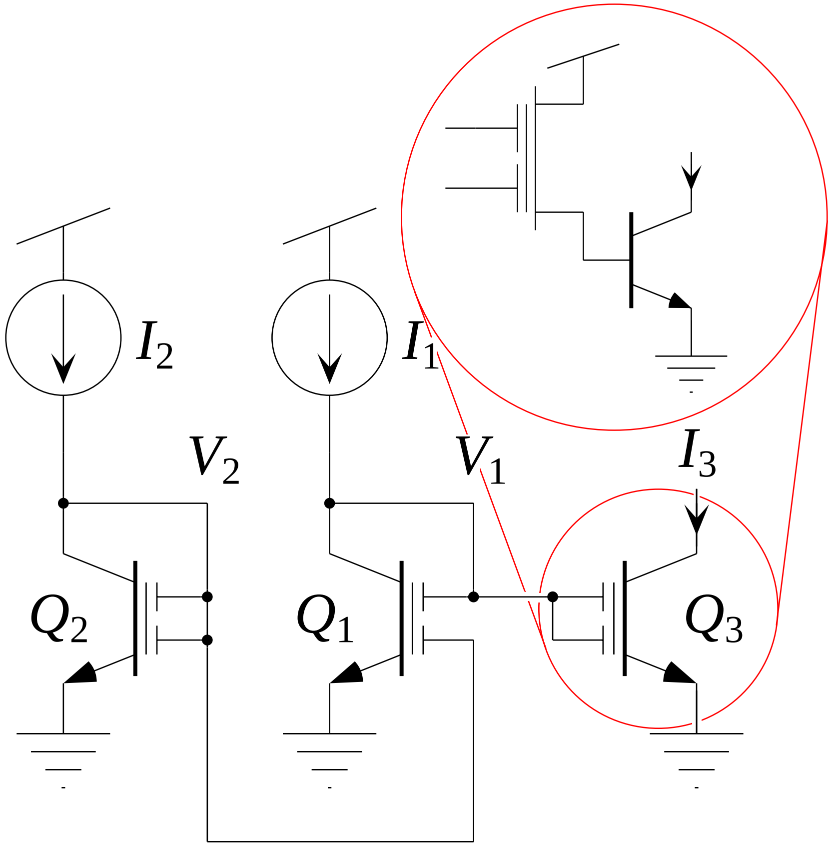


$$I_3 = \frac{I_1^2}{I_2}$$

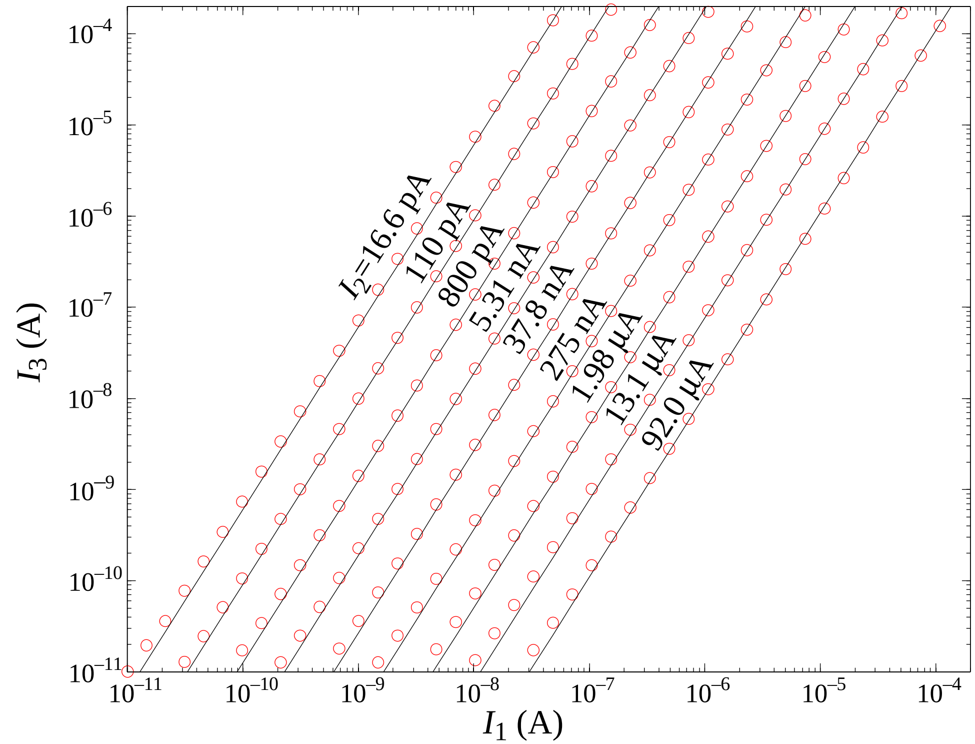


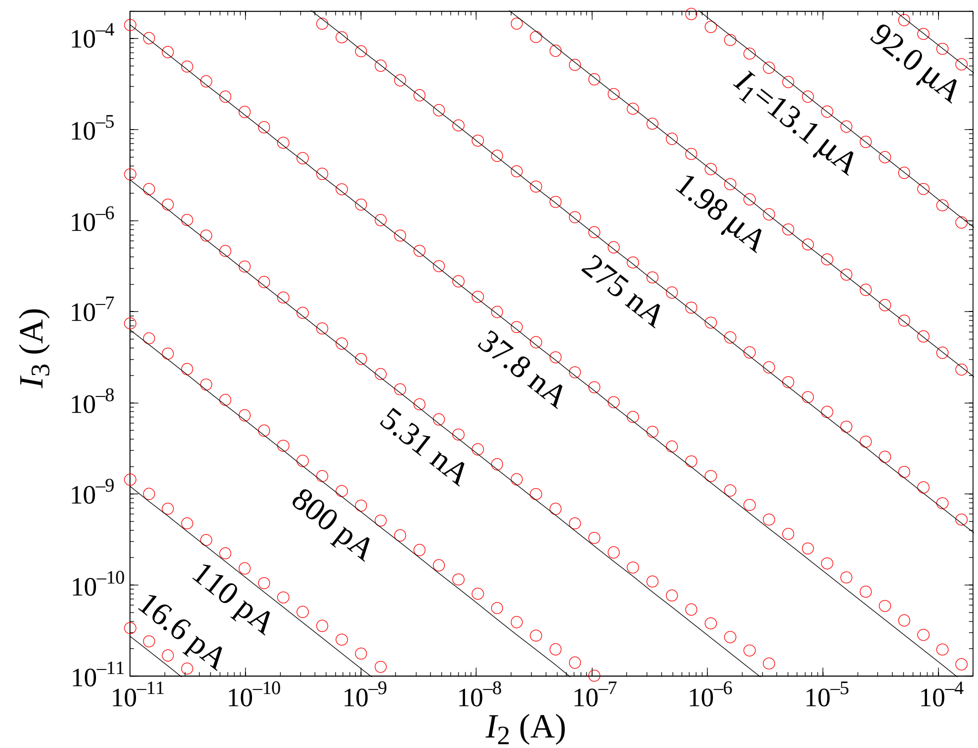


MITE Networks: Square/Reciprocal

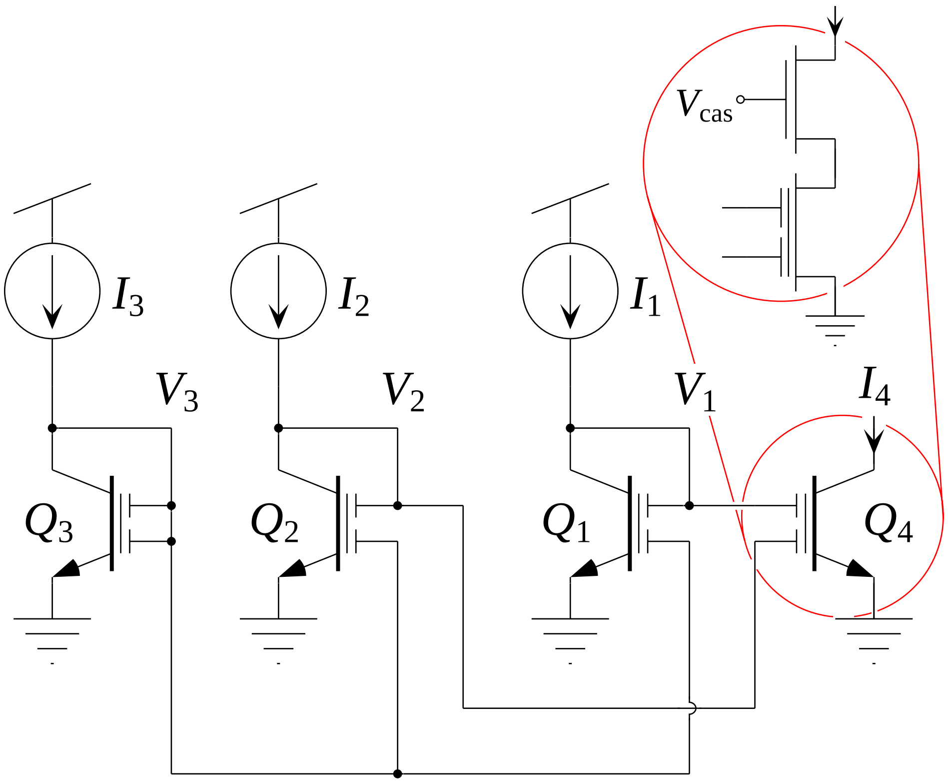


$$I_3 = \frac{I_1^2}{I_2}$$

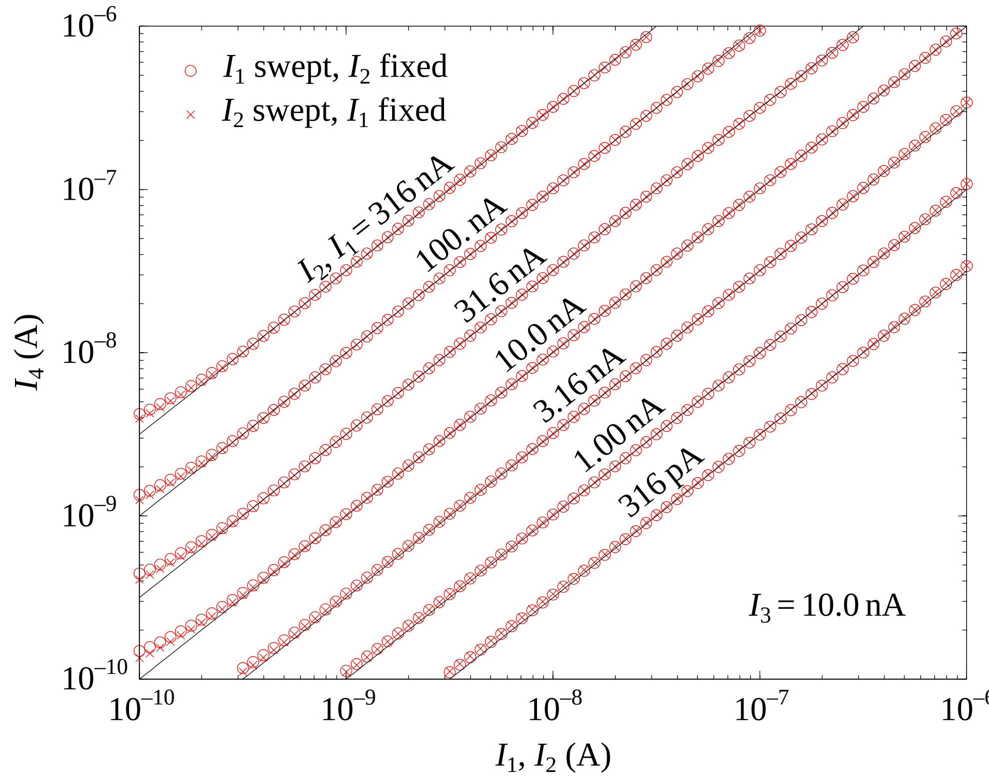


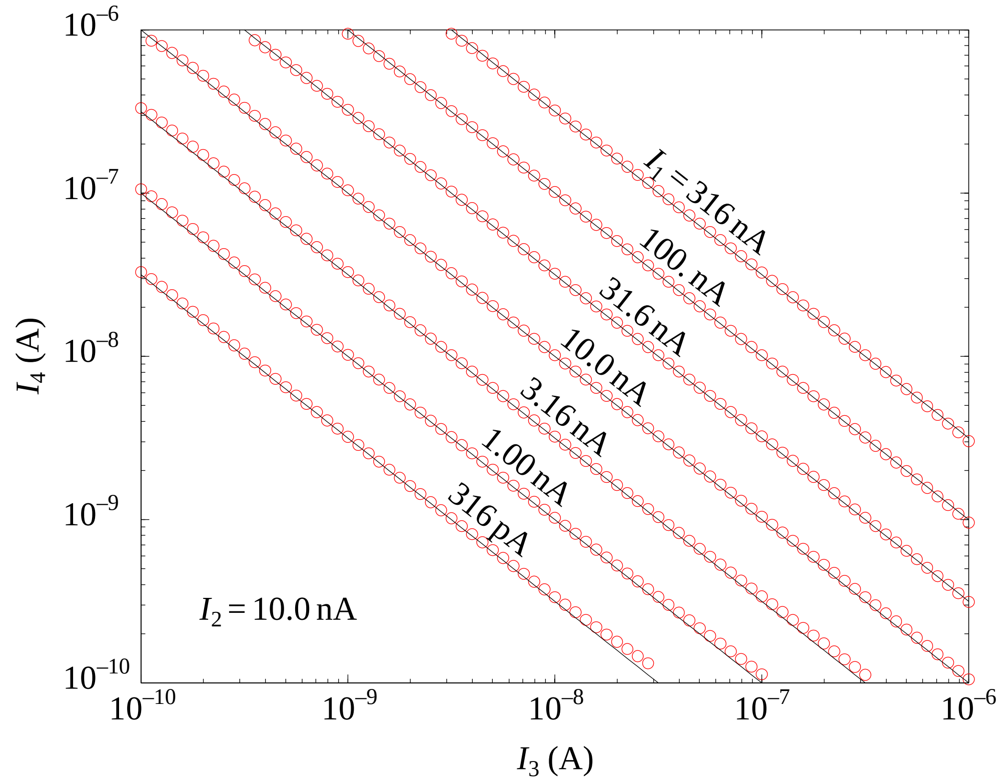


MITE Networks: Multiply/Reciprocal

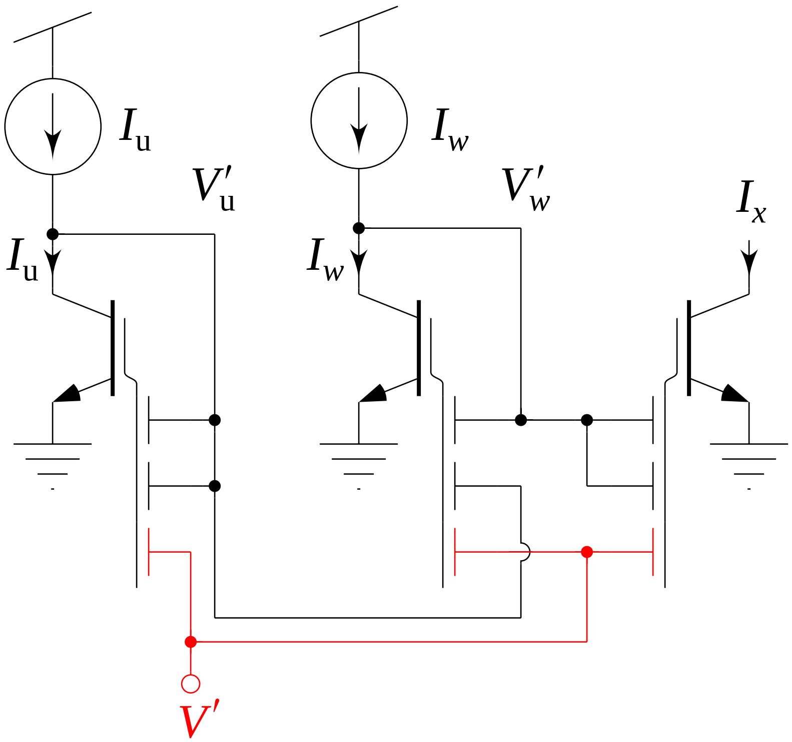


$$I_4 = \frac{I_1 I_2}{I_3}$$





Static MITE Networks: Augmentation



Let $\lambda' \equiv \lambda e^{wV'/U_T}$

$$I_u = \lambda' I_s e^{2wV'_u/U_T} \Rightarrow \frac{wV'_u}{U_T} = \log\sqrt{\frac{I_u}{\lambda' I_s}}$$

$$I_w = \lambda' I_s e^{w(V'_w + V'_u)/U_T} = \lambda' I_s e^{wV'_w/U_T} e^{\log\sqrt{I_u/\lambda' I_s}}$$

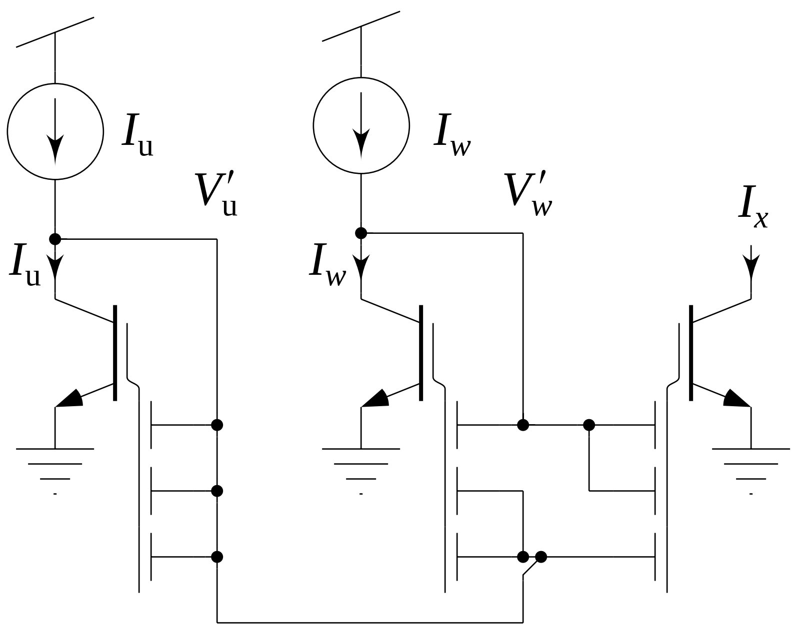
$$\Rightarrow \frac{wV'_w}{U_T} = \log\frac{I_w}{\sqrt{\lambda' I_s I_u}}$$

$$I_x = \lambda' I_s e^{2wV'_w/U_T} = \lambda' I_s e^{2\log I_w/\sqrt{\lambda' I_s I_u}} = \frac{I_w^2}{I_u}$$

$$\Rightarrow \underbrace{\left(\frac{I_x}{I_u}\right)}_{x} = \underbrace{\left(\frac{I_w}{I_u}\right)}_{w}^2 \Rightarrow \boxed{x = w^2}$$

$\Rightarrow$ Relationship holds regardless of V'

Static MITE Networks: Augmentation

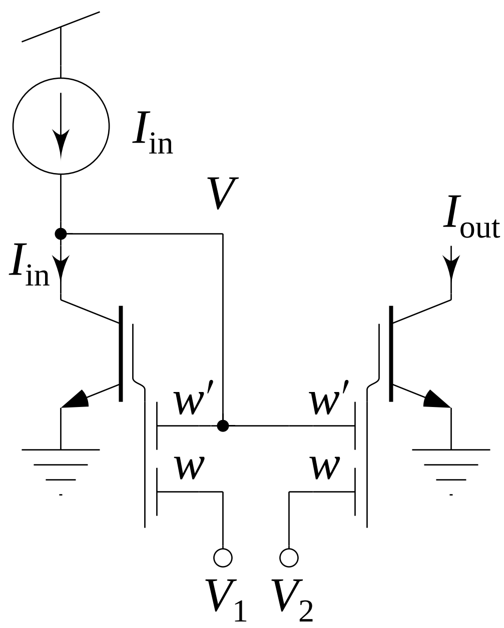


⇒ We can connect V'_u and V' together without changing the function of the MITE network...

Caveat emptor! To avoid the creation of (positive) feedback loops, choose a MITE that only has self connections.

If we had connected V'_w and V', we would have introduced a positive feedback loop into the MITE network.

MITE Log-Domain Filters: Building Block



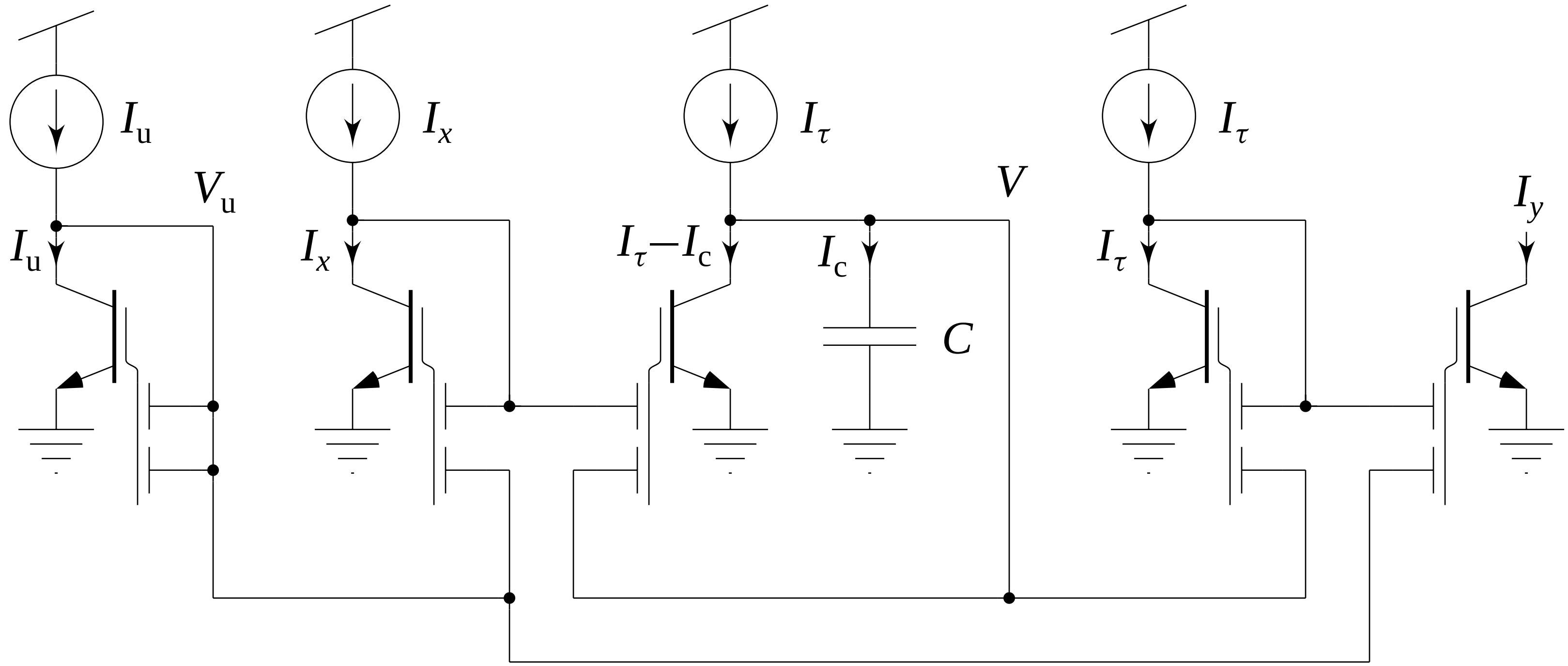
$$I_{\text{in}} = \lambda I_s e^{(w'V + wV_1)/U_T}$$

$$I_{\text{out}} = \lambda I_s e^{(w'V + wV_2)/U_T}$$

$$\Rightarrow \frac{I_{\text{out}}}{I_{\text{in}}} = \frac{\cancel{\lambda I_s}}{\cancel{\lambda I_s}} \frac{\cancel{e^{w'V/U_T}}}{\cancel{e^{w'V/U_T}}} \frac{e^{wV_2/U_T}}{e^{wV_1/U_T}}$$

$$\Rightarrow \boxed{I_{\text{out}} = I_{\text{in}} e^{w(V_2 - V_1)/U_T}}$$

MITE Log-Domain Filters: First-Order Low-Pass

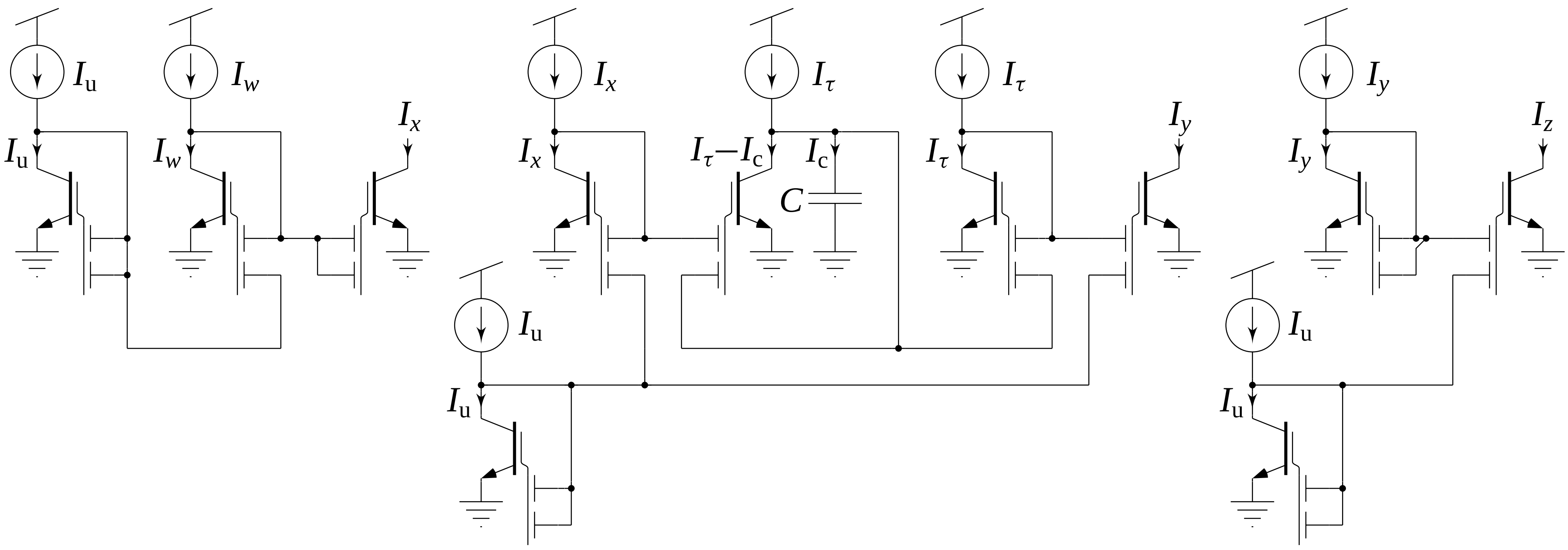


$$I_y = I_\tau e^{w(V_u - V)/U_T} \Rightarrow e^{w(V-V_u)/U_T} = \frac{I_\tau}{I_y} \text{ and } \frac{dV}{dt} = -\frac{U_T}{wI_y}\frac{dI_y}{dt}$$

$$\text{KCL} \Rightarrow C\frac{dV}{dt} = I_\tau - I_x e^{w(V-V_u)/U_T} \Rightarrow -\frac{CU_T}{wI_y}\frac{dI_y}{dt} = I_\tau - I_x\frac{I_\tau}{I_y}$$

$$\Rightarrow \frac{CU_T}{wI_\tau}\frac{dI_y}{dt} + I_y = I_x \Rightarrow \underbrace{\frac{CU_T}{wI_\tau}}_{\tau}\frac{d}{dt}\underbrace{\left(\frac{I_y}{I_u}\right)}_{y} + \underbrace{\left(\frac{I_y}{I_u}\right)}_{y} = \underbrace{\left(\frac{I_x}{I_u}\right)}_{x} \Rightarrow \boxed{\tau\frac{dy}{dt} + y = x}$$

Synthesis of Dynamic MITE Networks: RMS-to-DC Converter

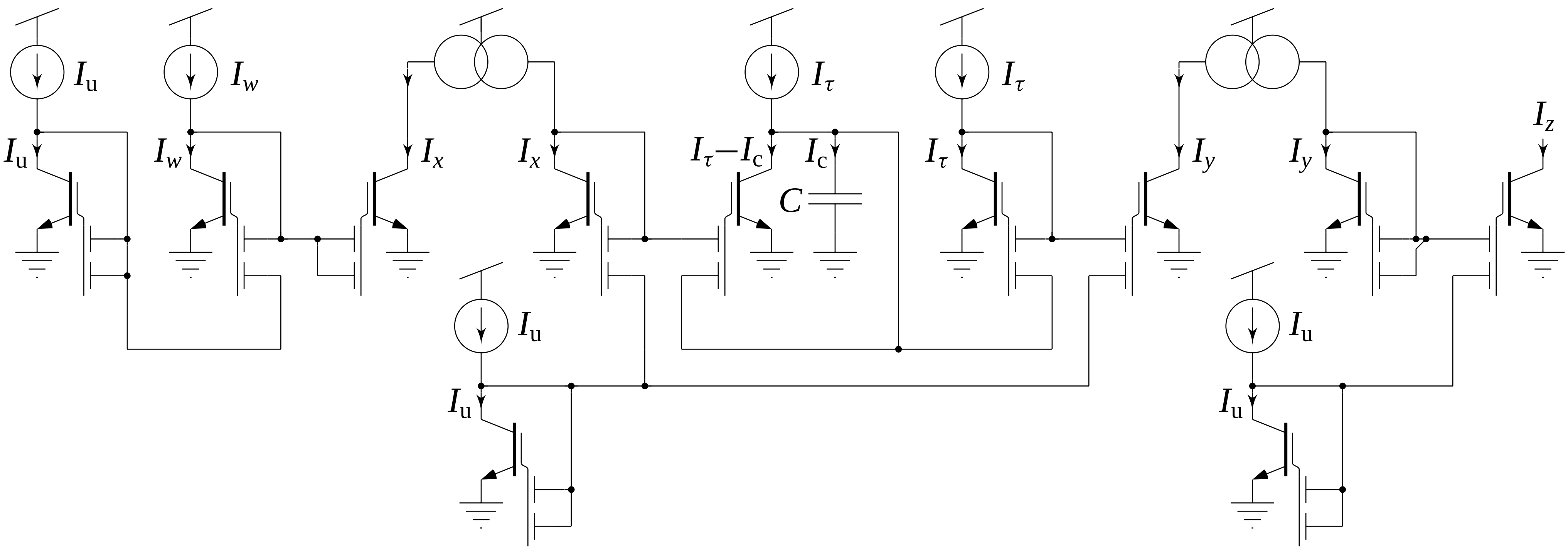


$$x = w^2$$

$$\tau \frac{dy}{dt} + y = x$$

$$z = \sqrt{y}$$

Synthesis of Dynamic MITE Networks: RMS-to-DC Converter



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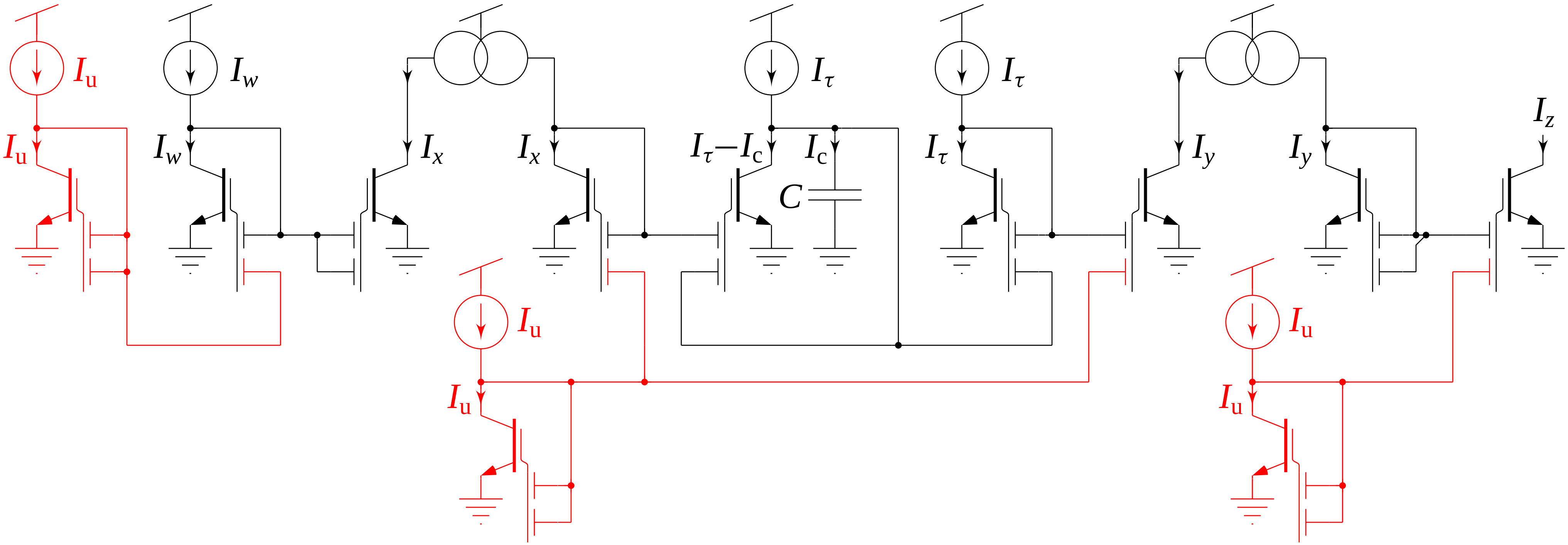
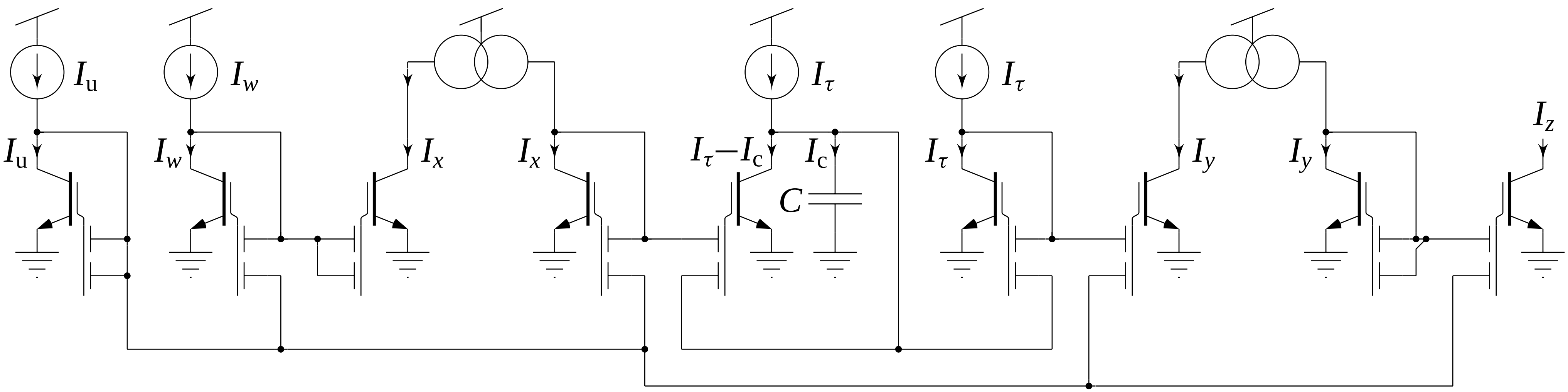


$$x = w^2 \qquad \tau \frac{dy}{dt} + y = x \qquad z = \sqrt{y}$$

Synthesis of Dynamic MITE Networks: RMS-to-DC Converter



$$x = w^2$$

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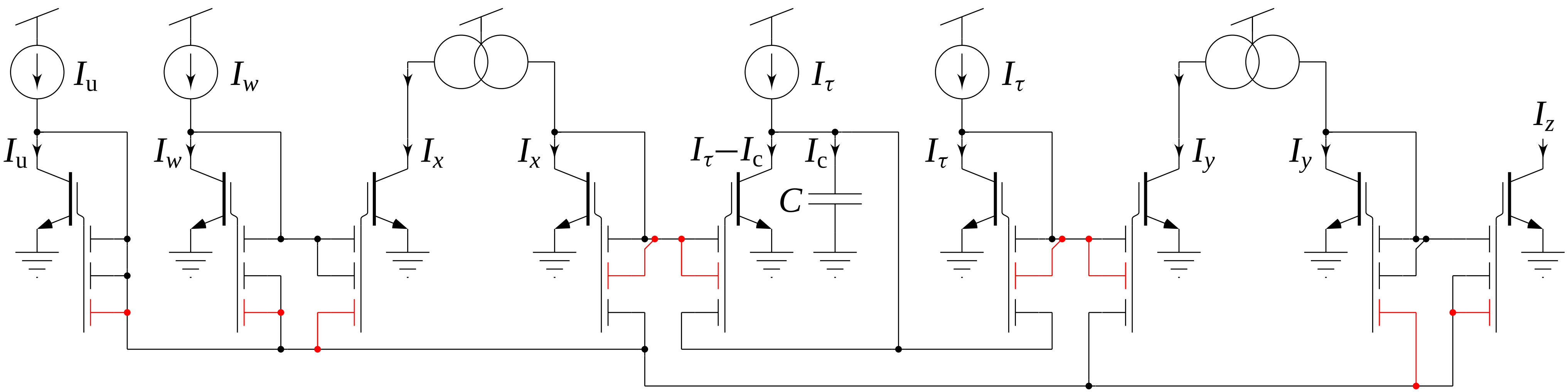

$$x = w^2$$

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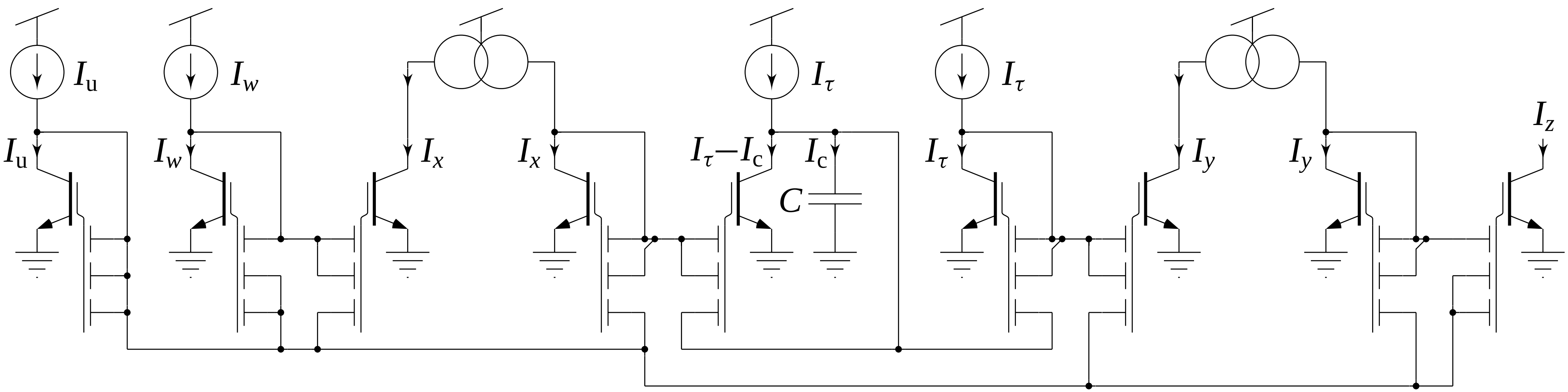

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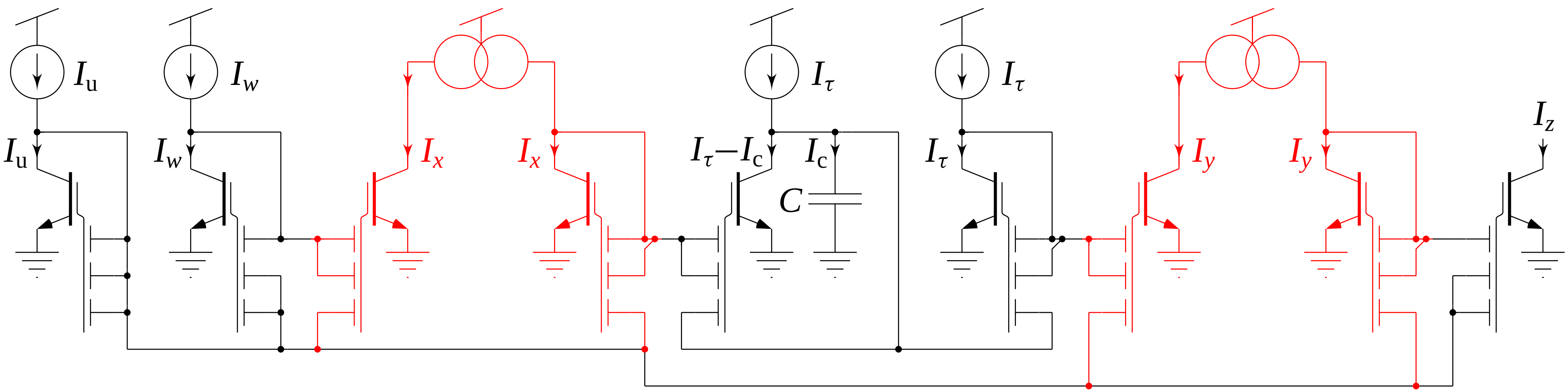

$$x = w^2$$

$$\tau \frac{dy}{dt} + y = x$$

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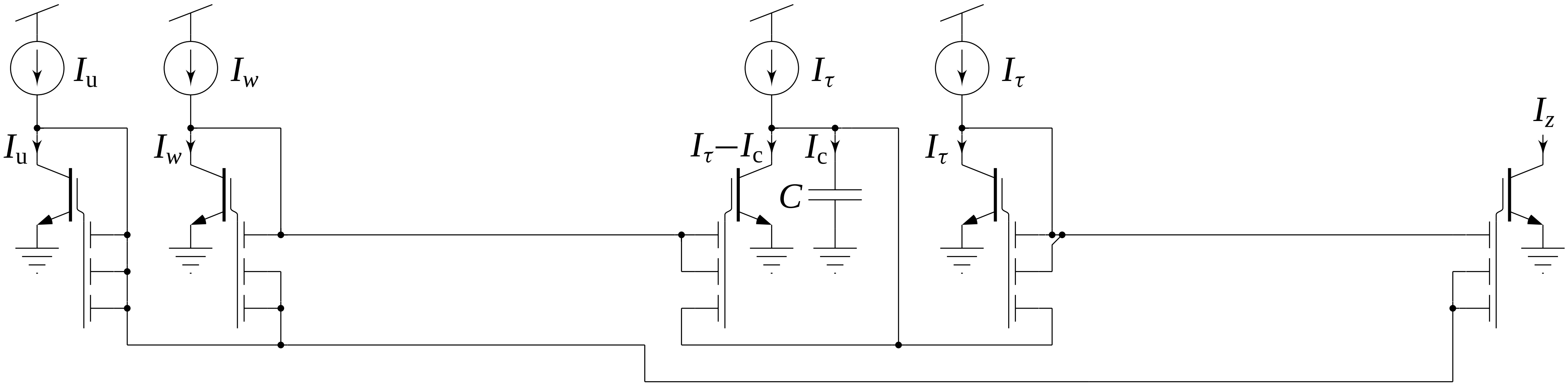
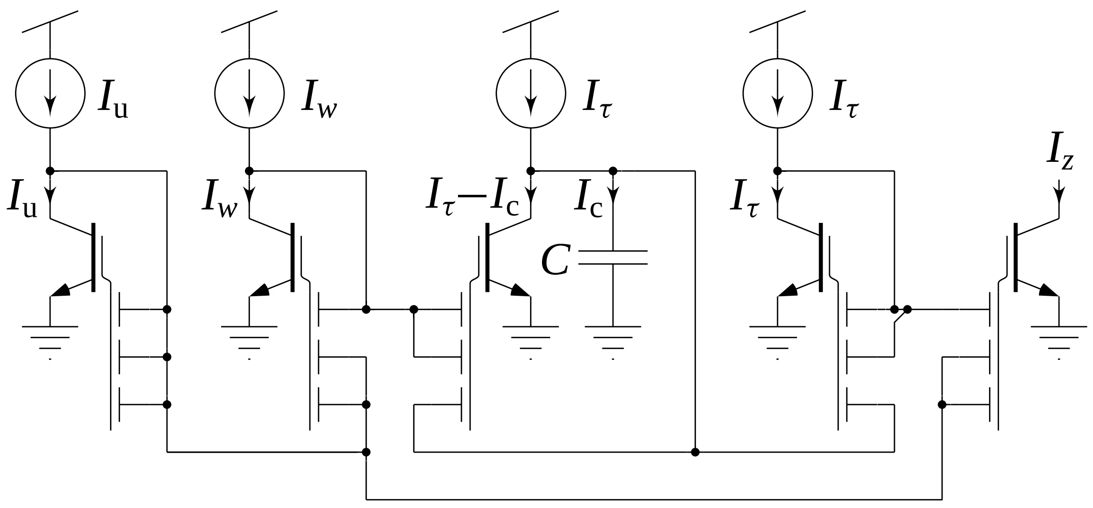
Synthesis of Dynamic MITE Networks: RMS-to-DC Converter



$$\left(2\tau \frac{dz}{dt} + z\right) z = w^2$$

Synthesis of Dynamic MITE Networks: RMS-to-DC Converter



$$\left(2\tau\frac{dz}{dt}+z\right)z=w^2$$