

Synthesis of Static and Dynamic **MITE** Networks

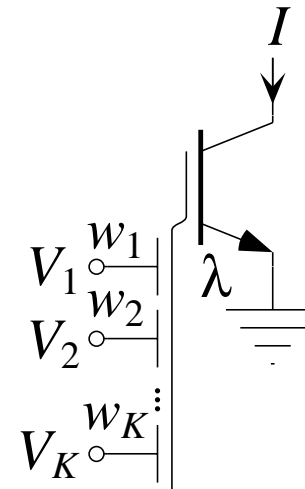
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The Multiple-Input Translinear Element

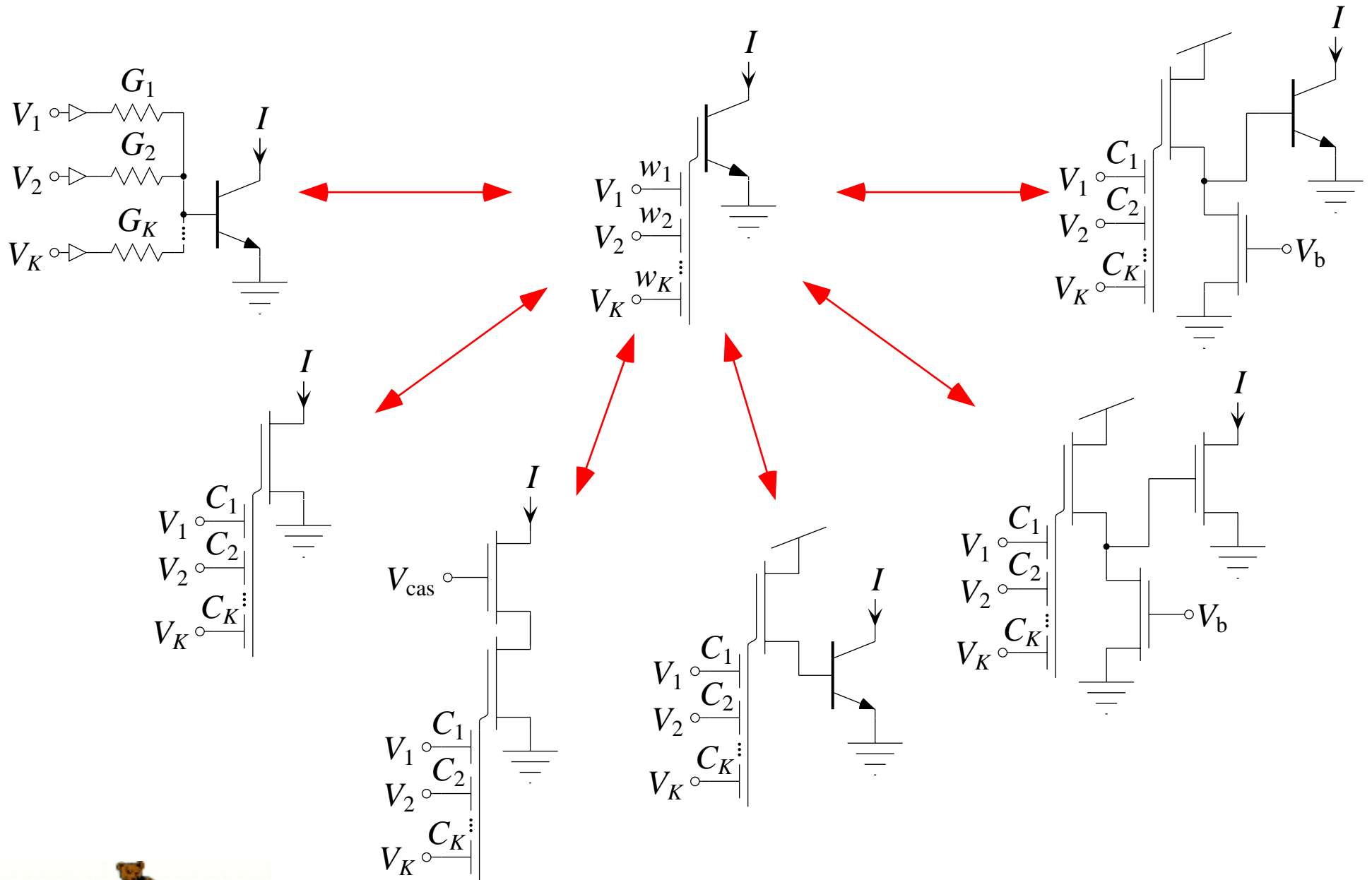
$$I = \lambda I_s \exp \left[\sum_{k=1}^K \frac{w_k V_k}{U_T} \right]$$



- ▶ The MITE has K *trans*conductances, each of which is *linear* in the output current, I :

$$\begin{aligned} g_k &= \frac{\partial I}{\partial V_k} \\ &= \frac{w_k}{U_T} \lambda I_s \exp \left[\sum_{k=1}^K \frac{w_k V_k}{U_T} \right] \\ &= \frac{w_k}{U_T} I \end{aligned}$$

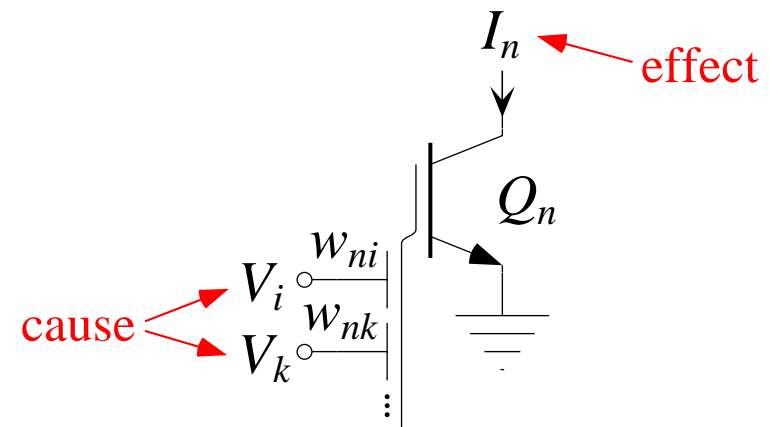
MITE Implementations



Basic MITE Configurations: Voltage-In, Current-Out

$$I_n \propto \exp\left[\frac{w_{ni}V_i + w_{nk}V_k + \dots}{U_T}\right]$$

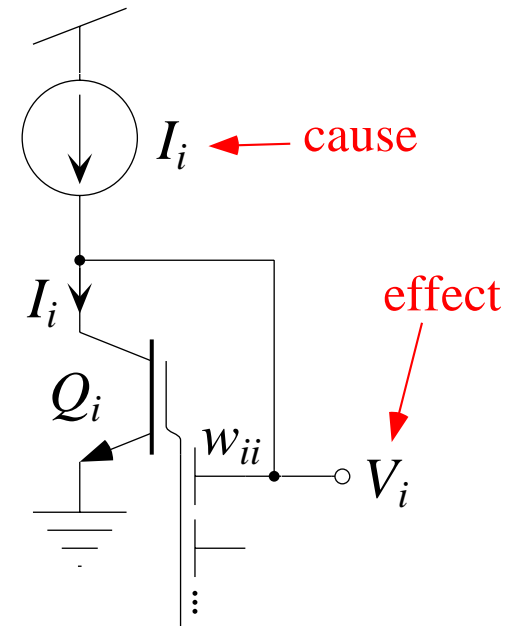
$$\Rightarrow I_n \propto \exp\left[\frac{w_{ni}V_i}{U_T}\right] \exp\left[\frac{w_{nk}V_k}{U_T}\right]$$



Basic MITE Configurations: Current-In, Voltage-Out

$$I_i \propto \exp\left[\frac{w_{ii} V_i + \dots}{U_T}\right]$$

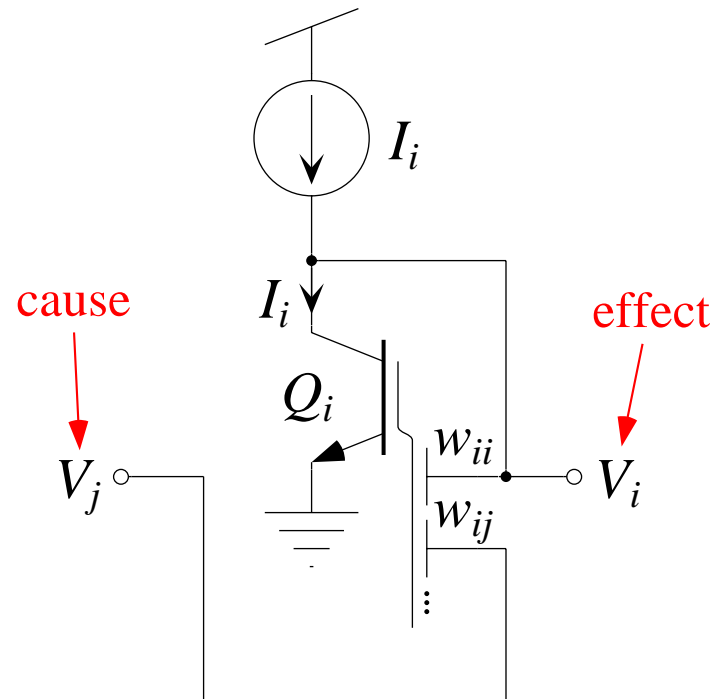
$$\Rightarrow V_i = \frac{U_T}{w_{ii}} \log I_i - \dots$$



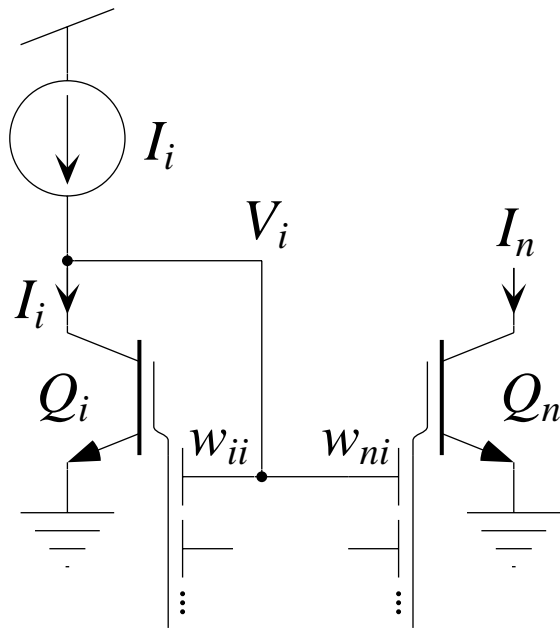
Basic MITE Configurations: Voltage-In, Voltage-Out

$$I_i \propto \exp\left[\frac{w_{ii} V_i + w_{ij} V_j + \dots}{U_T}\right]$$

$$\Rightarrow V_i = \frac{U_T}{w_{ii}} \log I_i - \frac{w_{ij}}{w_{ii}} V_j - \dots$$



Static MITE Networks: Power-Law Circuits



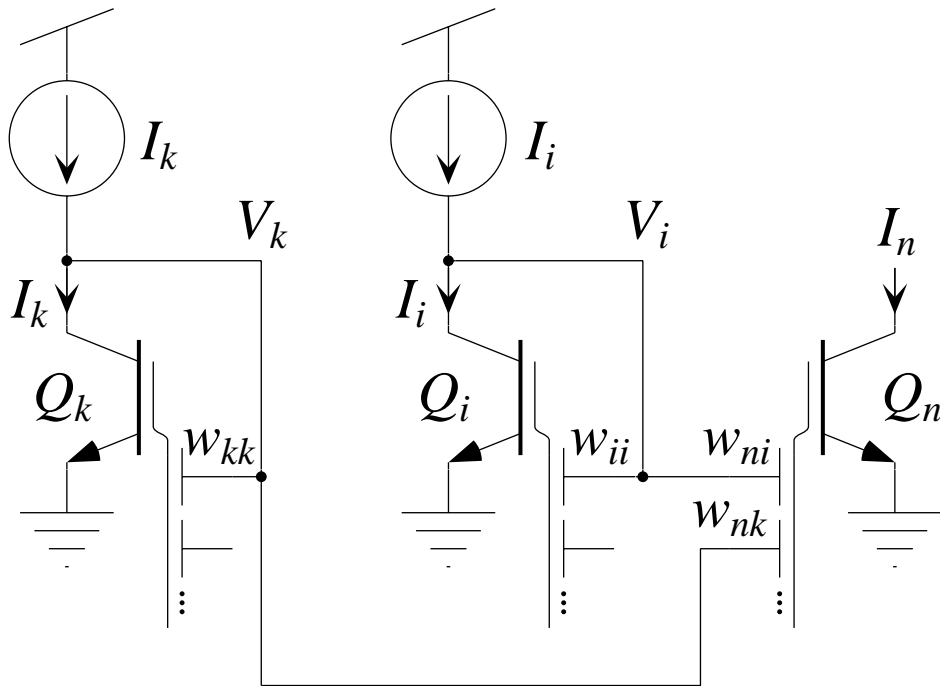
$$I_n \propto \exp\left[\frac{w_{ni} V_i + \dots}{U_T}\right]$$

$$\Rightarrow I_n \propto \exp\left[\frac{w_{ni}}{U_T} \left(\frac{U_T}{w_{ii}} \log I_i - \dots\right)\right]$$

$$\Rightarrow I_n \propto \exp\left[\frac{\cancel{U_T}}{\cancel{U_T}} \frac{w_{ni}}{w_{ii}} \log I_i\right]$$

$$\Rightarrow I_n \propto I_i^{\frac{w_{ni}}{w_{ii}}}$$

Static MITE Networks: Product-of-Power-Law Circuits



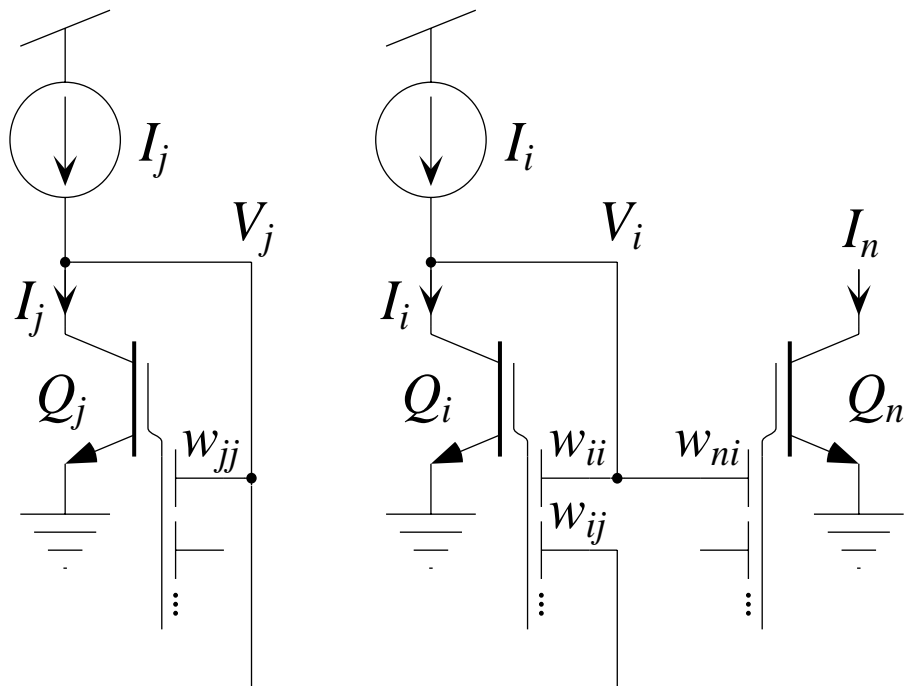
$$I_n \propto \exp\left[\frac{w_{ni} V_i}{U_T}\right] \exp\left[\frac{w_{nk} V_k}{U_T}\right]$$

$$\Rightarrow I_n \propto \exp\left[\frac{w_{ni}}{U_T} \left(\frac{U_T}{w_{ii}} \log I_i - \dots\right)\right] \times \exp\left[\frac{w_{nk}}{U_T} \left(\frac{U_T}{w_{kk}} \log I_k - \dots\right)\right]$$

$$\Rightarrow I_n \propto \exp\left[\frac{\cancel{U_T} w_{ni}}{\cancel{U_T} w_{ii}} \log I_i\right] \times \exp\left[\frac{\cancel{U_T} w_{nk}}{\cancel{U_T} w_{kk}} \log I_k\right]$$

$$\Rightarrow I_n \propto I_i^{\frac{w_{ni}}{w_{ii}}} \times I_k^{\frac{w_{nk}}{w_{kk}}}$$

Static MITE Networks: Quotient-of-Power-Law Circuits



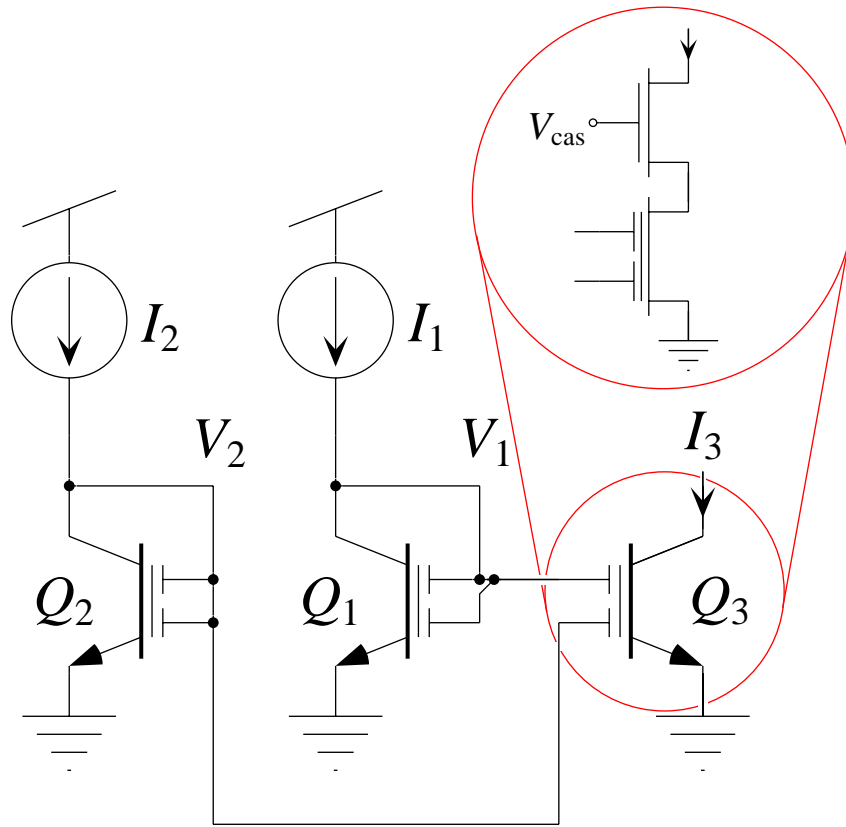
$$I_n \propto \exp\left[\frac{w_{ni} V_i + \dots}{U_T}\right]$$

$$\Rightarrow I_n \propto \exp\left[\frac{w_{ni}}{U_T} \left(\frac{U_T}{w_{ii}} \log I_i - \frac{w_{ij}}{w_{ii}} \left(\frac{U_T}{w_{jj}} \log I_j - \dots \right) \dots \right)\right]$$

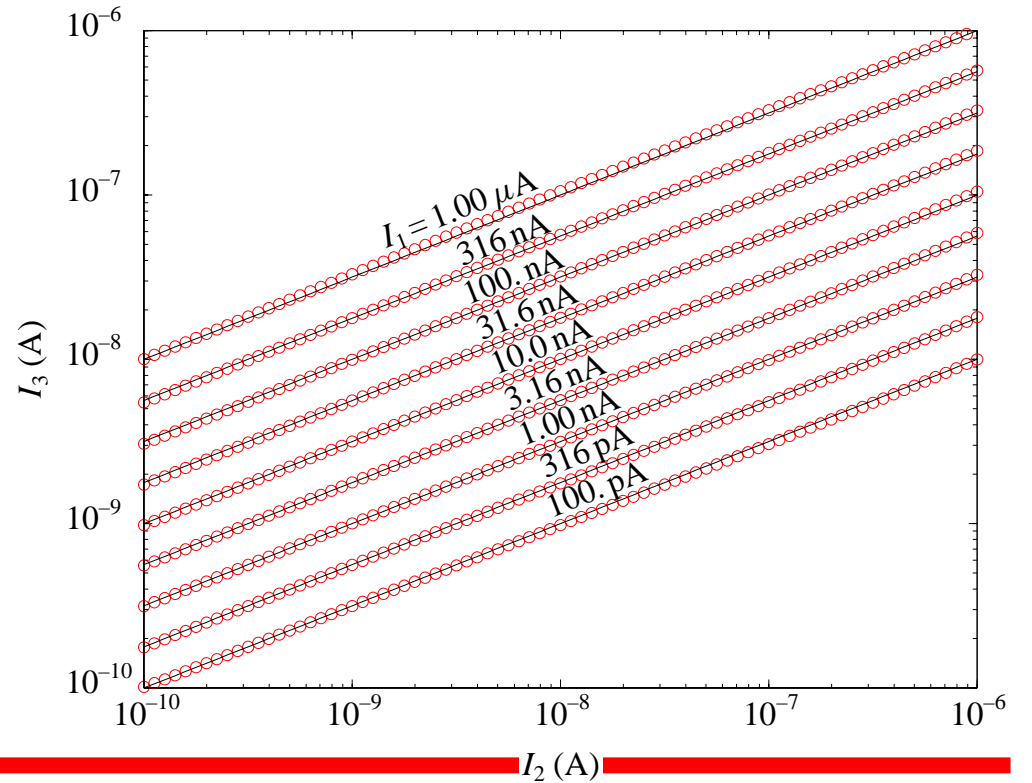
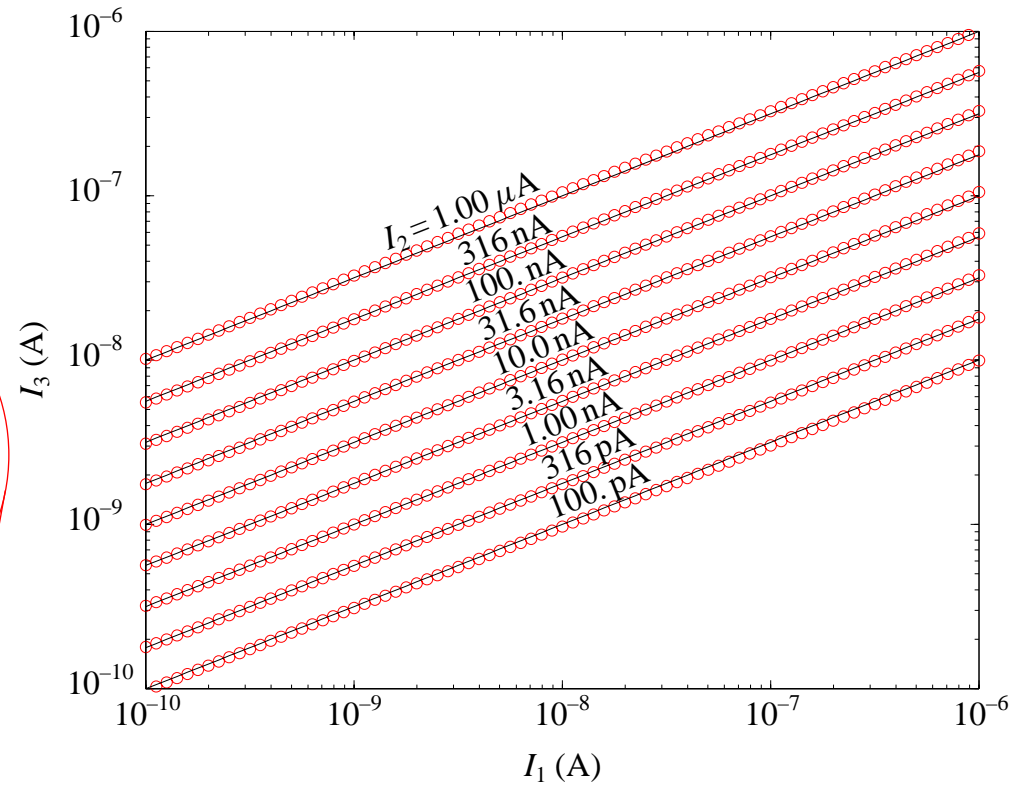
$$\Rightarrow I_n \propto \exp\left[\frac{\cancel{U_T} w_{ni}}{\cancel{U_T} w_{ii}} \log I_i\right] \times \exp\left[-\frac{\cancel{U_T} w_{ni} w_{ij}}{\cancel{U_T} w_{ii} w_{jj}} \log I_j\right]$$

$$\Rightarrow I_n \propto I_i^{\frac{w_{ni}}{w_{ii}}} \times I_j^{-\frac{w_{ni} w_{ij}}{w_{ii} w_{jj}}} \Rightarrow I_n \propto I_i^{\frac{w_{ni}}{w_{ii}}} \div I_j^{\frac{w_{ni} w_{ij}}{w_{ii} w_{jj}}}$$

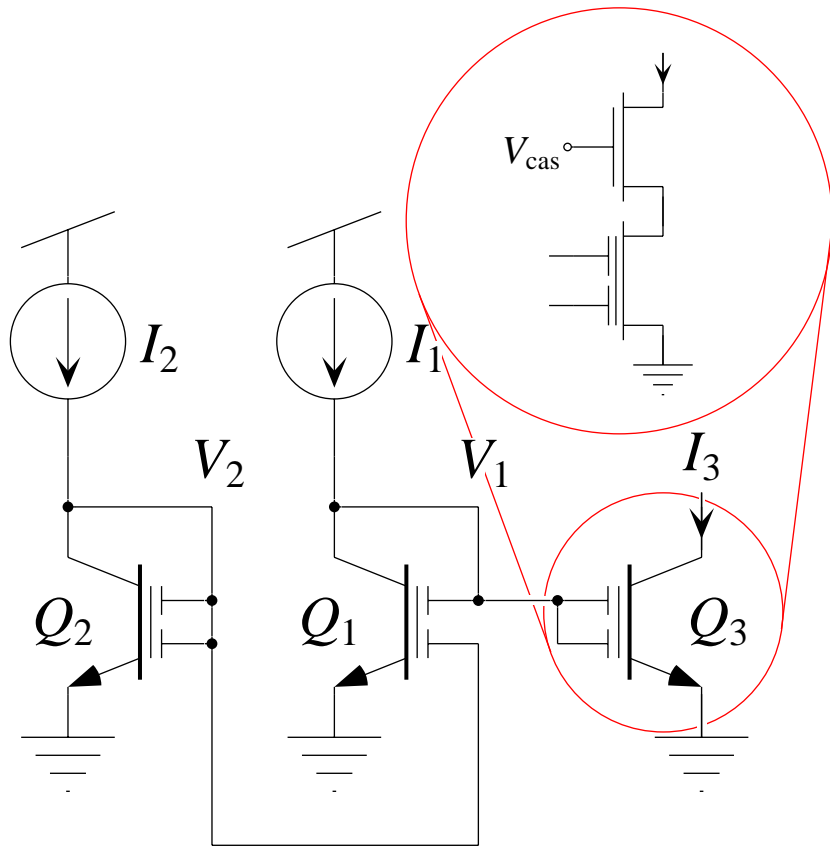
MITE Networks: Geometric Mean



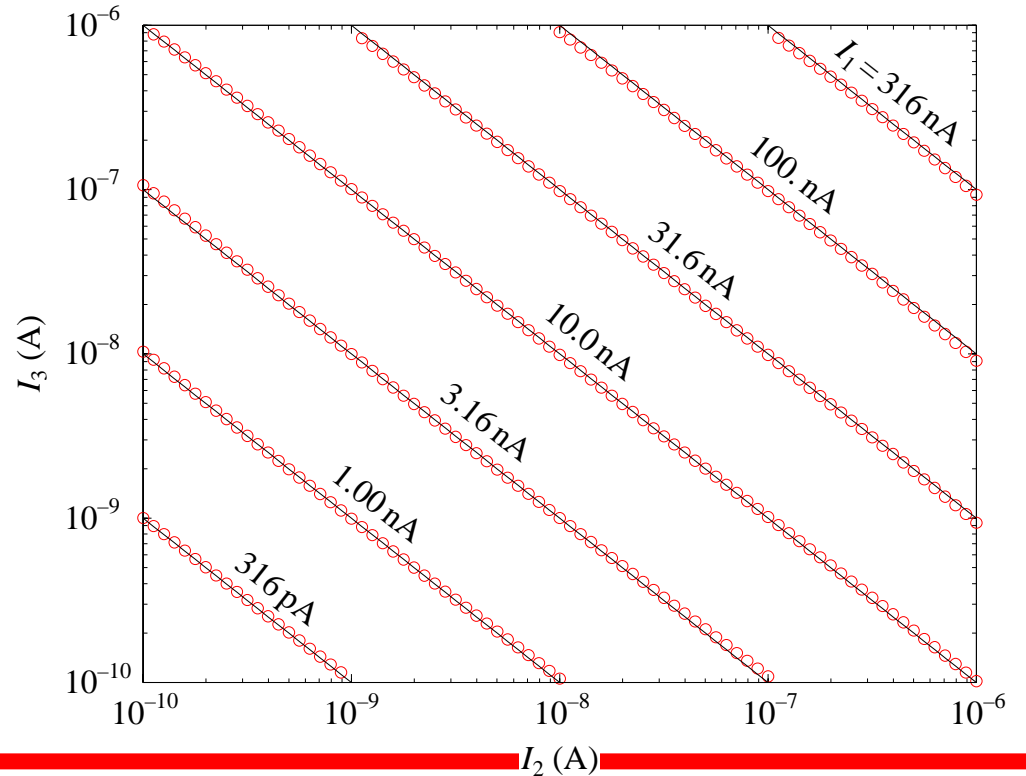
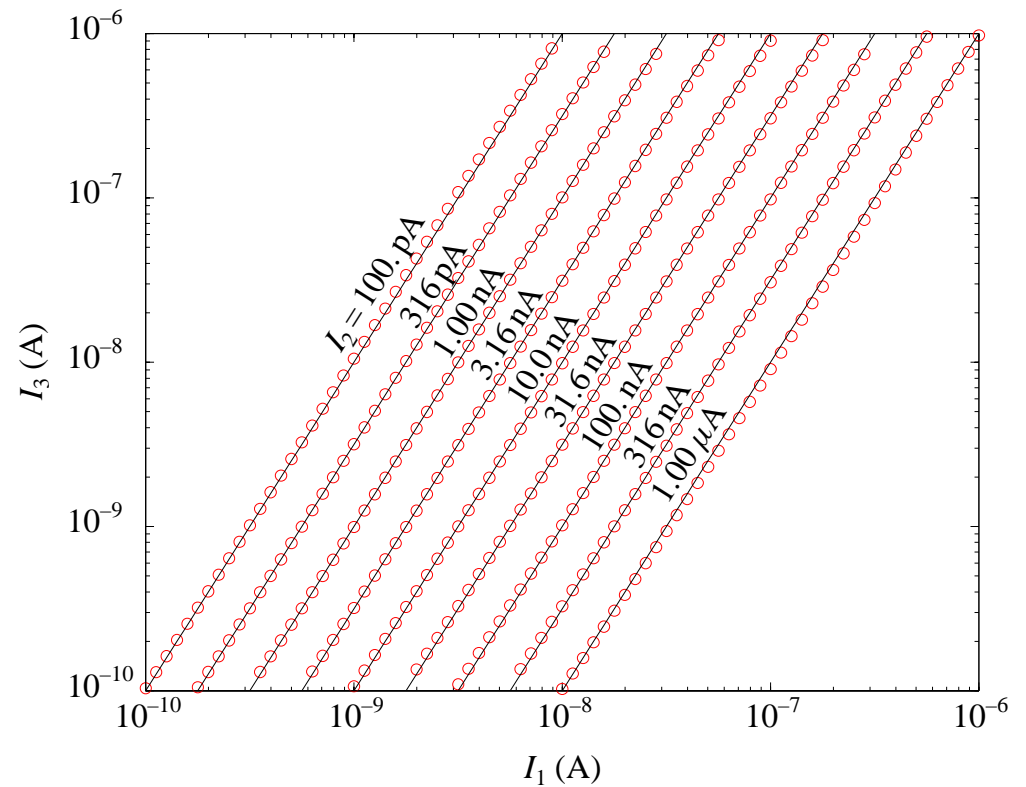
$$I_3 = \sqrt{I_1 I_2}$$



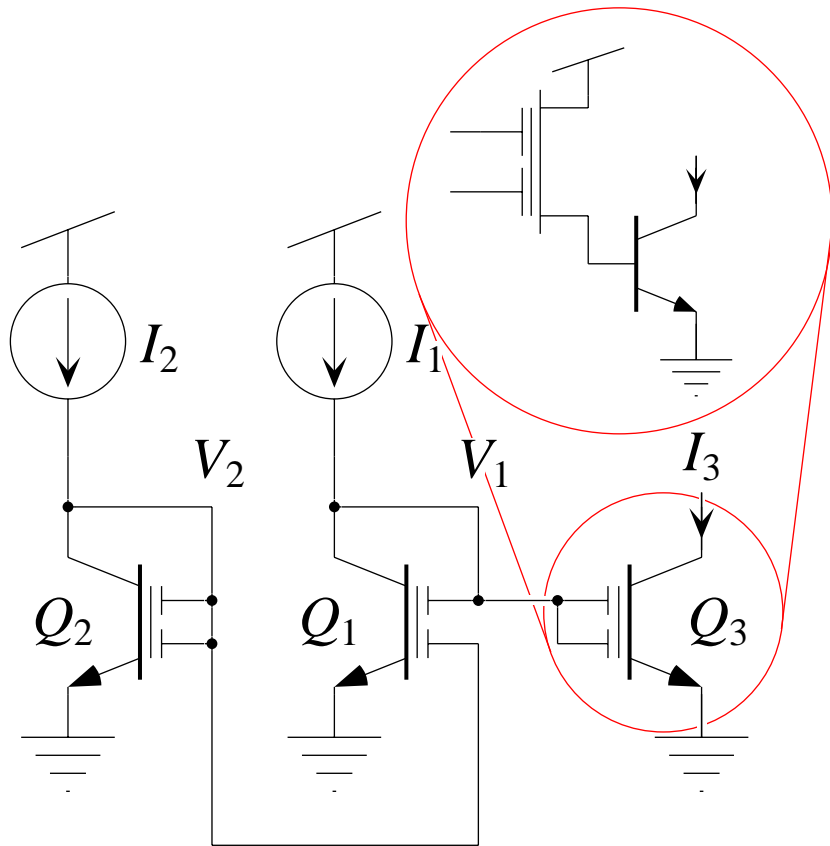
MITE Networks: Square/Reciprocal



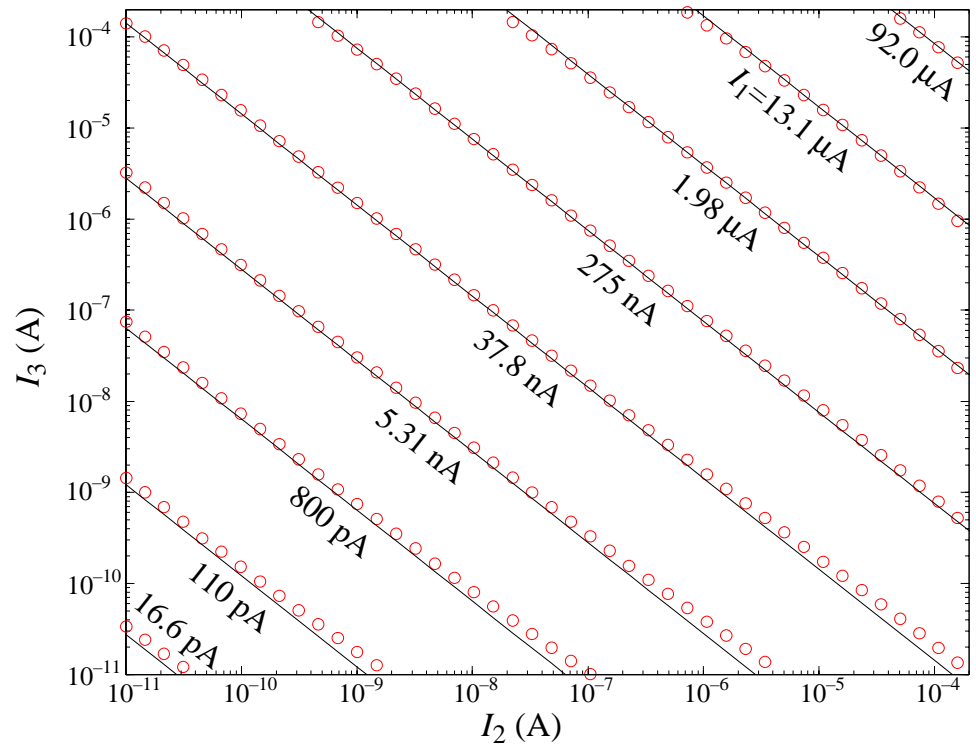
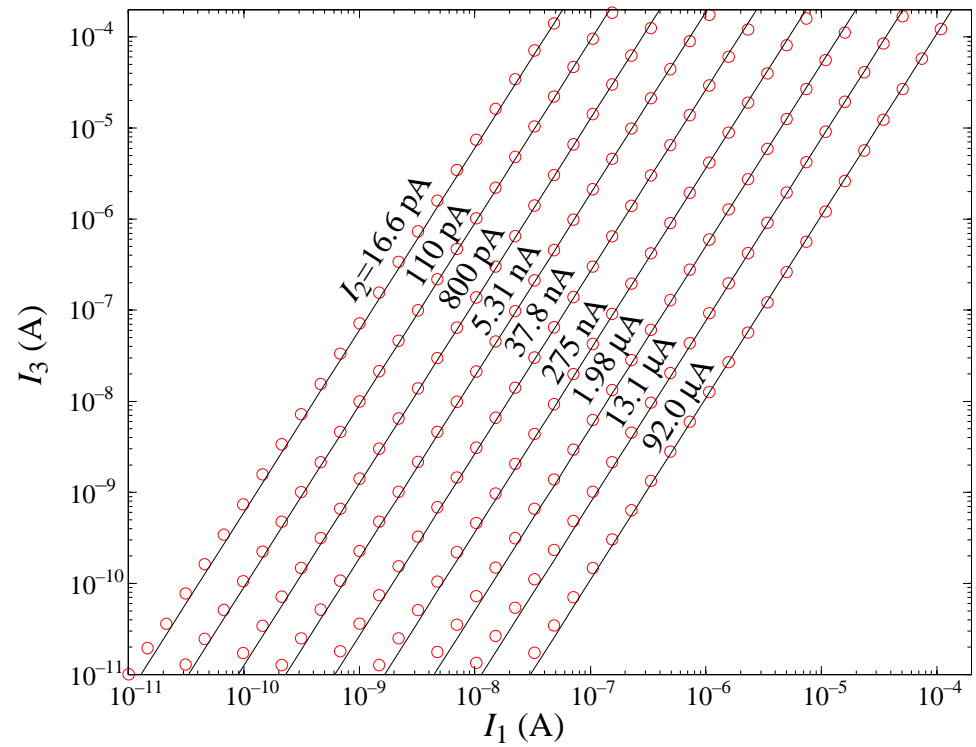
$$I_3 = \frac{I_1^2}{I_2}$$



MITE Networks: Square/Reciprocal



$$I_3 = \frac{I_1^2}{I_2}$$



ABC's of MITE Network Synthesis: Acquiring a set of TL Loop Equations

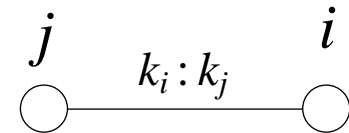
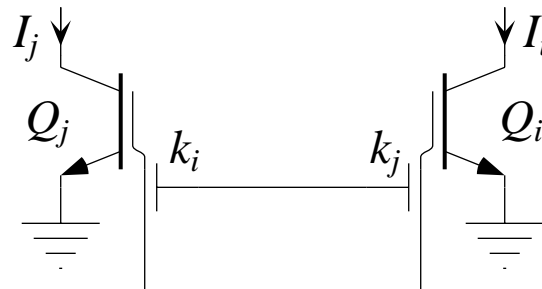
- ▶ Begin with a suitable relationship to implement using MITE networks.
- ▶ Represent the variables in terms of the ratio of positive signal currents to a unit current, I_u .
- ▶ From the original relationship and the signal representations, derive a set of TL loop equations.

ABC's of MITE Network Synthesis: Beginning the Network

- ▶ Begin with a TL loop equation:

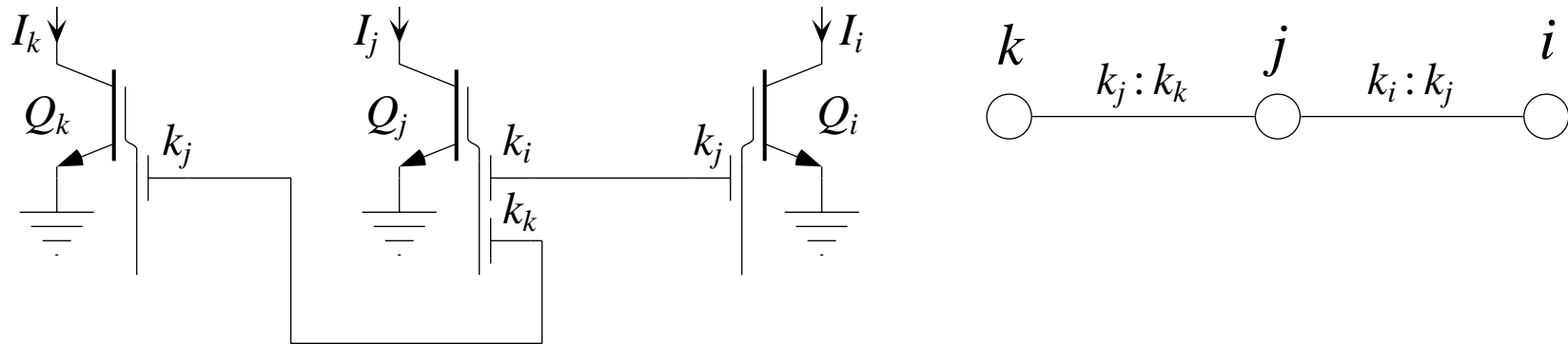
$$\prod_{n \in \text{"CW"}} I_n^{k_n} = \prod_{n \in \text{"CCW"}} I_n^{k_n}$$

- ▶ Pick a current from each set (e.g., I_i from "CW" and I_j from "CCW"), make a new MITE for each, make a new node in the circuit, and couple it into MITE Q_i through k_j unit inputs and into MITE Q_j through k_i unit inputs. If k_i and k_j have a factor in common, they can both be divided by that factor in determining the number of unit inputs.



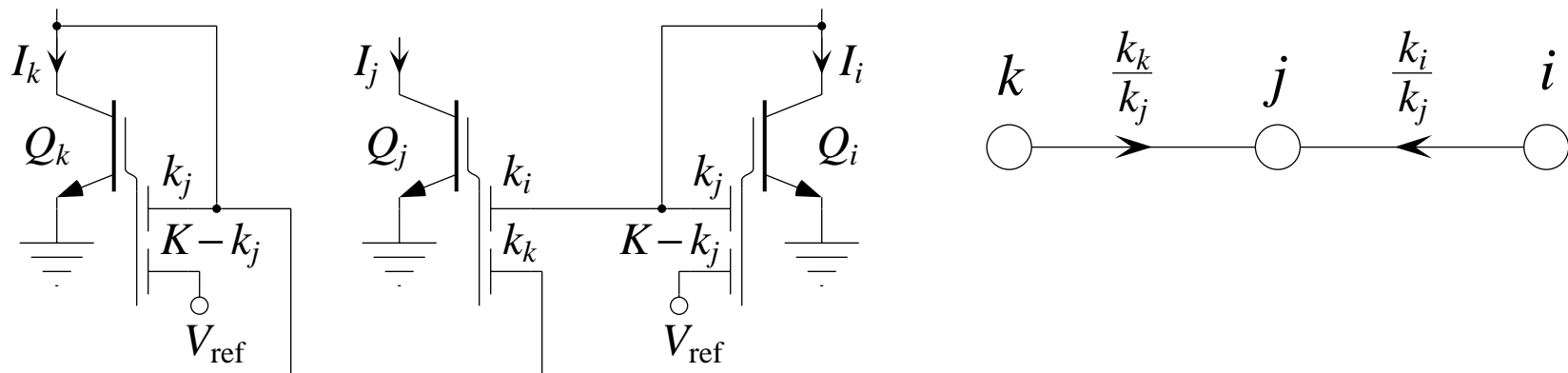
ABC's of MITE Network Synthesis: Building the Network

- ▶ For each additional current (e.g., I_k from “CW”), make a new MITE and connect it to an existing MITE whose current is from the **opposite** set (e.g., I_j from “CCW”), by making a new node and coupling it into MITE Q_k through k_j unit inputs and into MITE Q_j through k_k unit inputs. If k_j and k_k have a factor in common, they can both be divided by that factor in determining the number of unit inputs.



ABC's of MITE Network Synthesis: Biasing the Network

- Bias the MITE network by diode connecting those MITEs whose currents are inputs.

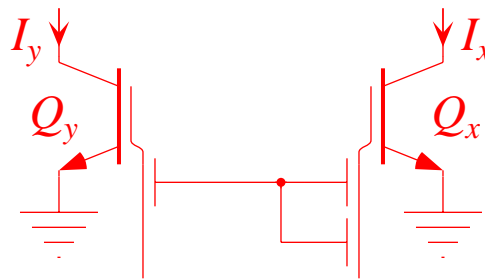


MITE Network Synthesis: Acquiring a TL Equation

$$x = y^2 \quad \Rightarrow \quad \left(\frac{I_x}{I_u} \right) = \left(\frac{I_y}{I_u} \right)^2 \quad \Rightarrow \quad \underbrace{I_x I_u}_{\text{“CW”}} = \underbrace{I_y^2}_{\text{“CCW”}}$$

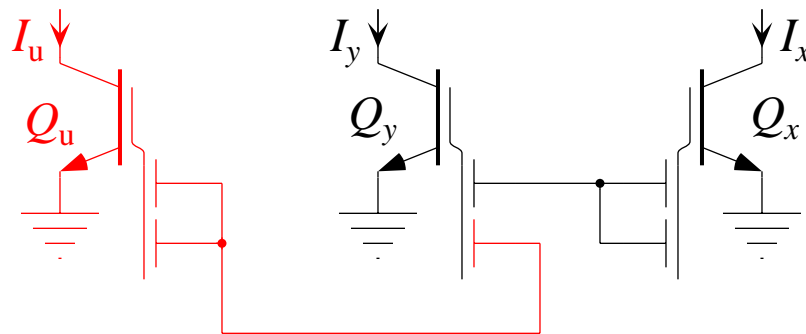
MITE Network Synthesis: Beginning the Network

$$x = y^2 \quad \Rightarrow \quad \left(\frac{I_x}{I_u} \right) = \left(\frac{I_y}{I_u} \right)^2 \quad \Rightarrow \quad \underbrace{I_x I_u}_{\text{“CW”}} = \underbrace{I_y^2}_{\text{“CCW”}}$$



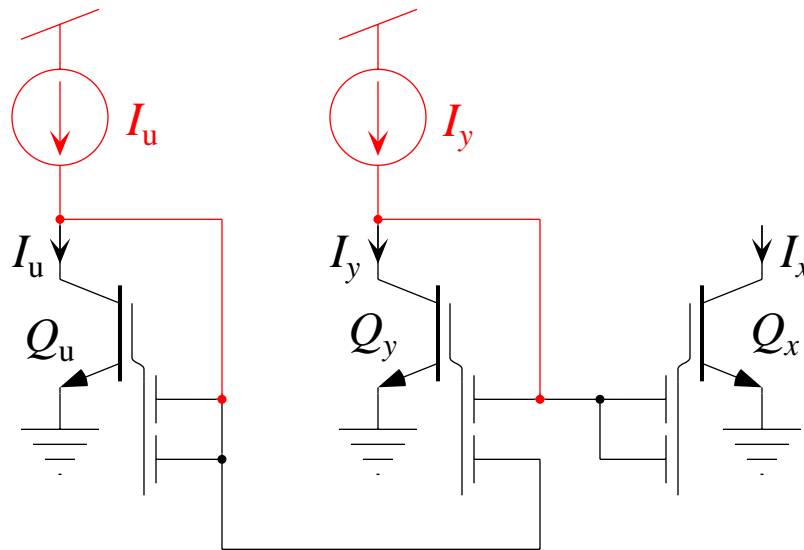
MITE Network Synthesis: Building the Network

$$x = y^2 \quad \Rightarrow \quad \left(\frac{I_x}{I_u} \right) = \left(\frac{I_y}{I_u} \right)^2 \quad \Rightarrow \quad \underbrace{I_x I_u}_{\text{“CW”}} = \underbrace{I_y^2}_{\text{“CCW”}}$$



MITE Network Synthesis: Biasing the Network

$$x = y^2 \quad \Rightarrow \quad \left(\frac{I_x}{I_u} \right) = \left(\frac{I_y}{I_u} \right)^2 \quad \Rightarrow \quad \boxed{I_x = \frac{I_y^2}{I_u}}$$

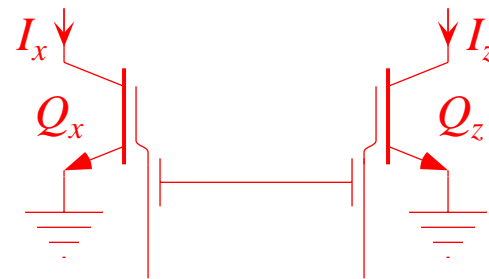


MITE Network Synthesis: Acquiring a TL Equation

$$z = xy \Rightarrow \begin{pmatrix} I_z \\ I_u \end{pmatrix} = \begin{pmatrix} I_x \\ I_u \end{pmatrix} \begin{pmatrix} I_y \\ I_u \end{pmatrix} \Rightarrow \underbrace{I_z I_u}_{\text{“CW”}} = \underbrace{I_x I_y}_{\text{“CCW”}}$$

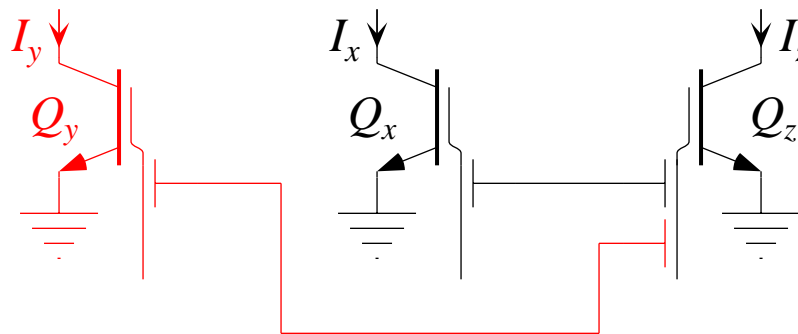
MITE Network Synthesis: Beginning the Network

$$z = xy \Rightarrow \left(\frac{I_z}{I_u} \right) = \left(\frac{I_x}{I_u} \right) \left(\frac{I_y}{I_u} \right) \Rightarrow \underbrace{I_z I_u}_{\text{“CW”}} = \underbrace{I_x I_y}_{\text{“CCW”}}$$



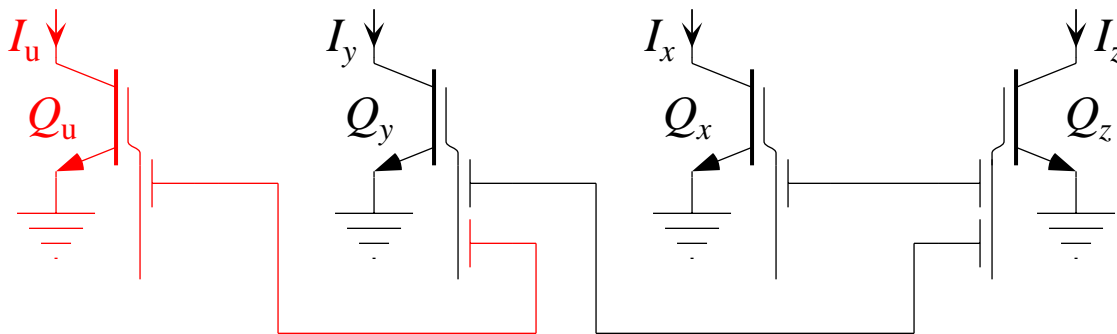
MITE Network Synthesis: Building the Network

$$z = xy \Rightarrow \left(\frac{I_z}{I_u} \right) = \left(\frac{I_x}{I_u} \right) \left(\frac{I_y}{I_u} \right) \Rightarrow \underbrace{I_z I_u}_{\text{“CW”}} = \underbrace{I_x I_y}_{\text{“CCW”}}$$



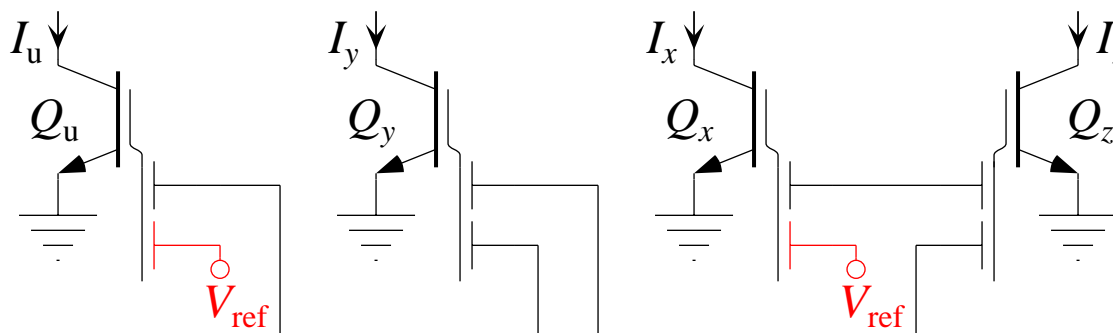
MITE Network Synthesis: Building the Network

$$z = xy \Rightarrow \begin{pmatrix} I_z \\ I_u \end{pmatrix} = \begin{pmatrix} I_x \\ I_u \end{pmatrix} \begin{pmatrix} I_y \\ I_u \end{pmatrix} \Rightarrow \underbrace{I_z I_u}_{\text{“CW”}} = \underbrace{I_x I_y}_{\text{“CCW”}}$$



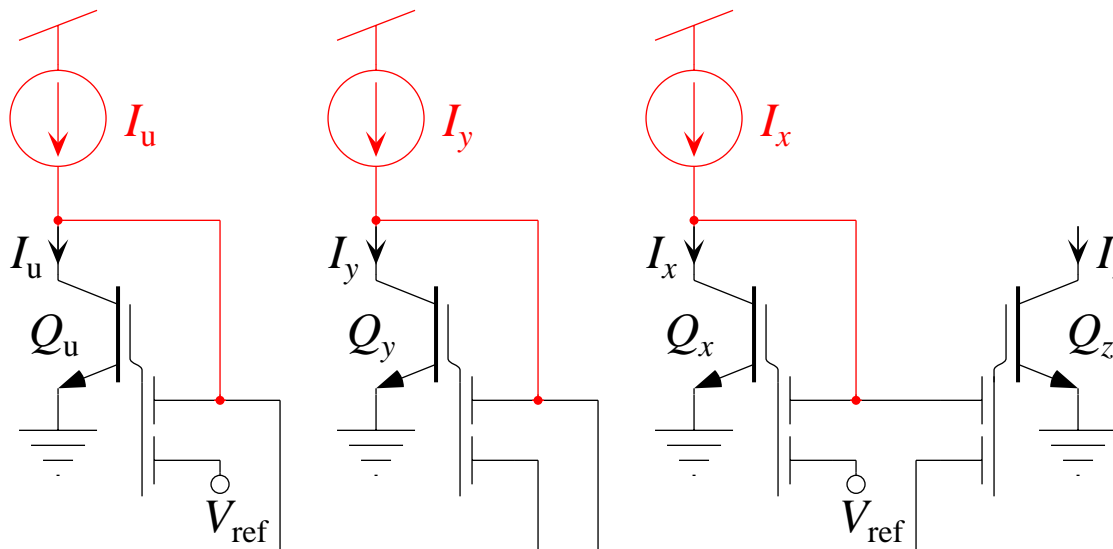
MITE Network Synthesis: Balancing the Network

$$z = xy \Rightarrow \left(\frac{I_z}{I_u} \right) = \left(\frac{I_x}{I_u} \right) \left(\frac{I_y}{I_u} \right) \Rightarrow \underbrace{I_z I_u}_{\text{“CW”}} = \underbrace{I_x I_y}_{\text{“CCW”}}$$



MITE Network Synthesis: Biasing the Network

$$z = xy \Rightarrow \begin{pmatrix} I_z \\ I_u \end{pmatrix} = \begin{pmatrix} I_x \\ I_u \end{pmatrix} \begin{pmatrix} I_y \\ I_u \end{pmatrix} \Rightarrow \boxed{I_z = \frac{I_x I_y}{I_u}}$$



MITE Network Synthesis: Vector Magnitude Circuit

$$r = \sqrt{x^2 + y^2} \Rightarrow \frac{I_r}{I_u} = \sqrt{\left(\frac{I_x}{I_u}\right)^2 + \left(\frac{I_y}{I_u}\right)^2} \Rightarrow I_r^2 = I_x^2 + I_y^2$$

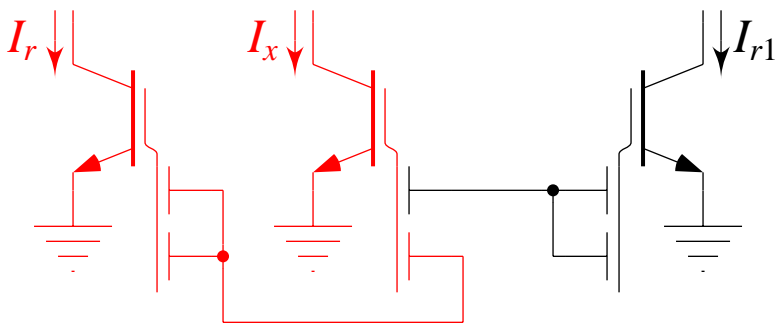
$$\Rightarrow I_r = \frac{I_x^2}{I_r} + \frac{I_y^2}{I_r} \Rightarrow I_r I_{r1} = I_x^2, I_r I_{r2} = I_y^2, \text{ and } I_r = I_{r1} + I_{r2}$$

$\underbrace{I_r}_{I_{r1}} \quad \underbrace{I_r}_{I_{r2}}$

MITE Network Synthesis: Vector Magnitude Circuit

$$r = \sqrt{x^2 + y^2} \Rightarrow \frac{I_r}{I_u} = \sqrt{\left(\frac{I_x}{I_u}\right)^2 + \left(\frac{I_y}{I_u}\right)^2} \Rightarrow I_r^2 = I_x^2 + I_y^2$$

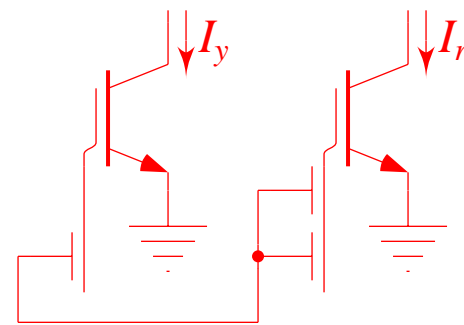
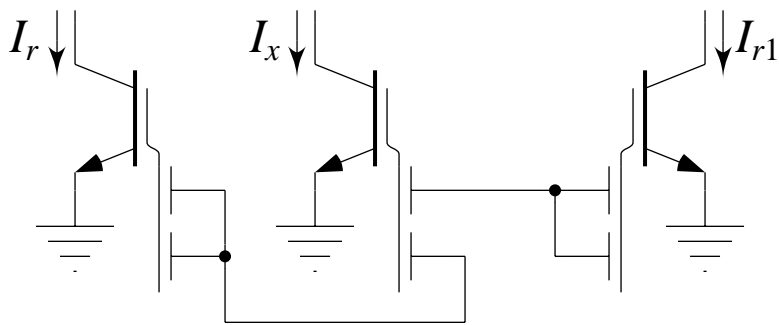
$$\Rightarrow I_r = \frac{I_x^2}{I_r} + \frac{I_y^2}{I_r} \Rightarrow I_r I_{r1} = I_x^2, I_r I_{r2} = I_y^2, \text{ and } I_r = I_{r1} + I_{r2}$$



MITE Network Synthesis: Vector Magnitude Circuit

$$r = \sqrt{x^2 + y^2} \Rightarrow \frac{I_r}{I_u} = \sqrt{\left(\frac{I_x}{I_u}\right)^2 + \left(\frac{I_y}{I_u}\right)^2} \Rightarrow I_r^2 = I_x^2 + I_y^2$$

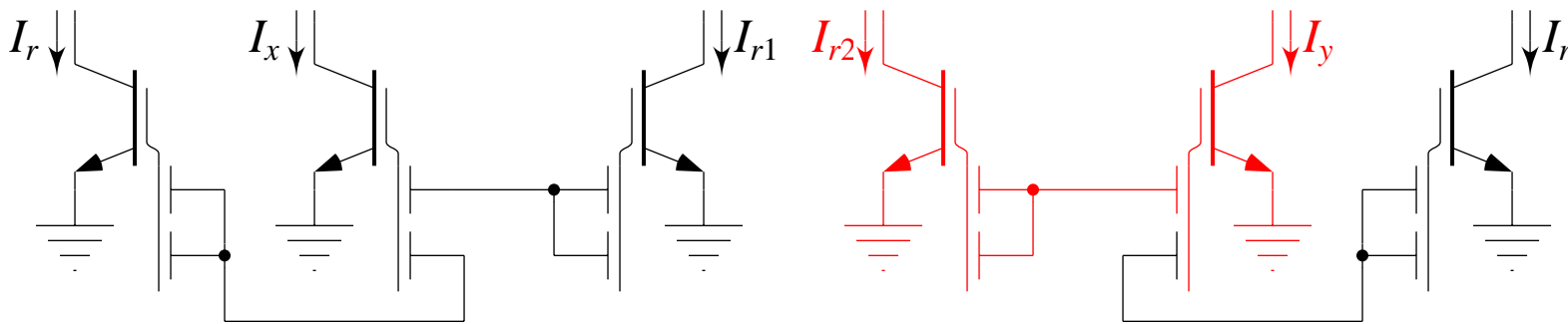
$$\Rightarrow I_r = \underbrace{\frac{I_x^2}{I_r}}_{I_{r1}} + \underbrace{\frac{I_y^2}{I_r}}_{I_{r2}} \Rightarrow I_r I_{r1} = I_x^2, I_r I_{r2} = I_y^2, \text{ and } I_r = I_{r1} + I_{r2}$$



MITE Network Synthesis: Vector Magnitude Circuit

$$r = \sqrt{x^2 + y^2} \Rightarrow \frac{I_r}{I_u} = \sqrt{\left(\frac{I_x}{I_u}\right)^2 + \left(\frac{I_y}{I_u}\right)^2} \Rightarrow I_r^2 = I_x^2 + I_y^2$$

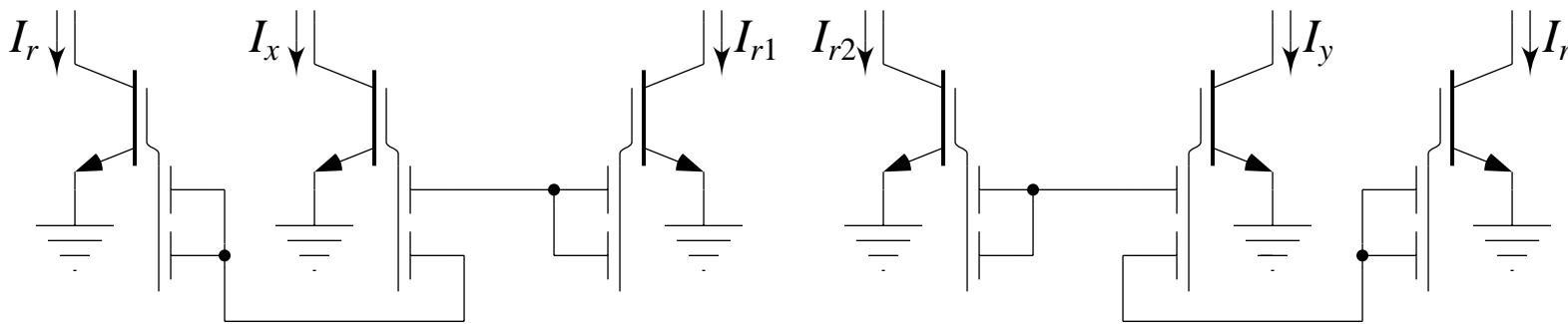
$$\Rightarrow I_r = \underbrace{\frac{I_x^2}{I_r}}_{I_{r1}} + \underbrace{\frac{I_y^2}{I_r}}_{I_{r2}} \Rightarrow I_r I_{r1} = I_x^2, I_r I_{r2} = I_y^2, \text{ and } I_r = I_{r1} + I_{r2}$$



MITE Network Synthesis: Vector Magnitude Circuit

$$r = \sqrt{x^2 + y^2} \Rightarrow \frac{I_r}{I_u} = \sqrt{\left(\frac{I_x}{I_u}\right)^2 + \left(\frac{I_y}{I_u}\right)^2} \Rightarrow I_r^2 = I_x^2 + I_y^2$$

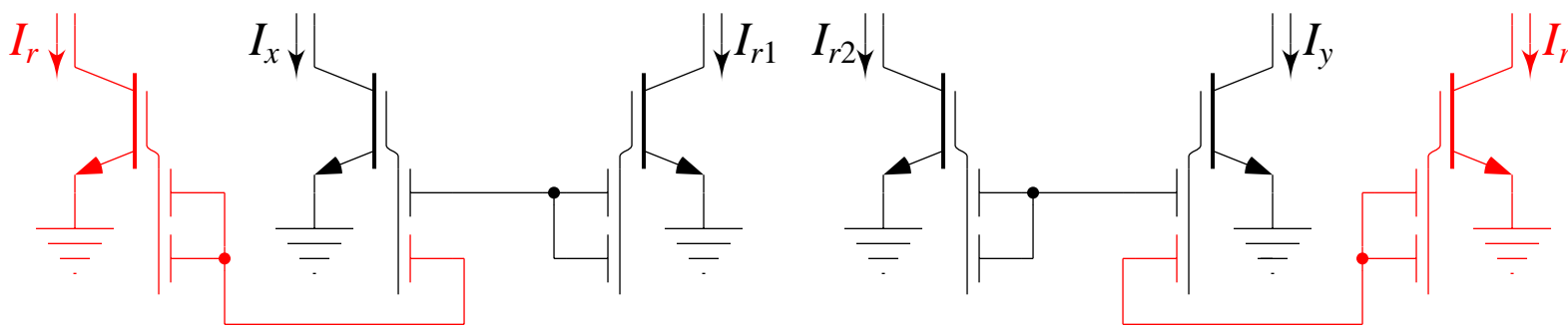
$$\Rightarrow I_r = \underbrace{\frac{I_x^2}{I_r}}_{I_{r1}} + \underbrace{\frac{I_y^2}{I_r}}_{I_{r2}} \Rightarrow I_r I_{r1} = I_x^2, I_r I_{r2} = I_y^2, \text{ and } I_r = I_{r1} + I_{r2}$$



MITE Network Synthesis: Vector Magnitude Circuit

$$r = \sqrt{x^2 + y^2} \Rightarrow \frac{I_r}{I_u} = \sqrt{\left(\frac{I_x}{I_u}\right)^2 + \left(\frac{I_y}{I_u}\right)^2} \Rightarrow I_r^2 = I_x^2 + I_y^2$$

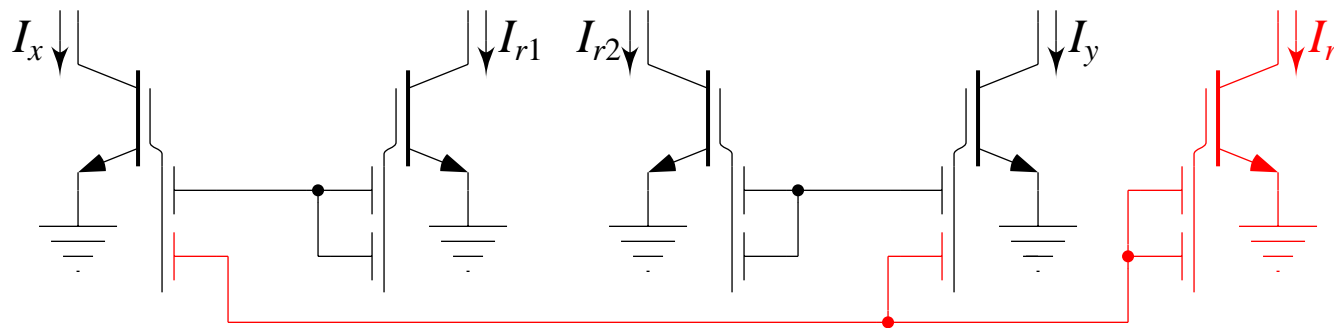
$$\Rightarrow I_r = \frac{I_x^2}{I_r} + \frac{I_y^2}{I_r} \Rightarrow I_r I_{r1} = I_x^2, I_r I_{r2} = I_y^2, \text{ and } I_r = I_{r1} + I_{r2}$$



MITE Network Synthesis: Vector Magnitude Circuit

$$r = \sqrt{x^2 + y^2} \Rightarrow \frac{I_r}{I_u} = \sqrt{\left(\frac{I_x}{I_u}\right)^2 + \left(\frac{I_y}{I_u}\right)^2} \Rightarrow I_r^2 = I_x^2 + I_y^2$$

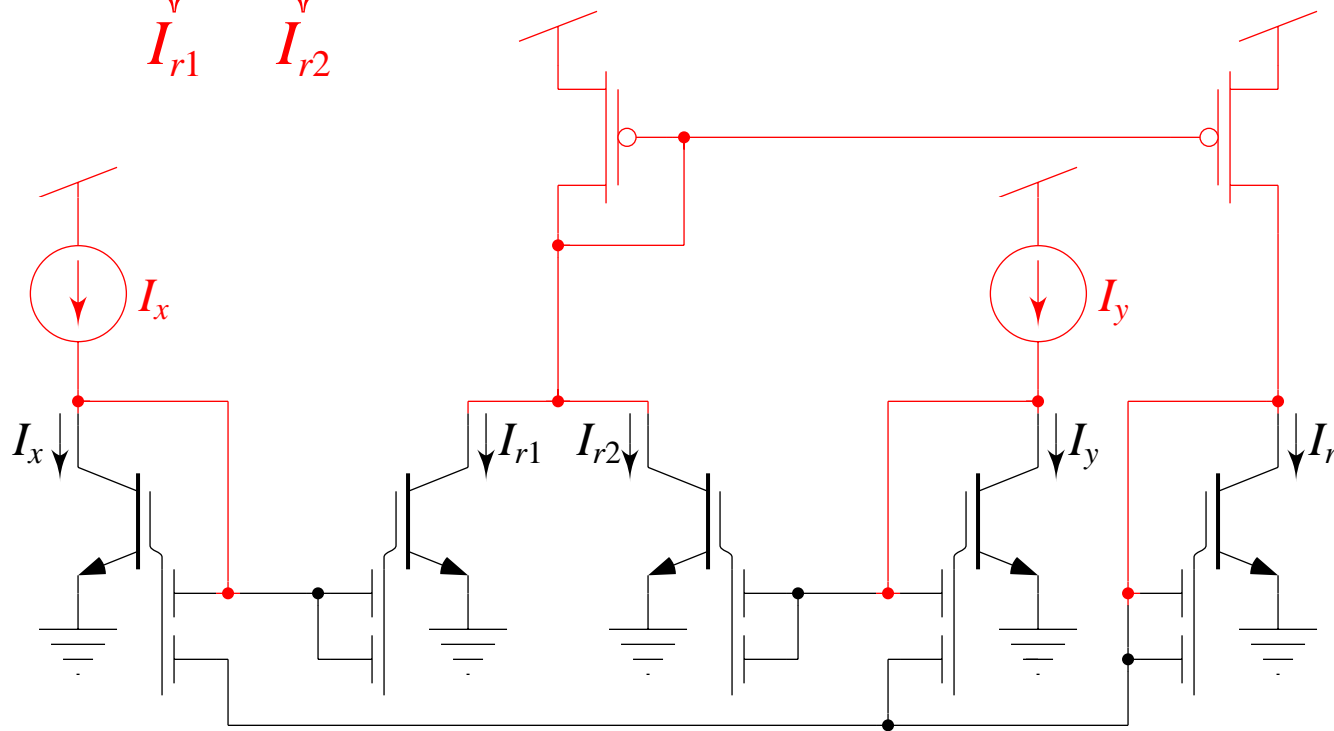
$$\Rightarrow I_r = \underbrace{\frac{I_x^2}{I_r}}_{I_{r1}} + \underbrace{\frac{I_y^2}{I_r}}_{I_{r2}} \Rightarrow I_r I_{r1} = I_x^2, I_r I_{r2} = I_y^2, \text{ and } I_r = I_{r1} + I_{r2}$$



MITE Network Synthesis: Vector Magnitude Circuit

$$r = \sqrt{x^2 + y^2} \Rightarrow \frac{I_r}{I_u} = \sqrt{\left(\frac{I_x}{I_u}\right)^2 + \left(\frac{I_y}{I_u}\right)^2} \Rightarrow I_r^2 = I_x^2 + I_y^2$$

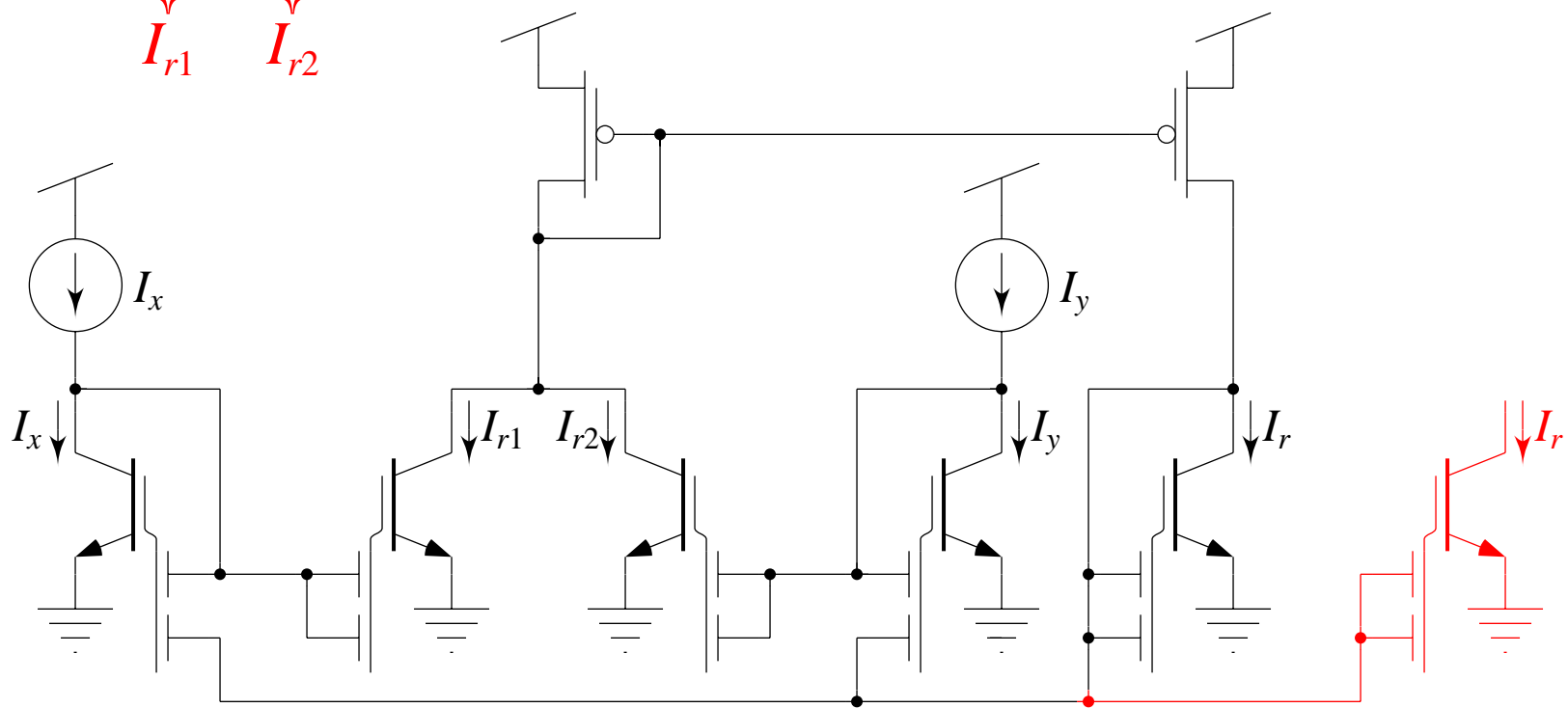
$$\Rightarrow I_r = \underbrace{\frac{I_x^2}{I_r}}_{I_{r1}} + \underbrace{\frac{I_y^2}{I_r}}_{I_{r2}} \Rightarrow I_r I_{r1} = I_x^2, I_r I_{r2} = I_y^2, \text{ and } I_r = I_{r1} + I_{r2}$$



MITE Network Synthesis: Vector Magnitude Circuit

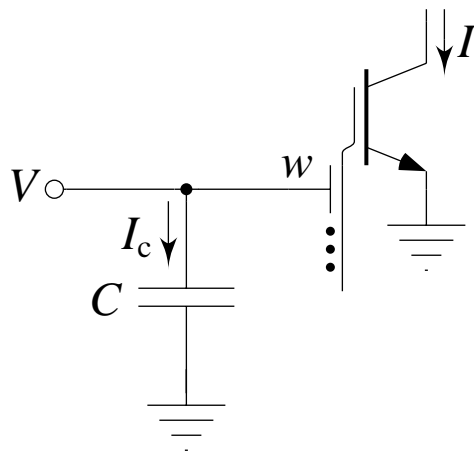
$$r = \sqrt{x^2 + y^2} \Rightarrow \frac{I_r}{I_u} = \sqrt{\left(\frac{I_x}{I_u}\right)^2 + \left(\frac{I_y}{I_u}\right)^2} \Rightarrow I_r^2 = I_x^2 + I_y^2$$

$$\Rightarrow I_r = \frac{I_x^2}{I_r} + \frac{I_y^2}{I_r} \Rightarrow I_r I_{r1} = I_x^2, I_r I_{r2} = I_y^2, \text{ and } I_r = I_{r1} + I_{r2}$$



Implementing **Dynamic** Constraints: Output Structures

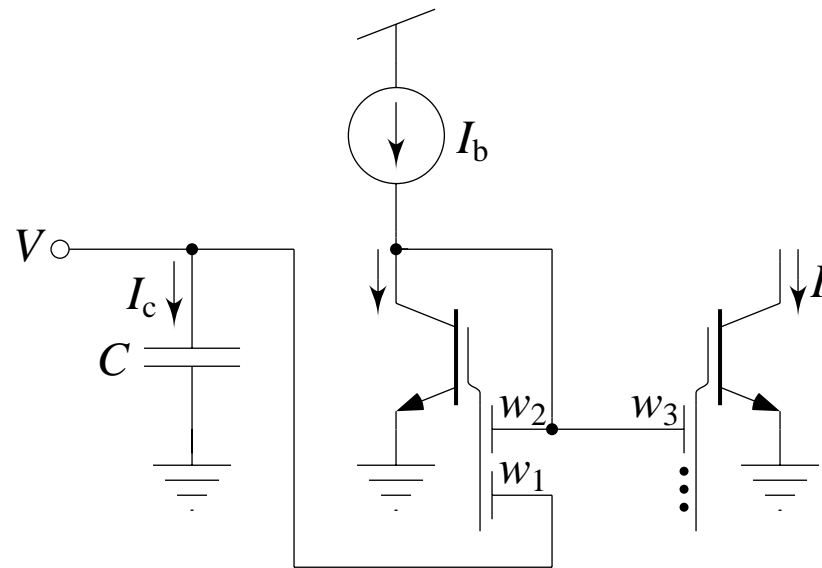
Noninverting



$$I \propto e^{wV/U_T}$$

$$\Rightarrow \frac{\partial I}{\partial V} = \frac{wI}{U_T}$$

Inverting



$$I \propto e^{-wV/U_T}$$

$$\Rightarrow \frac{\partial I}{\partial V} = -\frac{wI}{U_T}$$

$$\text{where } w = \frac{w_1 w_3}{w_2}$$

Dynamic MITE Network Synthesis: First-Order Lowpass Filter

$$\tau \frac{dy}{dt} + y = x \Rightarrow \tau \frac{d\left(\frac{I_y}{I_u}\right)}{dt} + \left(\frac{I_y}{I_u}\right) = \left(\frac{I_x}{I_u}\right) \Rightarrow \tau \frac{dI_y}{dt} + I_y = I_x$$

$$\Rightarrow \tau \frac{dI_y}{dV} \cdot \frac{dV}{dt} + I_y = I_x \Rightarrow \tau \left(-\frac{wI_y}{U_T} \right) \frac{dV}{dt} + I_y = I_x \Rightarrow -\frac{w\tau \overbrace{C}^{I_c}}{U_T C} \cdot \frac{dV}{dt} + 1 = \frac{I_x}{I_y}$$

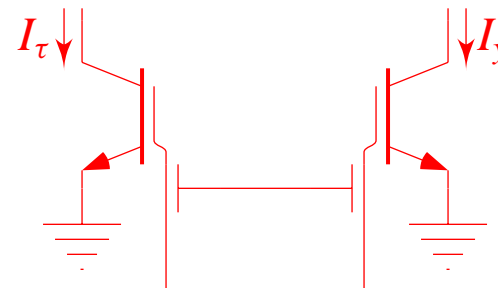
$\underbrace{\hspace{10em}}_{1/I_\tau}$

$$\Rightarrow -\frac{I_c}{I_\tau} + 1 = \frac{I_x}{I_y} \Rightarrow \underbrace{I_\tau - I_c}_{I_p} = \frac{I_\tau I_x}{I_y} \Rightarrow I_p I_y = I_\tau I_x \text{ and } I_\tau = I_p + I_c$$

Dynamic MITE Network Synthesis: First-Order Lowpass Filter

$$\tau \frac{dy}{dt} + y = x \Rightarrow \tau \frac{d\left(\frac{I_y}{I_u}\right) + \left(\frac{I_y}{I_u}\right) = \left(\frac{I_x}{I_u}\right) \Rightarrow \tau \frac{dI_y}{dt} + I_y = I_x$$

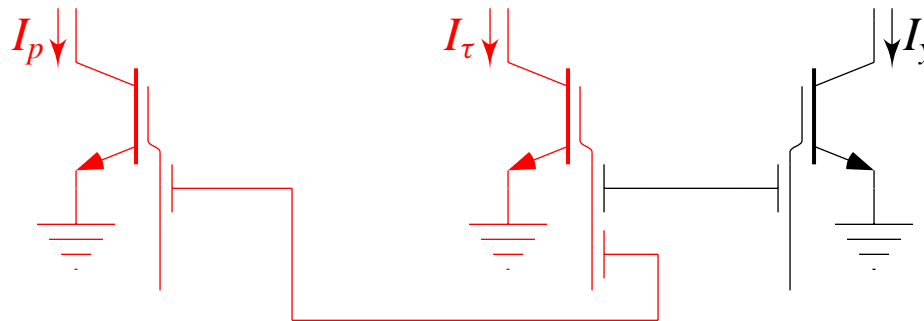
$$\Rightarrow -\underbrace{\frac{w\tau C}{U_T C}}_{1/I_\tau} \cdot \frac{dV}{dt} + 1 = \frac{I_x}{I_y} \Rightarrow \underbrace{I_\tau - I_c}_{I_p} = \frac{I_\tau I_x}{I_y} \Rightarrow I_p I_y = I_\tau I_x \text{ and } I_\tau = I_p + I_c$$



Dynamic MITE Network Synthesis: First-Order Lowpass Filter

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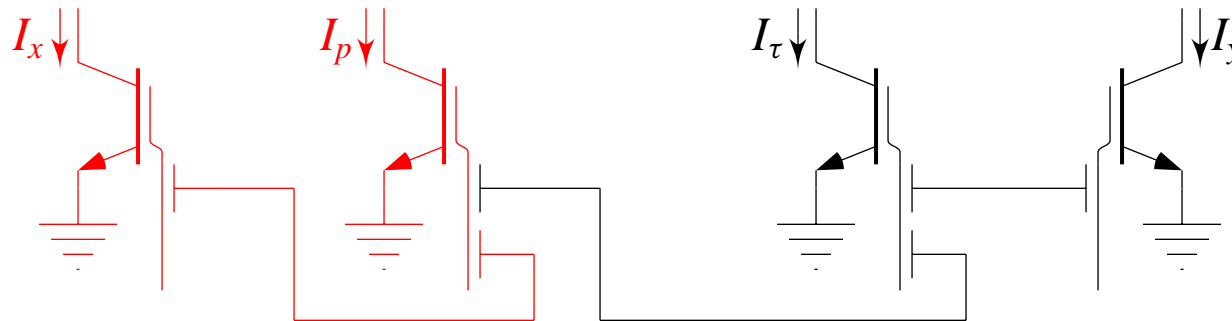
$$\Rightarrow -\underbrace{\frac{w\tau C}{U_T C}}_{1/I_\tau} \cdot \frac{dV}{dt} + 1 = \frac{I_x}{I_y} \Rightarrow \underbrace{I_\tau - I_c}_{I_p} = \frac{I_\tau I_x}{I_y} \Rightarrow I_p I_y = I_\tau I_x \text{ and } I_\tau = I_p + I_c$$



Dynamic MITE Network Synthesis: First-Order Lowpass Filter

$$\tau \frac{dy}{dt} + y = x \Rightarrow \tau \frac{d\left(\frac{I_y}{I_u}\right)}{dt} + \left(\frac{I_y}{I_u}\right) = \left(\frac{I_x}{I_u}\right) \Rightarrow \tau \frac{dI_y}{dt} + I_y = I_x$$

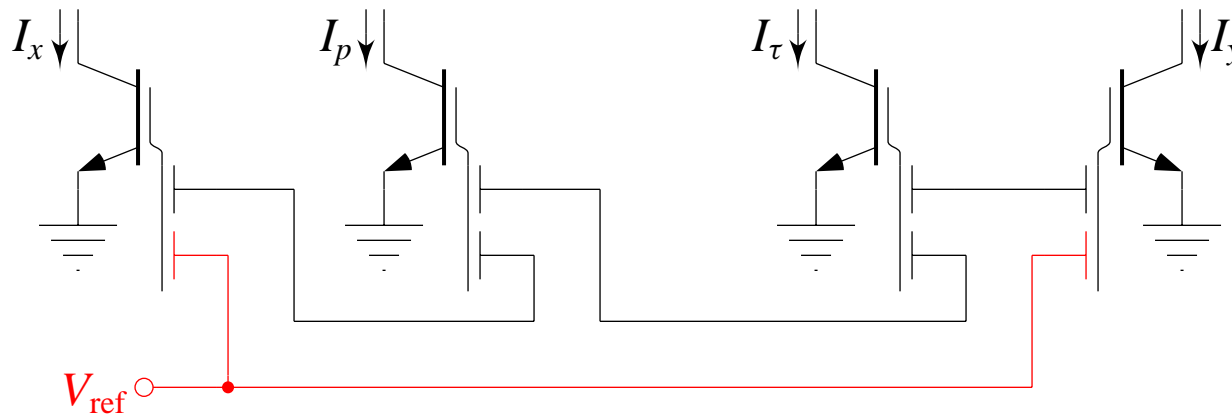
$$\Rightarrow - \underbrace{\frac{w\tau C}{U_T C}}_{1/I_\tau} \cdot \frac{dV}{dt} + 1 = \frac{I_x}{I_y} \Rightarrow \underbrace{I_\tau - I_c}_{I_p} = \frac{I_\tau I_x}{I_y} \Rightarrow I_p I_y = I_\tau I_x \text{ and } I_\tau = I_p + I_c$$



Dynamic MITE Network Synthesis: First-Order Lowpass Filter

$$\tau \frac{dy}{dt} + y = x \Rightarrow \tau \frac{d\left(\frac{I_y}{I_u}\right) + \left(\frac{I_y}{I_u}\right) = \left(\frac{I_x}{I_u}\right) \Rightarrow \tau \frac{dI_y}{dt} + I_y = I_x$$

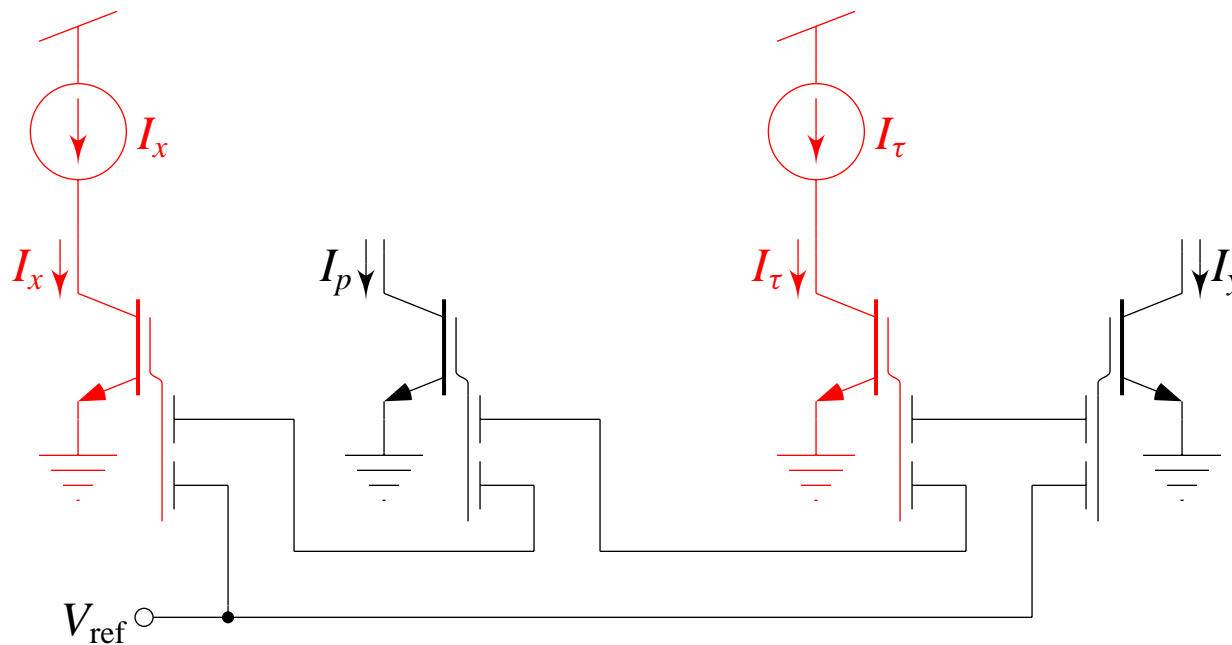
$$\Rightarrow -\underbrace{\frac{w\tau C}{U_T C}}_{1/I_\tau} \cdot \frac{dV}{dt} + 1 = \frac{I_x}{I_y} \Rightarrow \underbrace{I_\tau - I_c}_{I_p} = \frac{I_\tau I_x}{I_y} \Rightarrow I_p I_y = I_\tau I_x \text{ and } I_\tau = I_p + I_c$$



Dynamic MITE Network Synthesis: First-Order Lowpass Filter

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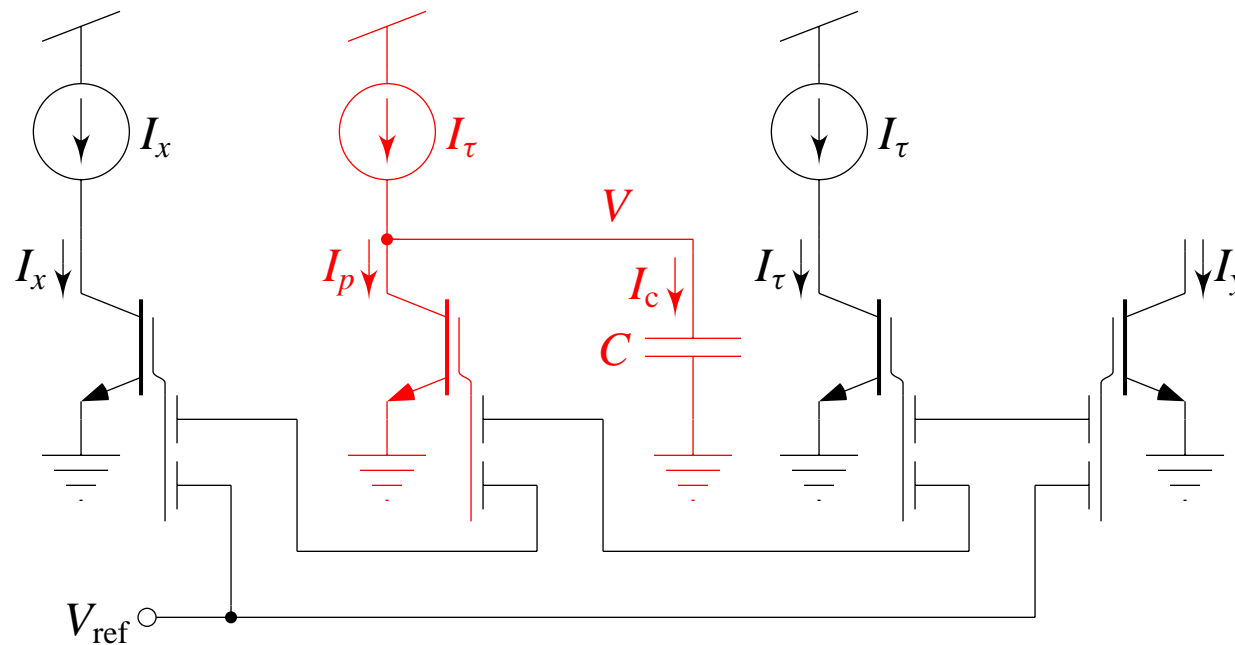
$$\Rightarrow - \underbrace{\frac{w\tau C}{U_T C}}_{1/I_\tau} \cdot \frac{dV}{dt} + 1 = \frac{I_x}{I_y} \Rightarrow \underbrace{I_\tau - I_c}_{I_p} = \frac{I_\tau I_x}{I_y} \Rightarrow I_p I_y = I_\tau I_x \text{ and } I_\tau = I_p + I_c$$



Dynamic MITE Network Synthesis: First-Order Lowpass Filter

$$\tau \frac{dy}{dt} + y = x \Rightarrow \tau \frac{d\left(\frac{I_y}{I_u}\right)}{dt} + \left(\frac{I_y}{I_u}\right) = \left(\frac{I_x}{I_u}\right) \Rightarrow \tau \frac{dI_y}{dt} + I_y = I_x$$

$$\Rightarrow - \underbrace{\frac{w\tau C}{U_T C}}_{1/I_\tau} \cdot \frac{dV}{dt} + 1 = \frac{I_x}{I_y} \Rightarrow \underbrace{I_\tau - I_c}_{I_p} = \frac{I_\tau I_x}{I_y} \Rightarrow I_p I_y = I_\tau I_x \text{ and } I_\tau = I_p + I_c$$



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