Synthesis of Multiple-Input Tanslinear Element Log-Domain Filters

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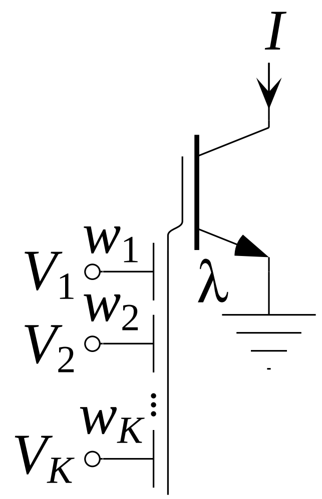
The Multiple-Input Translinear Element

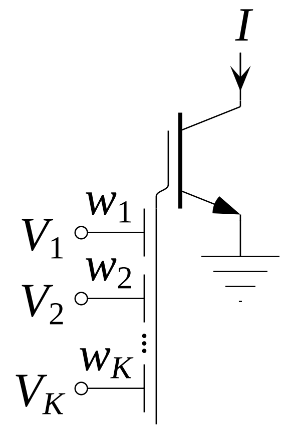
$$I = \lambda I_s \exp\left[\sum_{k=1}^{K} \frac{w_k V_k}{U_T}\right]$$



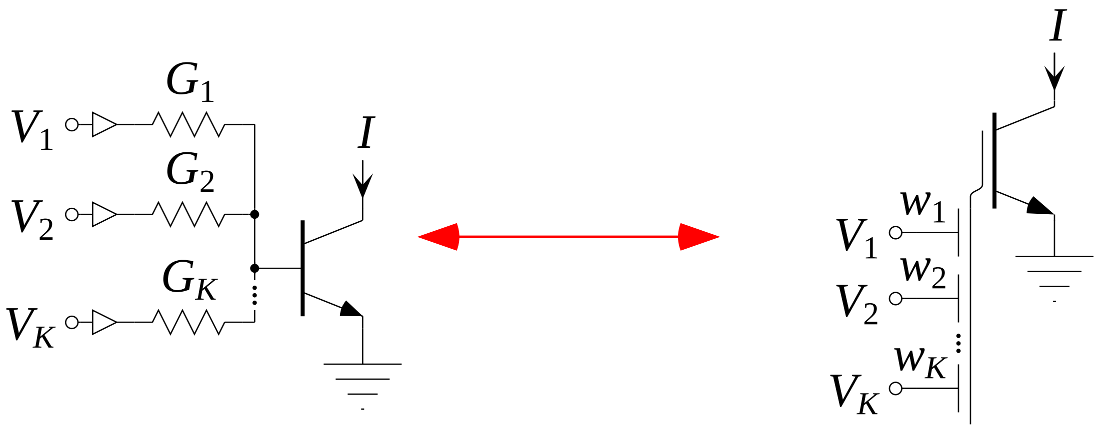
- The MITE has *K* *trans*conductances, each of which is *linear* in the output current, *I*:

$$g_k = \frac{\partial I}{\partial V_k} = \frac{w_k}{U_T}\lambda I_s \exp\left[\sum_{k=1}^{K} \frac{w_k V_k}{U_T}\right] = \frac{w_k}{U_T} I$$

MITE Implementations



MITE Implementations



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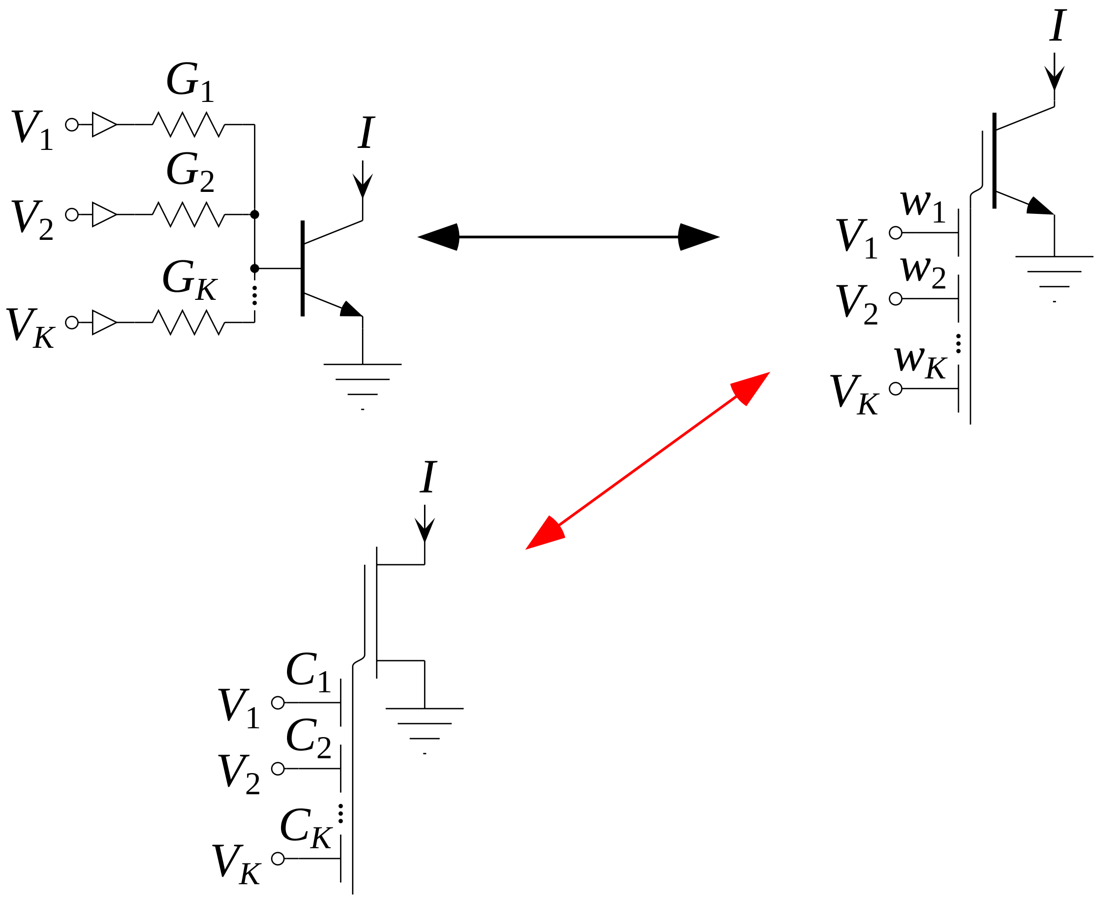

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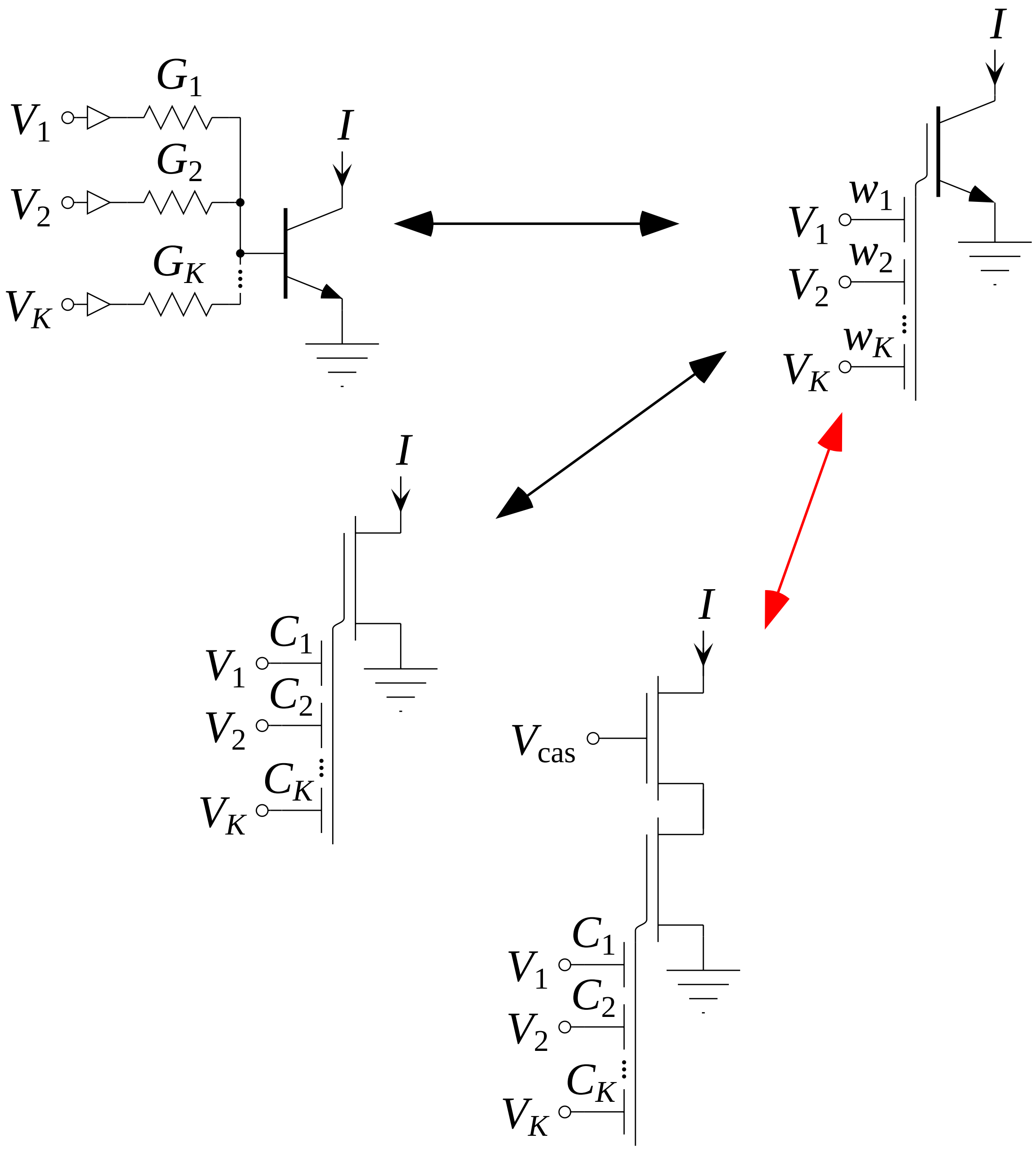

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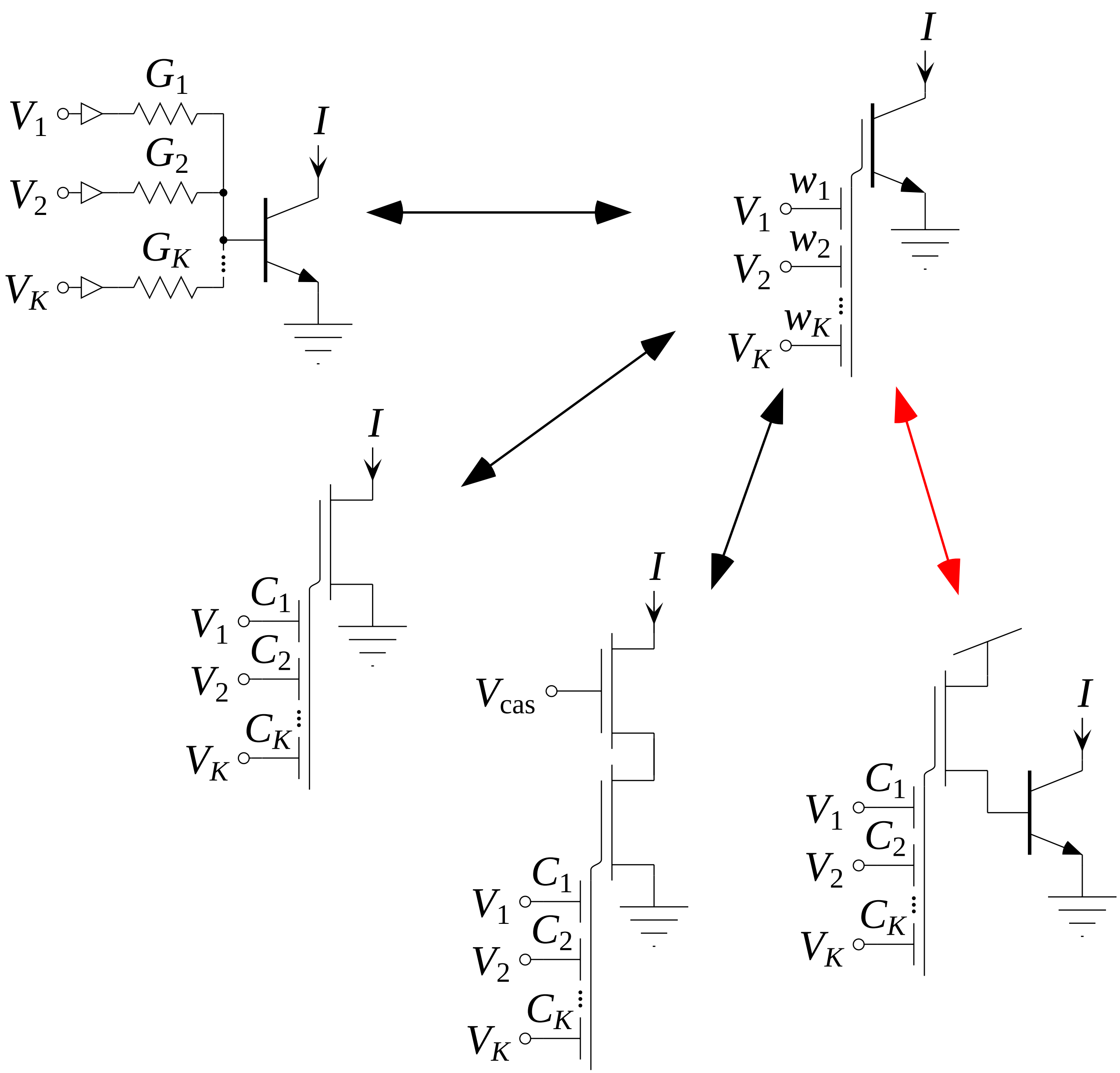

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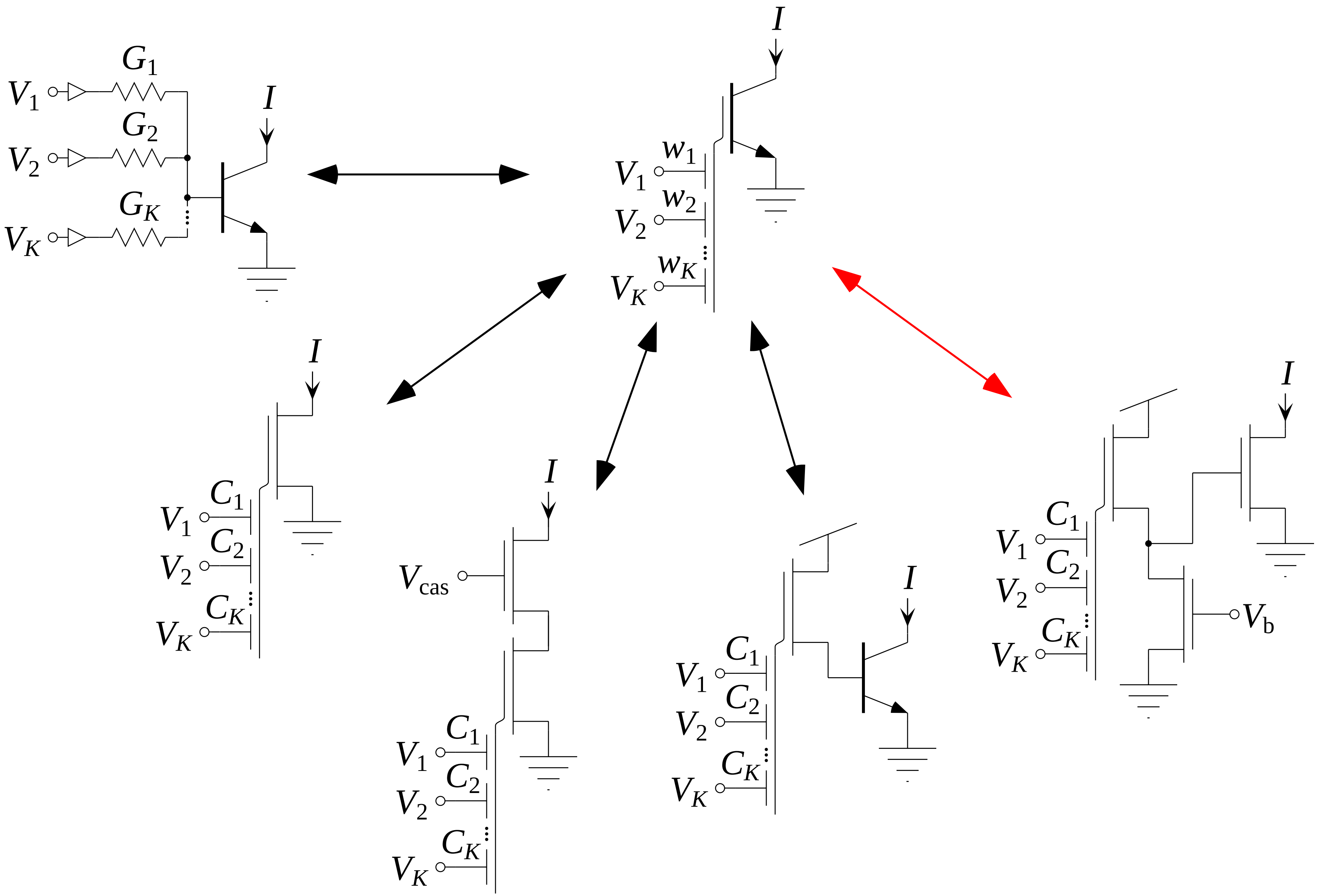

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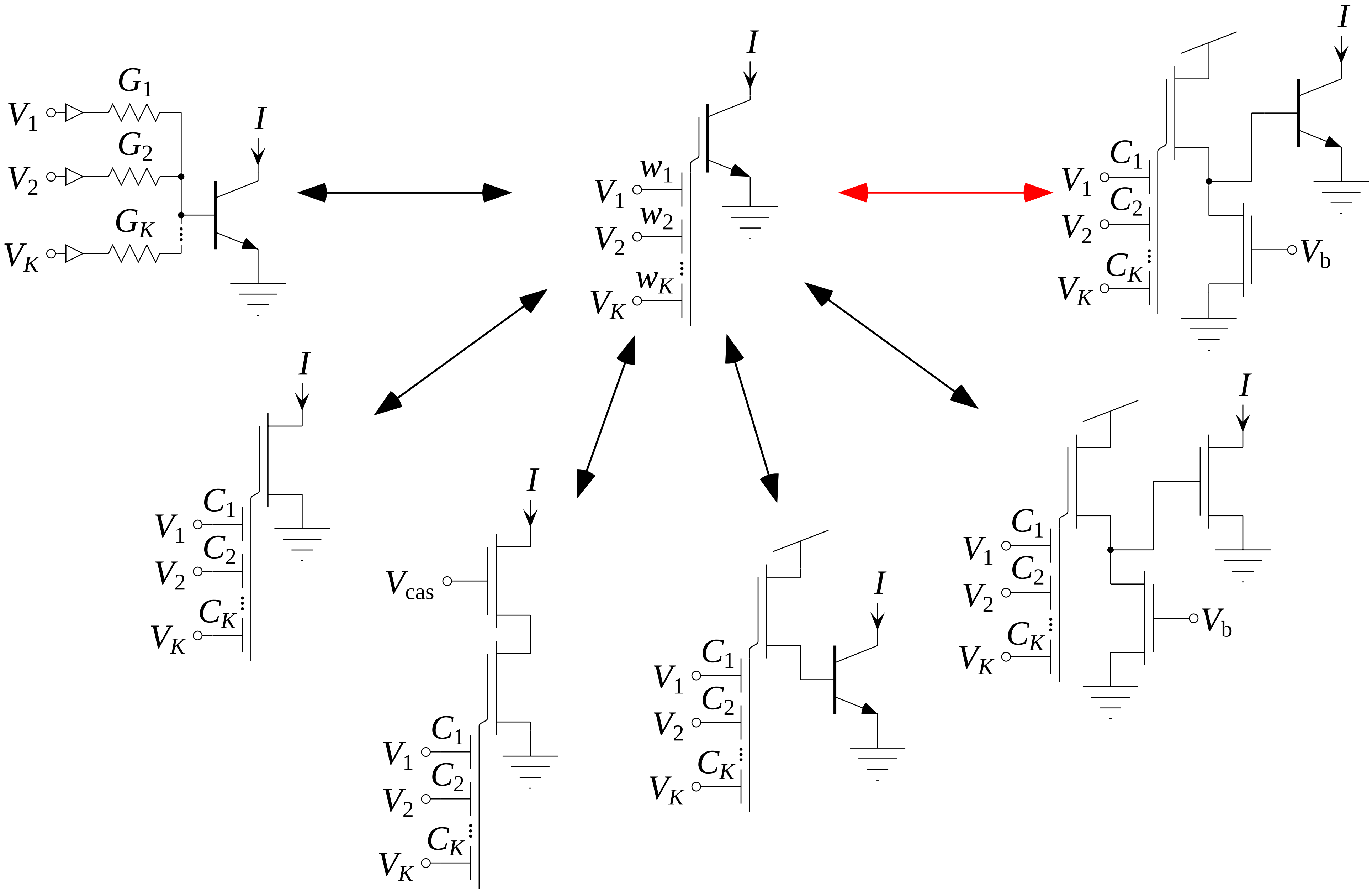

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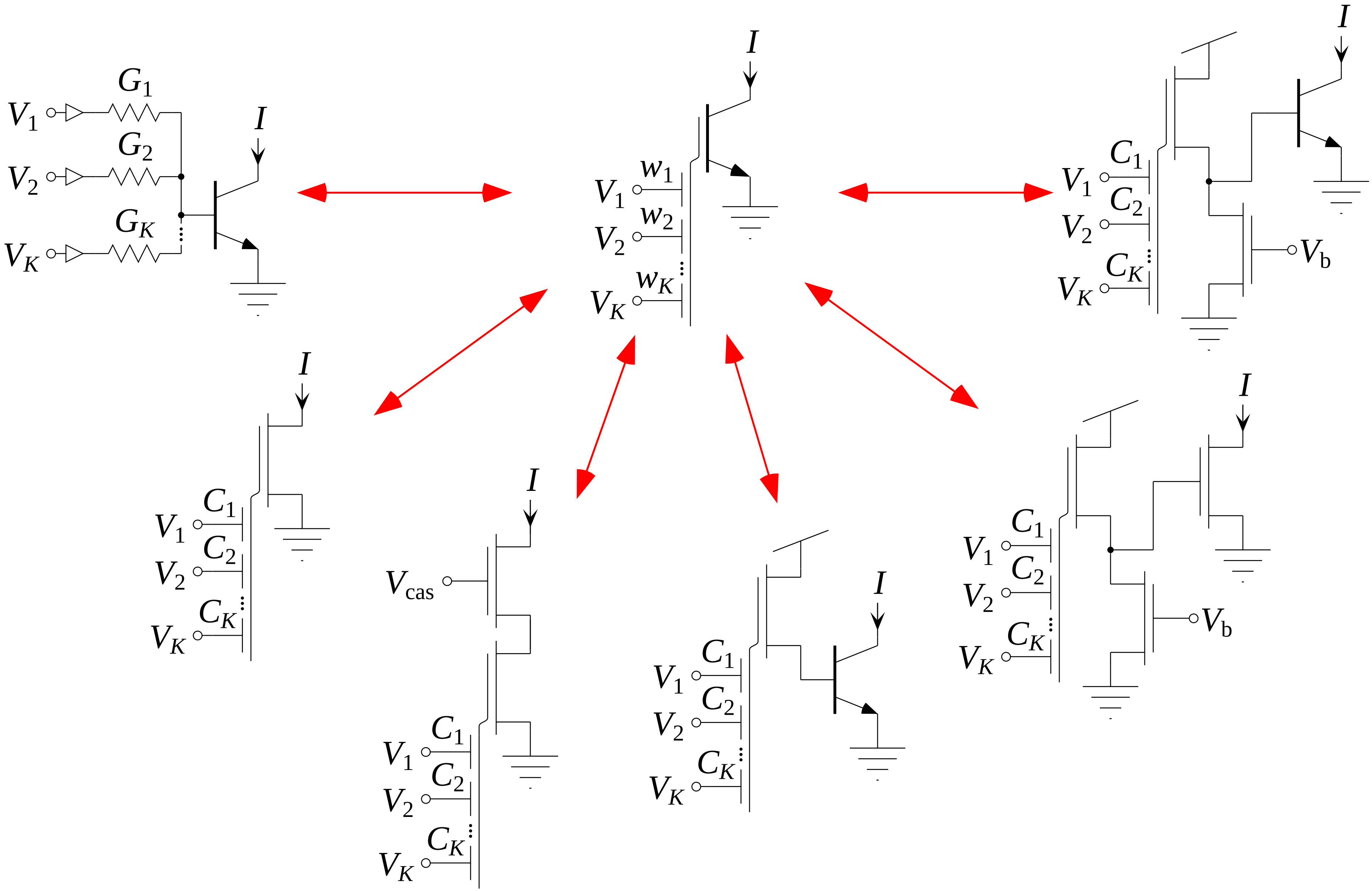
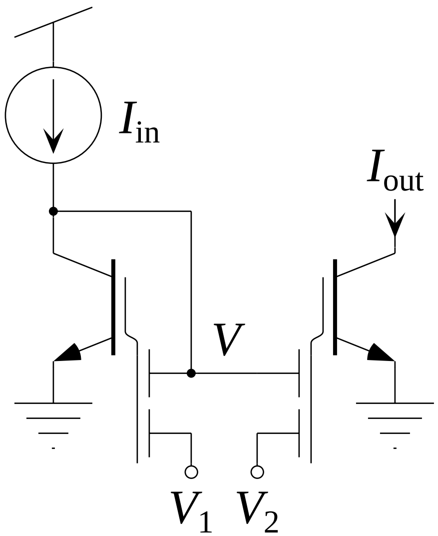


MITE Log-Domain Filters: Building Block



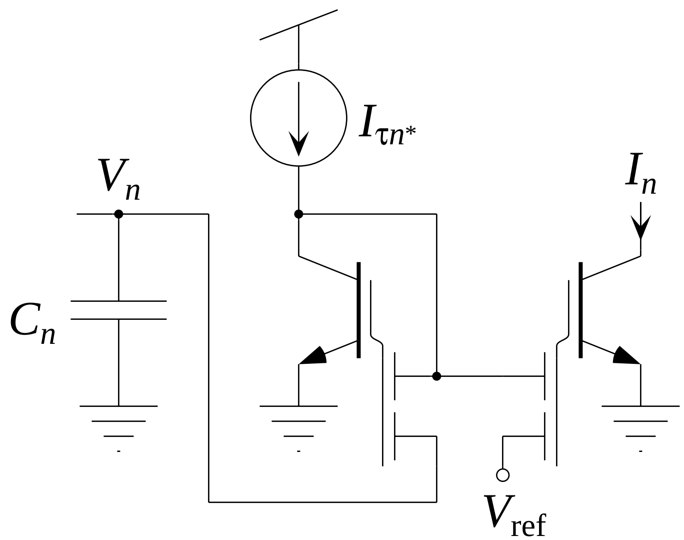
$$I_{in} = I_s e^{w(V+V_1)/U_T}$$

$$I_{out} = I_s e^{w(V+V_2)/U_T}$$

$$\Rightarrow \frac{I_{out}}{I_{in}} = \frac{\cancel{I_s}}{\cancel{I_s}} \frac{\cancel{e^{wV/U_T}}}{\cancel{e^{wV/U_T}}} \frac{e^{wV_2/U_T}}{e^{wV_1/U_T}}$$

$$\Rightarrow \boxed{I_{out} = I_{in} e^{w(V_2 - V_1)/U_T}}$$

MITE Log-Domain Filters:
Output Structure



$$\tau_n = \frac{C_n U_{\mathrm{T}}}{w I_{\tau n}}$$

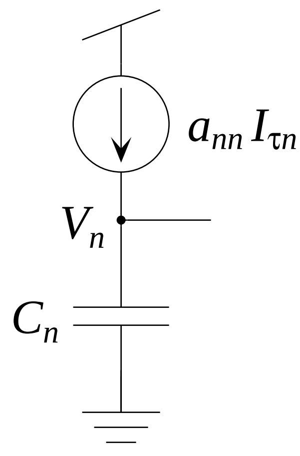
$$I_n = I_{\tau n^*} e^{w(V_{\mathrm{ref}} - V_n)/U_{\mathrm{T}}}$$

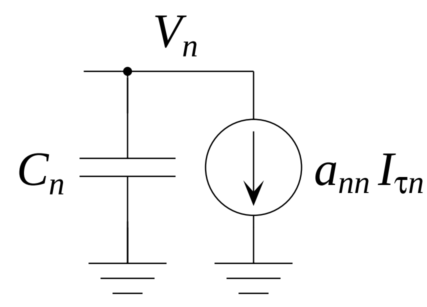
$$\frac{dV_n}{dt} = -\frac{U_{\mathrm{T}}}{w}\frac{1}{I_n}\frac{dI_n}{dt}$$

$$\tau_n \frac{dI_n}{dt} = \ldots$$

Note: n^* is the index of the state that is excited by the external input.

MITE Log-Domain Filters: Diagonal Terms

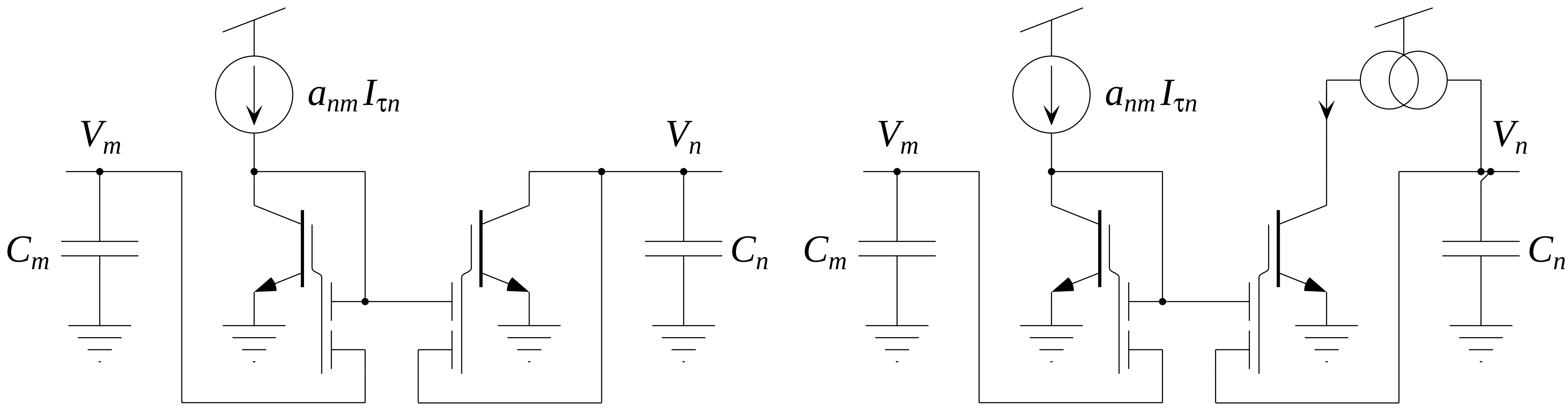




$$\tau_n \frac{dI_n}{dt} = \ldots - a_{nn} I_n - \ldots$$

$$\tau_n \frac{dI_n}{dt} = \ldots + a_{nn} I_n - \ldots$$

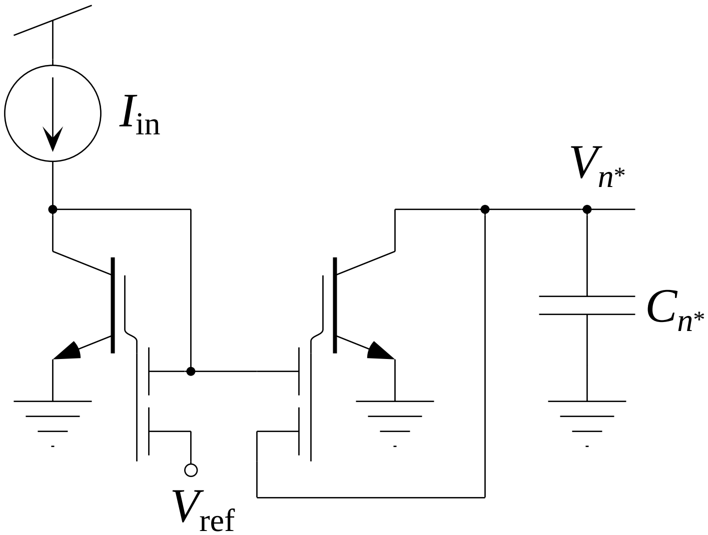
MITE Log-Domain Filters:
Off-Diagonal Terms

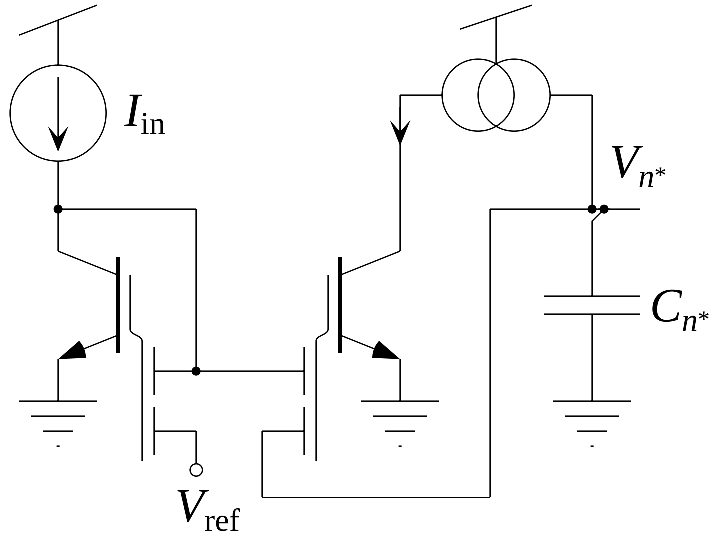


$$\tau_n \frac{dI_n}{dt} = \ldots + a_{nm} I_m - \ldots$$

$$\tau_n \frac{dI_n}{dt} = \ldots - a_{nm} I_m - \ldots$$

MITE Log-Domain Filters:
Input Structure

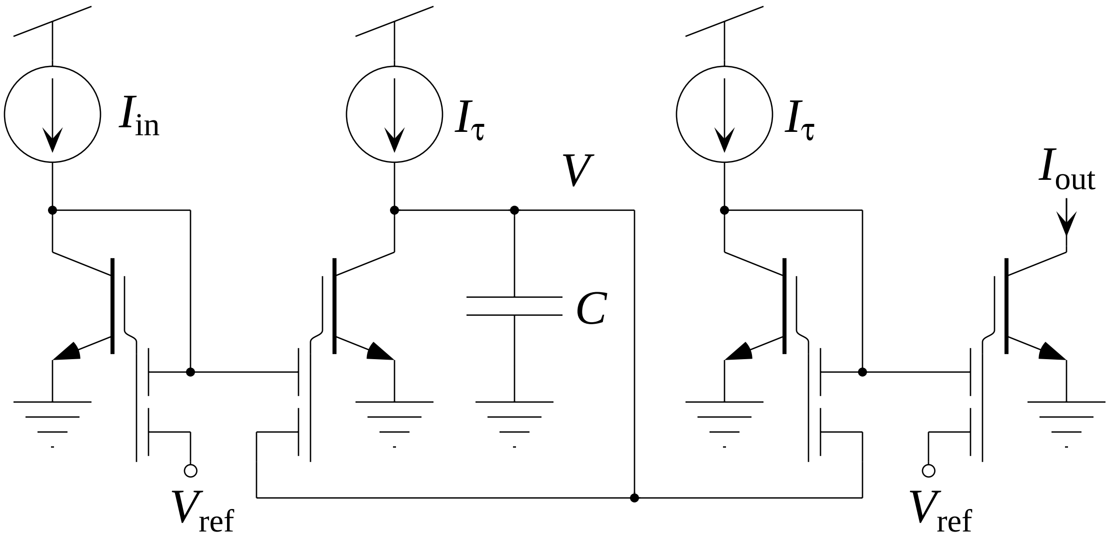




$$\tau_n \frac{dI_n}{dt} = \ldots + I_{in} - \ldots$$

$$\tau_n \frac{dI_n}{dt} = \ldots - I_{in} - \ldots$$

MITE Log-Domain Filters:
First-Order Low-Pass Filter



$$I_{out} = I_\tau e^{w(V_{ref} - V)/U_T}$$

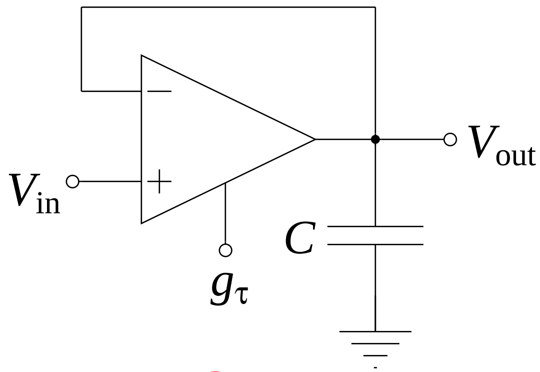
$$\Rightarrow \frac{dV}{dt} = -\frac{U_T}{w}\frac{1}{I_{out}}\frac{dI_{out}}{dt} \text{ and } e^{w(V - V_{ref})/U_T} = \frac{I_\tau}{I_{out}}$$

$$\text{KCL} \Rightarrow C\frac{dV}{dt} = I_\tau - I_{in} e^{w(V - V_{ref})/U_T}$$

$$\Rightarrow -\frac{CU_T}{w}\frac{1}{I_{out}}\frac{dI_{out}}{dt} = I_\tau - I_{in}\frac{I_\tau}{I_{out}}$$

$$\tau \equiv \frac{CU_T}{wI_\tau} \Rightarrow \boxed{\tau\frac{dI_{out}}{dt} + I_{out} = I_{in}}$$

MITE Log-Domain Filter Synthesis: First-Order Low-Pass Filter



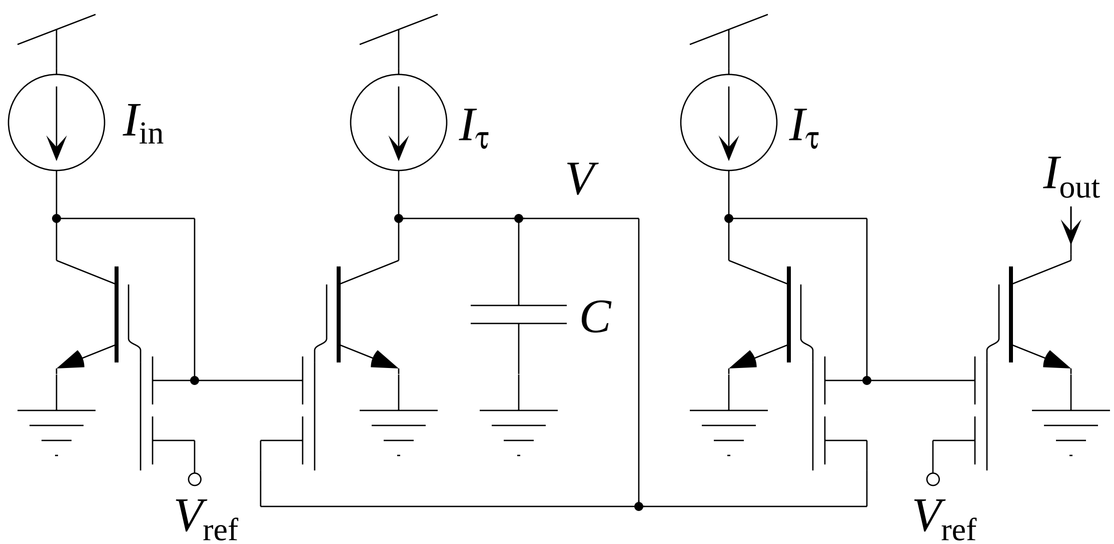
①

② $$C\frac{dV_{out}}{dt} = g_\tau\left(V_{in} - V_{out}\right)$$

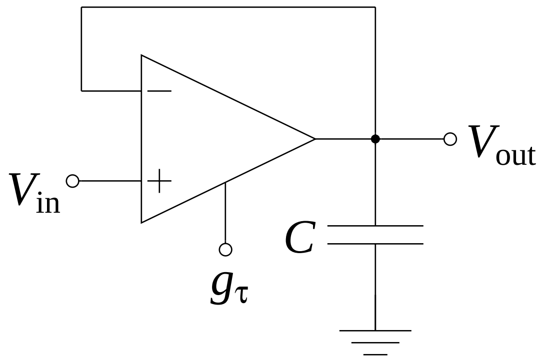
③ $$\tau\frac{dV_{out}}{dt} = V_{in} - V_{out}$$

④ $$\tau\frac{dI_{out}}{dt} = I_{in} - I_{out}$$

⑤



MITE Log-Domain Filter Synthesis: First-Order Low-Pass Filter



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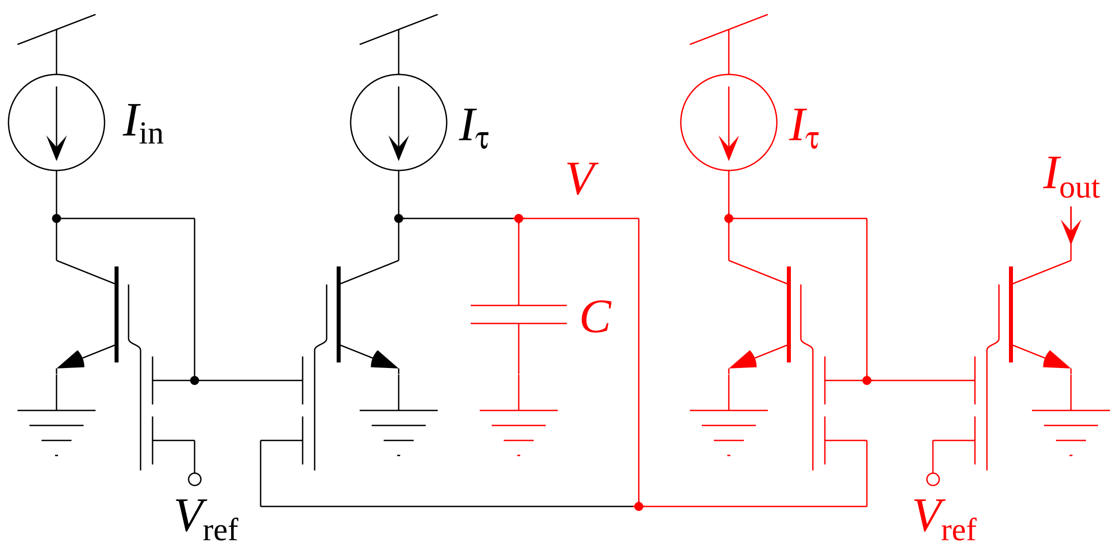
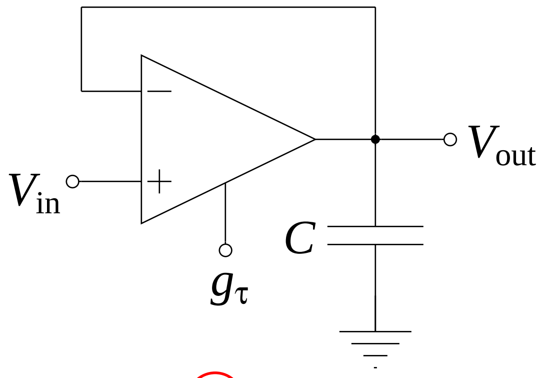
④ $$\tau\frac{dI_{out}}{dt} = I_{in} - I_{out}$$

⑤



MITE Log-Domain Filter Synthesis: First-Order Low-Pass Filter



①

② $$C\frac{dV_{out}}{dt} = g_\tau(V_{in} - V_{out})$$

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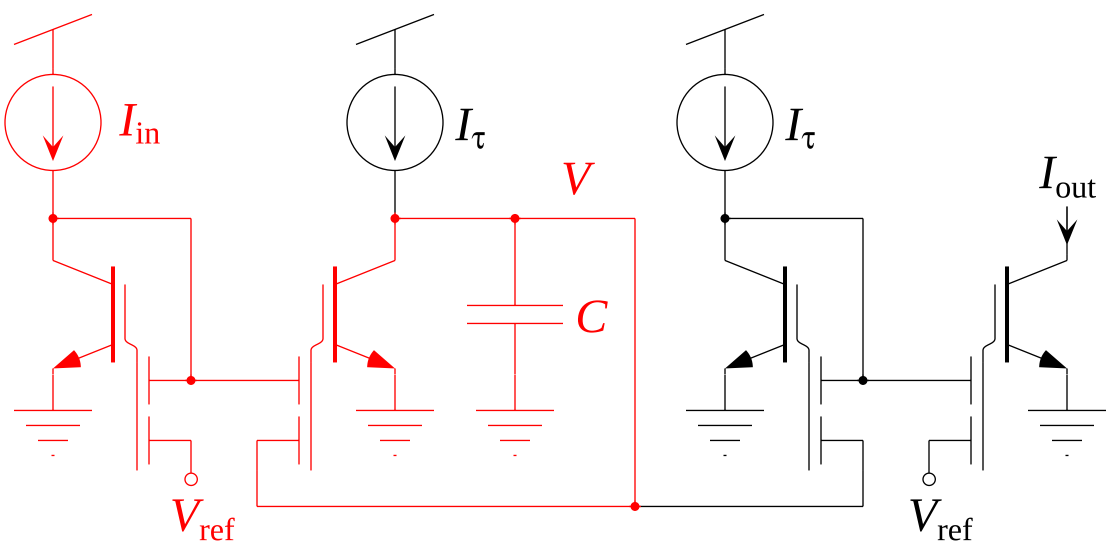
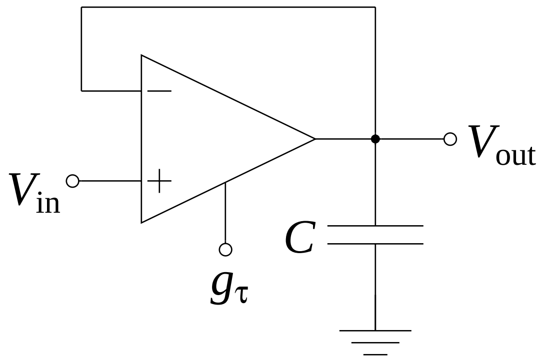
⑤



MITE Log-Domain Filter Synthesis: First-Order Low-Pass Filter



①

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③ $$\tau\frac{dV_{out}}{dt} = V_{in} - V_{out}$$

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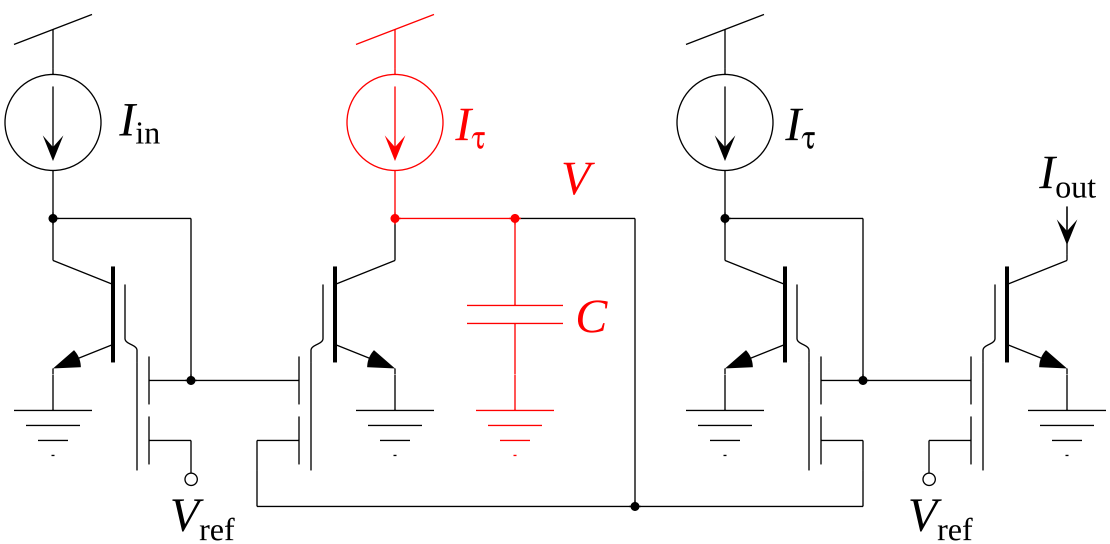
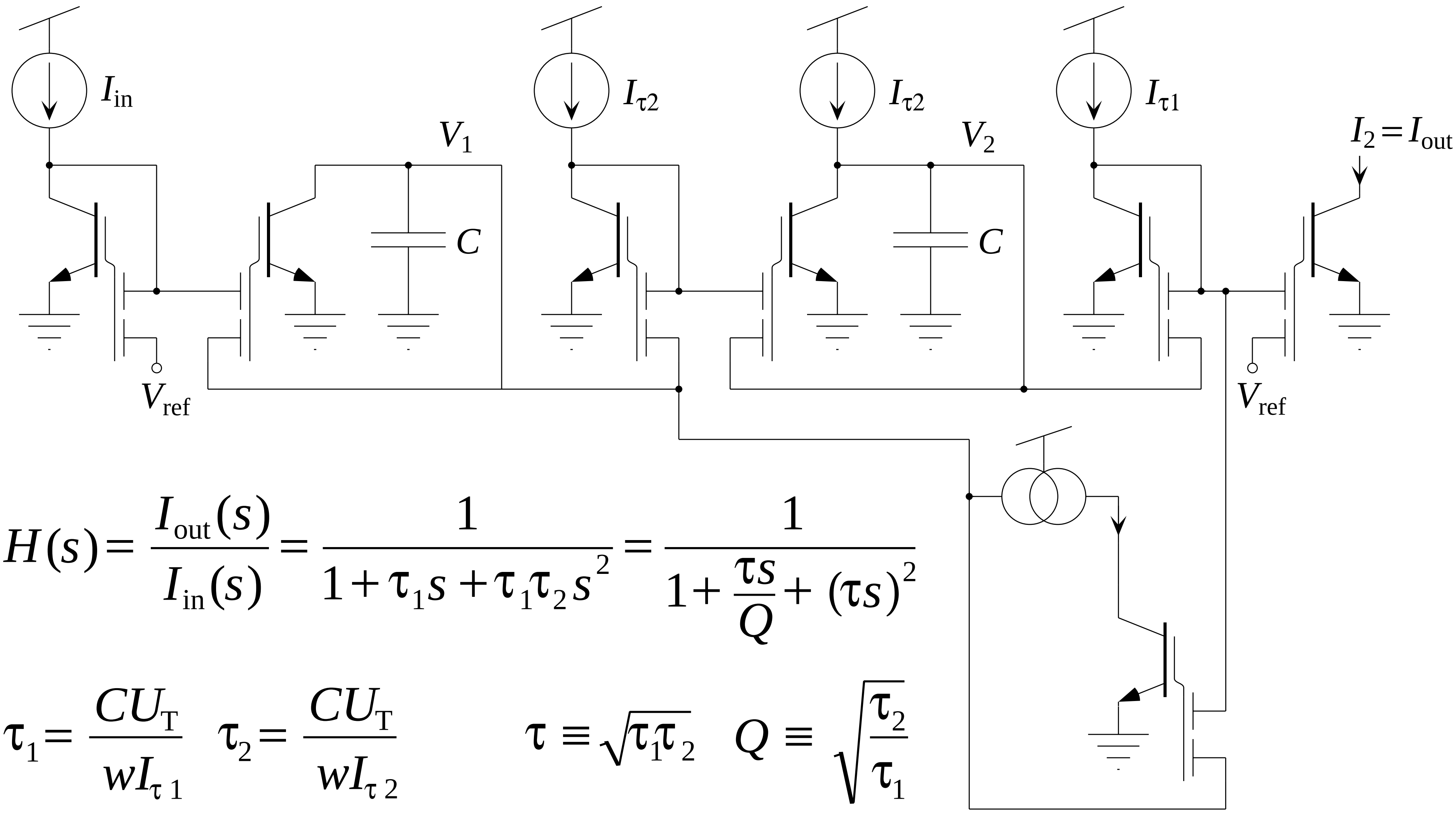
⑤



MITE Log-Domain Filter:
Second-Order Low-Pass Filter

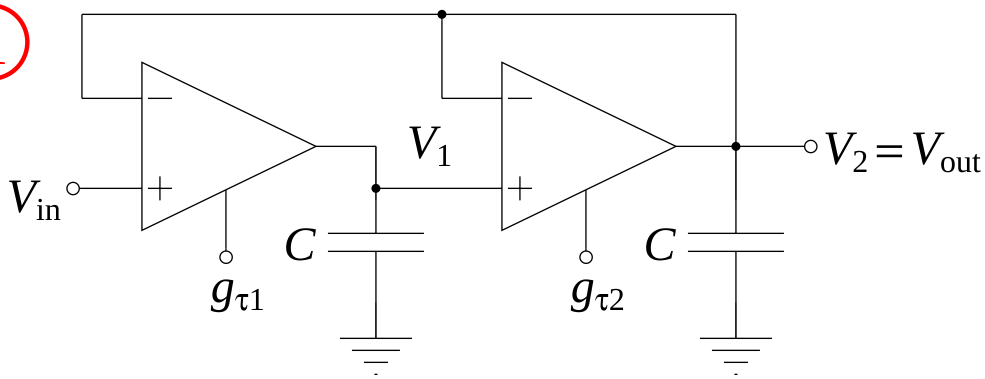


$$H(s) = \frac{I_{out}(s)}{I_{in}(s)} = \frac{1}{1 + \tau_1 s + \tau_1 \tau_2 s^2} = \frac{1}{1 + \frac{\tau s}{Q} + (\tau s)^2}$$

$$\tau_1 = \frac{CU_T}{wI_{\tau 1}} \quad \tau_2 = \frac{CU_T}{wI_{\tau 2}} \qquad \tau \equiv \sqrt{\tau_1 \tau_2} \quad Q \equiv \sqrt{\frac{\tau_2}{\tau_1}}$$

MITE Log-Domain Filter Synthesis: Second-Order Low-Pass Filter

①



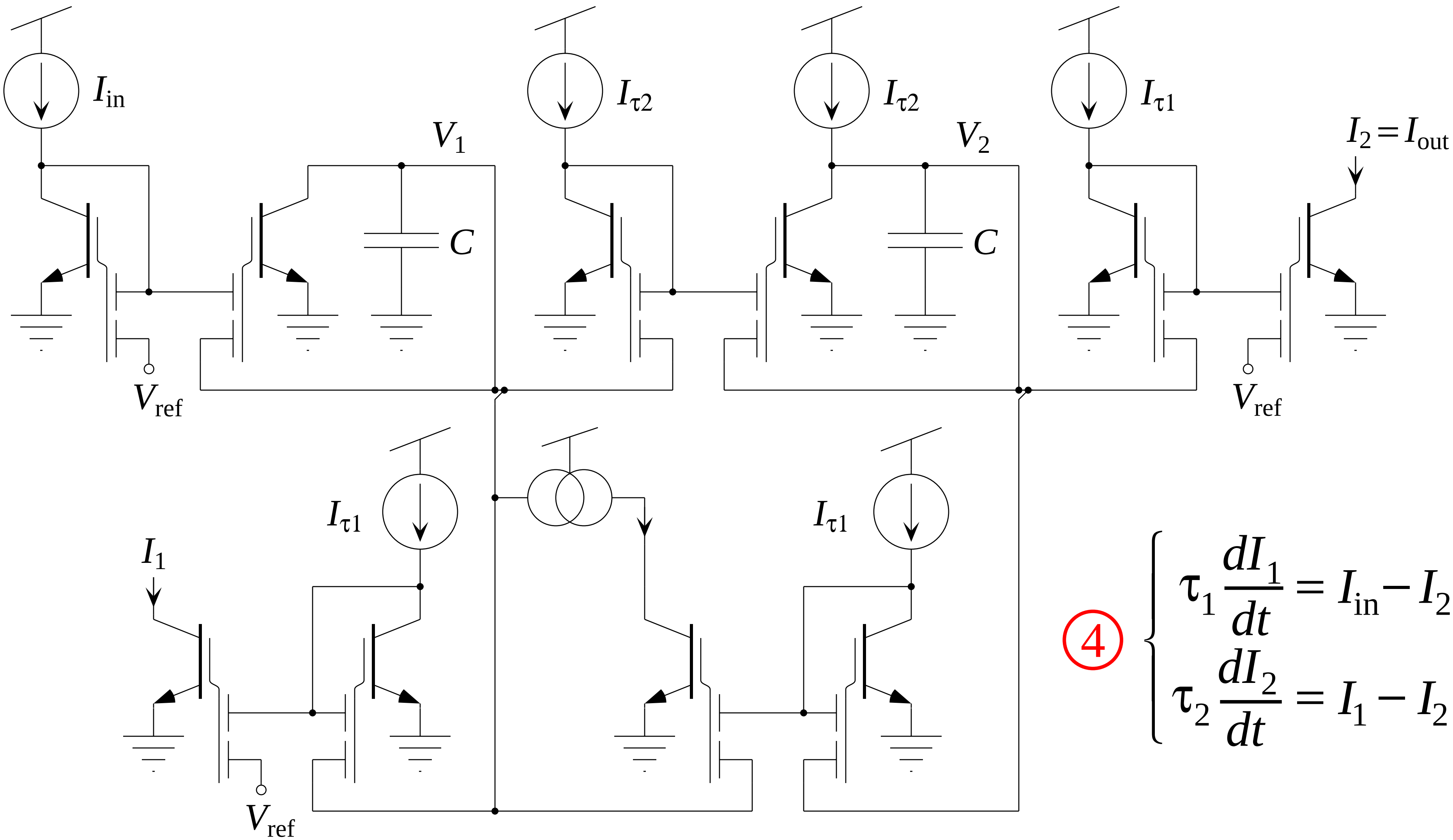
②
$$\begin{cases} C\dfrac{dV_1}{dt} = g_{\tau 1}(V_{in} - V_2) \\ C\dfrac{dV_2}{dt} = g_{\tau 2}(V_1 - V_2) \end{cases}$$

③
$$\begin{cases} \tau_1\dfrac{dV_1}{dt} = V_{in} - V_2 \\ \tau_2\dfrac{dV_2}{dt} = V_1 - V_2 \end{cases}$$

④
$$\begin{cases} \tau_1\dfrac{dI_1}{dt} = I_{in} - I_2 \\ \tau_2\dfrac{dI_2}{dt} = I_1 - I_2 \end{cases}$$

MITE Log-Domain Filter Synthesis: Second-Order Low-Pass Filter

⑤

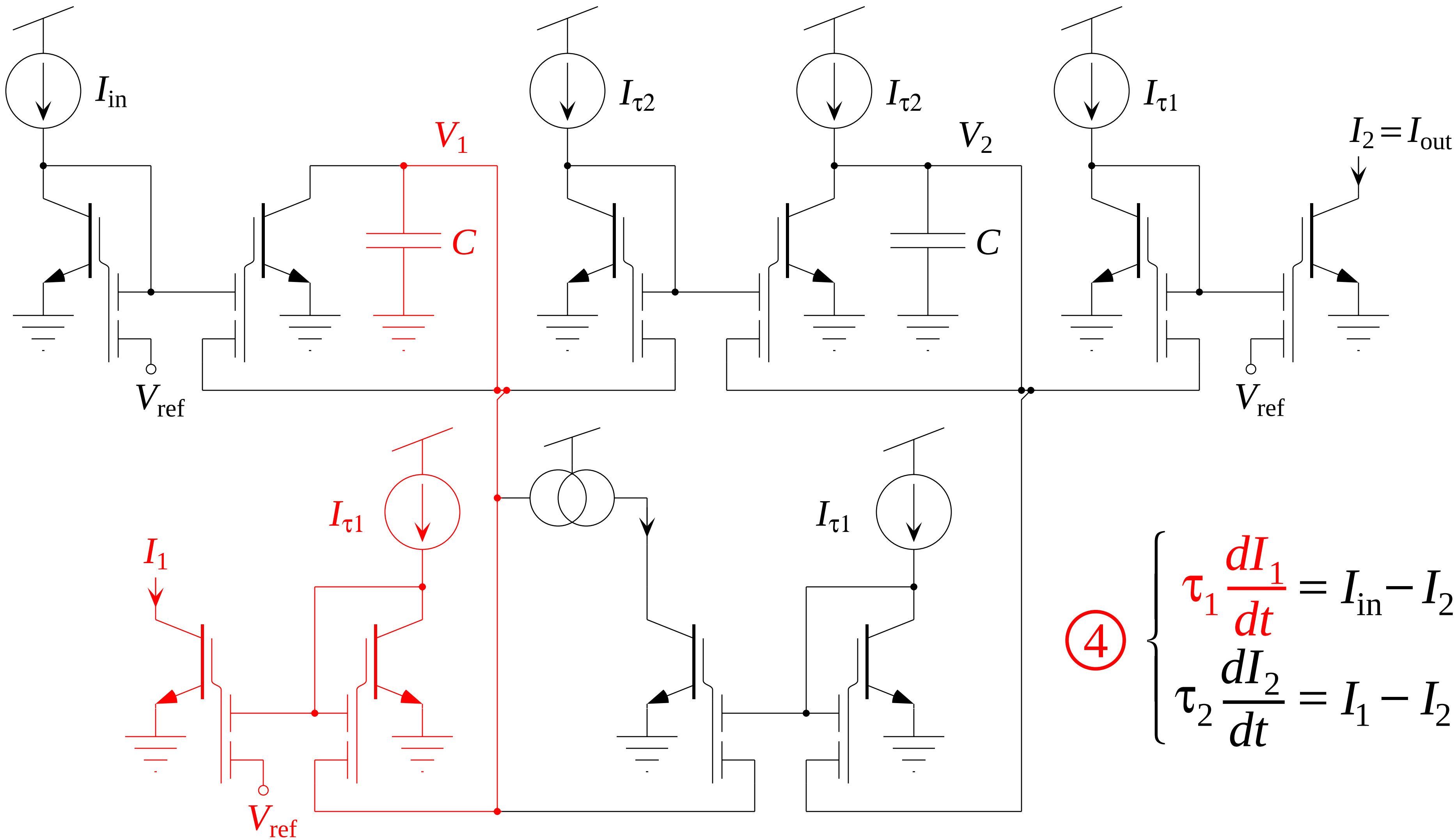


④

$$\begin{cases} \tau_1 \dfrac{dI_1}{dt} = I_{in} - I_2 \\ \tau_2 \dfrac{dI_2}{dt} = I_1 - I_2 \end{cases}$$

MITE Log-Domain Filter Synthesis: Second-Order Low-Pass Filter

5

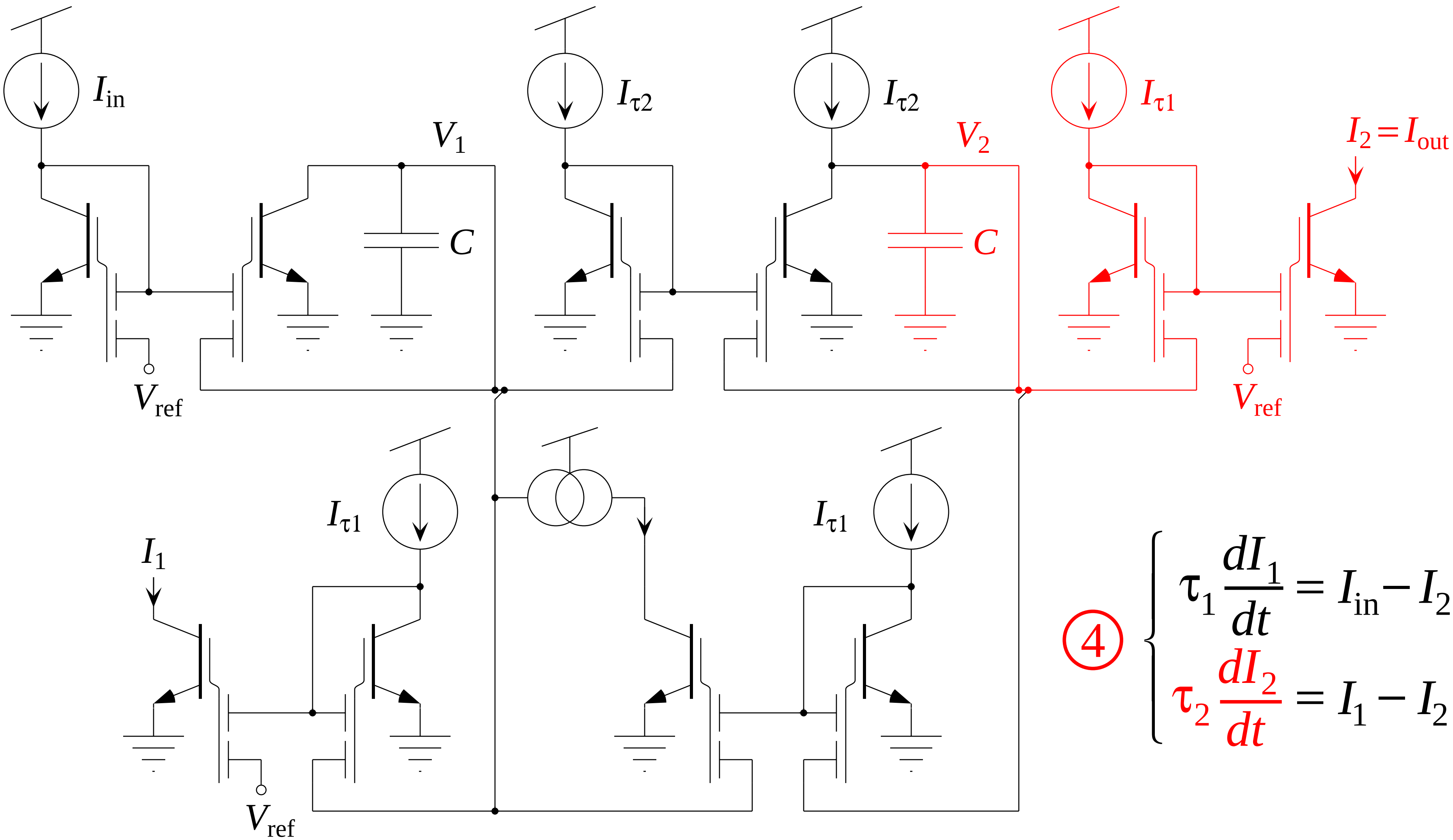


4

$$\begin{cases} \tau_1 \dfrac{dI_1}{dt} = I_{in} - I_2 \\ \tau_2 \dfrac{dI_2}{dt} = I_1 - I_2 \end{cases}$$

MITE Log-Domain Filter Synthesis: Second-Order Low-Pass Filter

⑤

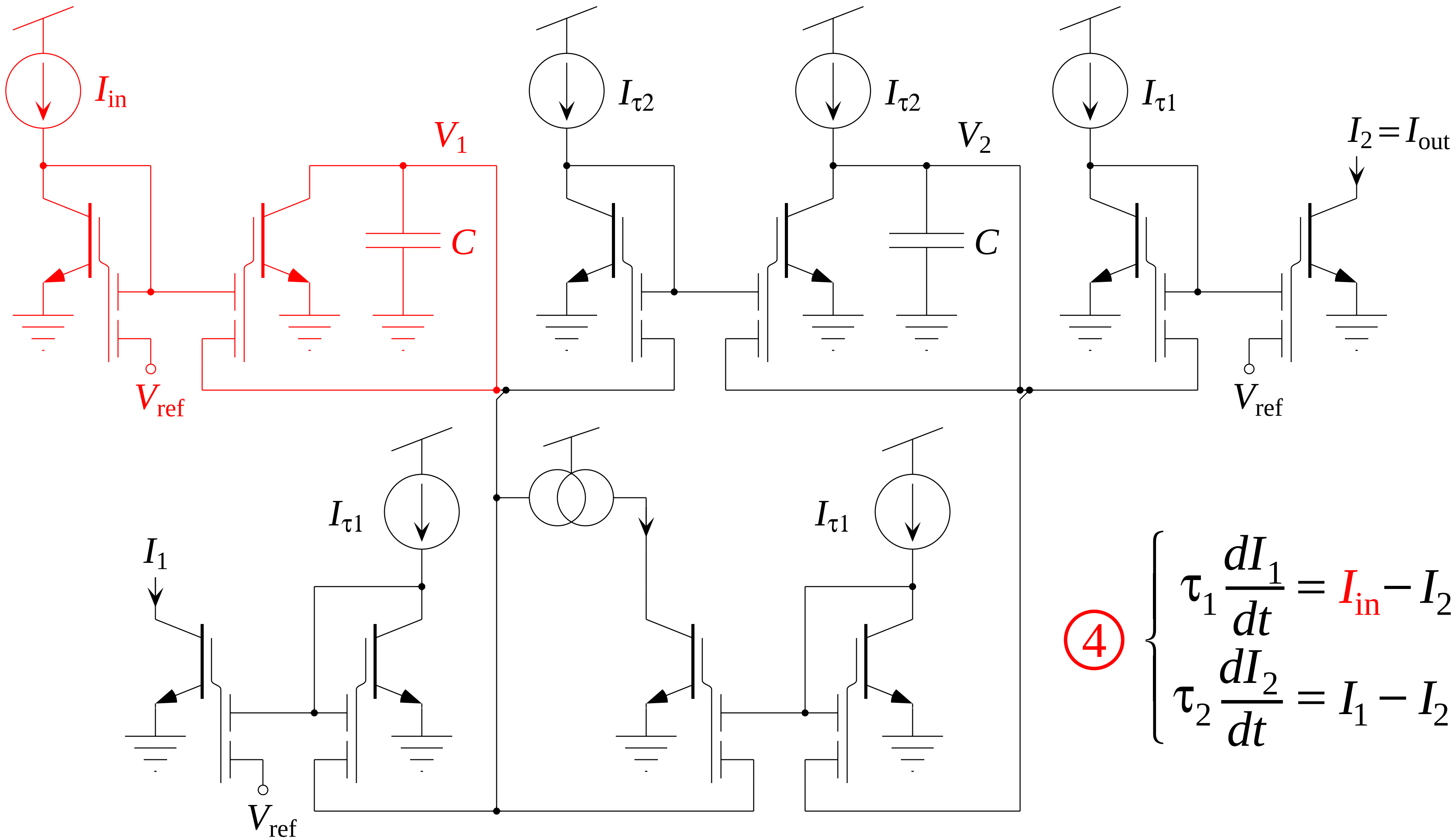


④

$$\begin{cases} \tau_1 \dfrac{dI_1}{dt} = I_{in} - I_2 \\ \tau_2 \dfrac{dI_2}{dt} = I_1 - I_2 \end{cases}$$

MITE Log-Domain Filter Synthesis: Second-Order Low-Pass Filter

⑤

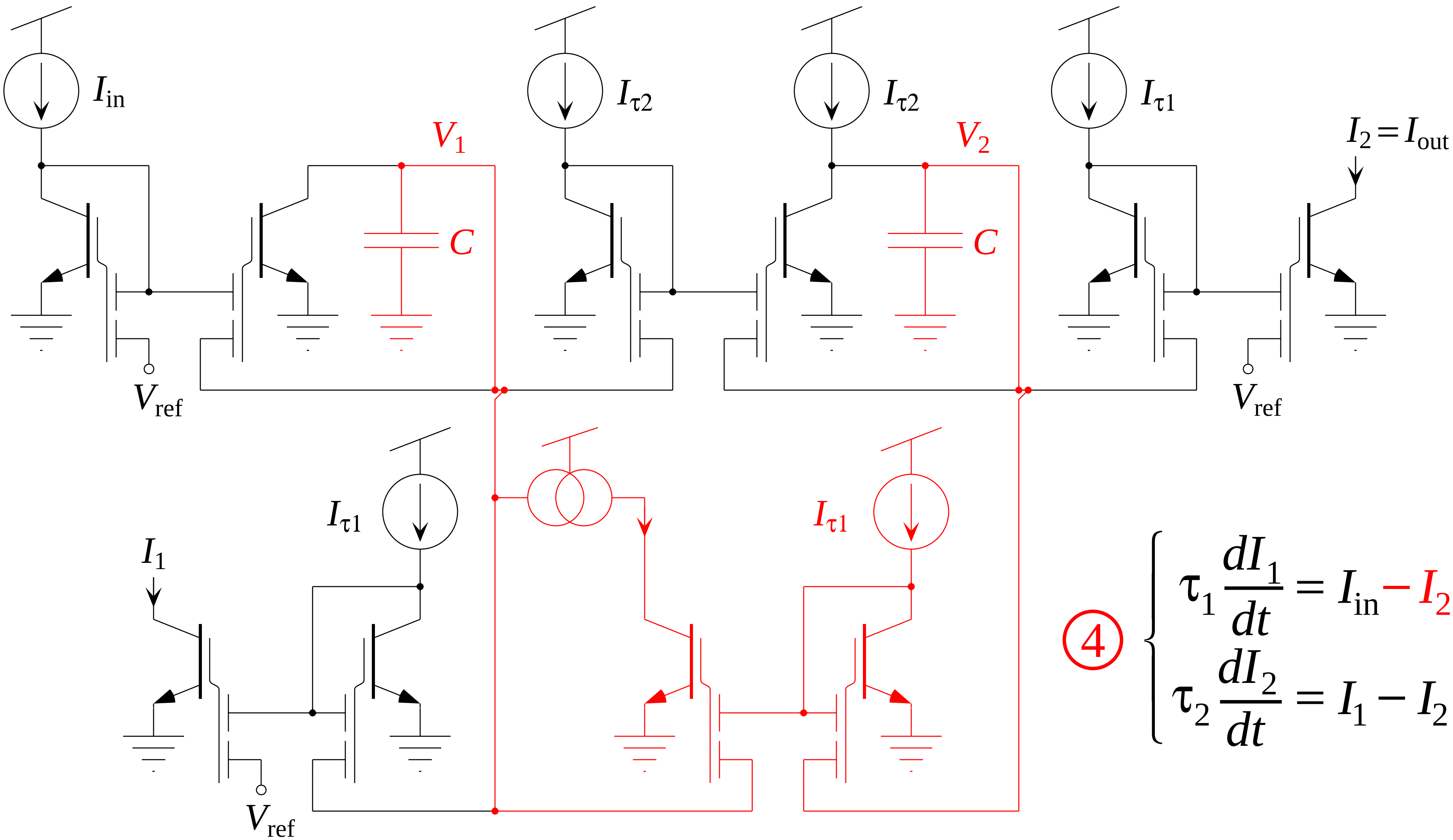


④

$$\begin{cases} \tau_1 \dfrac{dI_1}{dt} = I_{in} - I_2 \\ \tau_2 \dfrac{dI_2}{dt} = I_1 - I_2 \end{cases}$$

MITE Log-Domain Filter Synthesis: Second-Order Low-Pass Filter

5

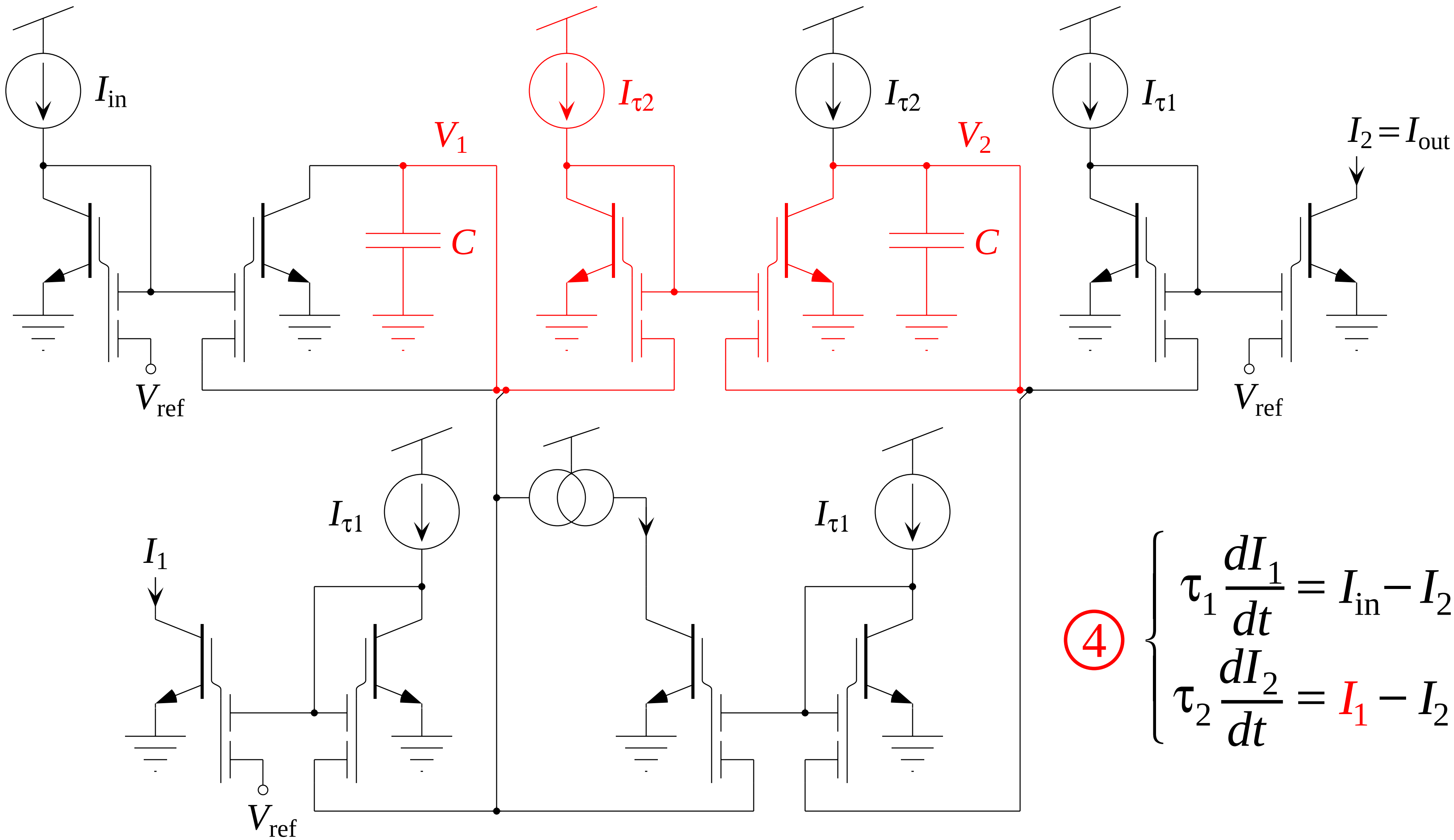


4

$$\begin{cases} \tau_1 \dfrac{dI_1}{dt} = I_{in} - I_2 \\ \tau_2 \dfrac{dI_2}{dt} = I_1 - I_2 \end{cases}$$

MITE Log-Domain Filter Synthesis: Second-Order Low-Pass Filter

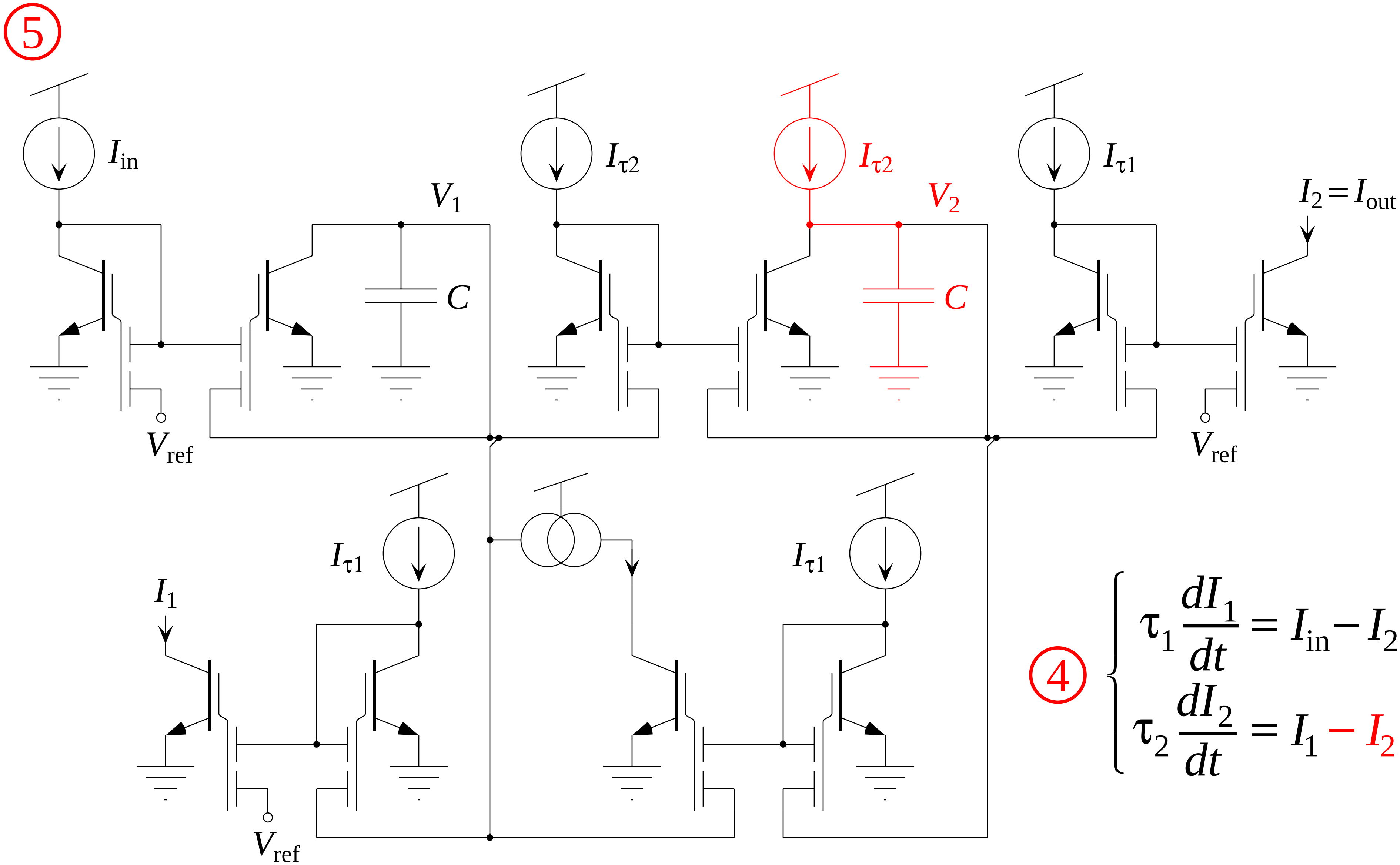
⑤



④

$$\begin{cases} \tau_1 \dfrac{dI_1}{dt} = I_{in} - I_2 \\ \tau_2 \dfrac{dI_2}{dt} = I_1 - I_2 \end{cases}$$

MITE Log-Domain Filter Synthesis: Second-Order Low-Pass Filter



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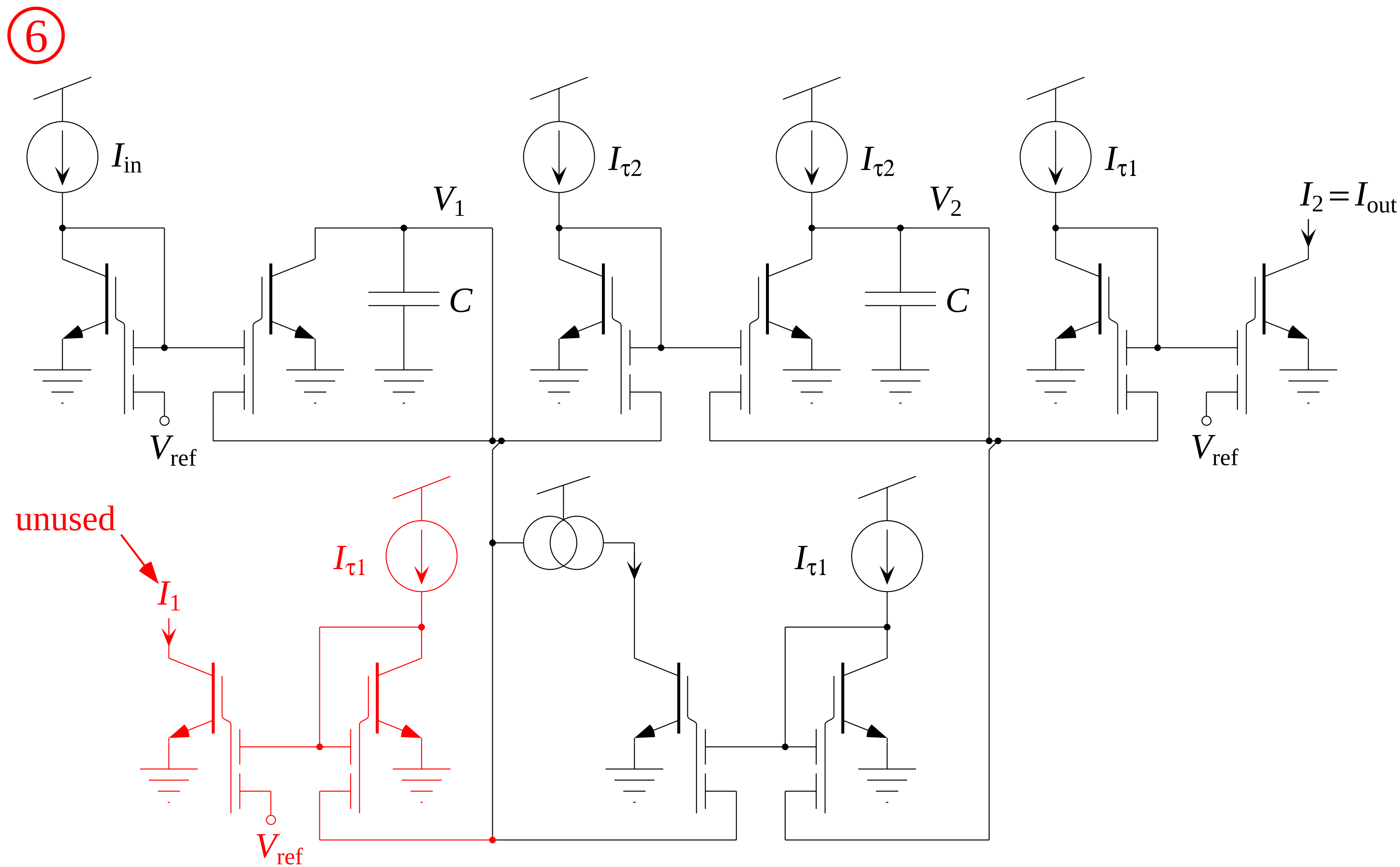
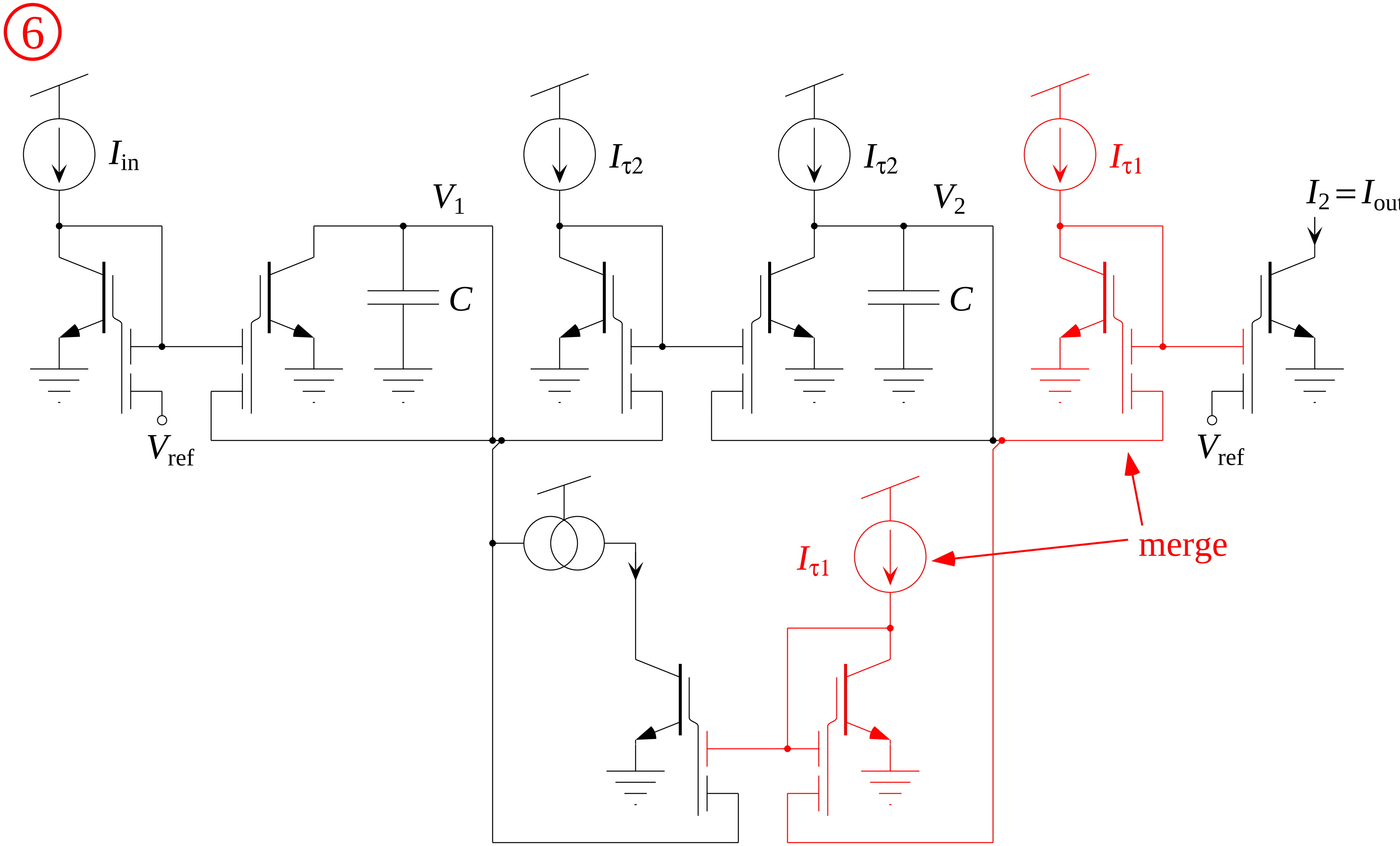
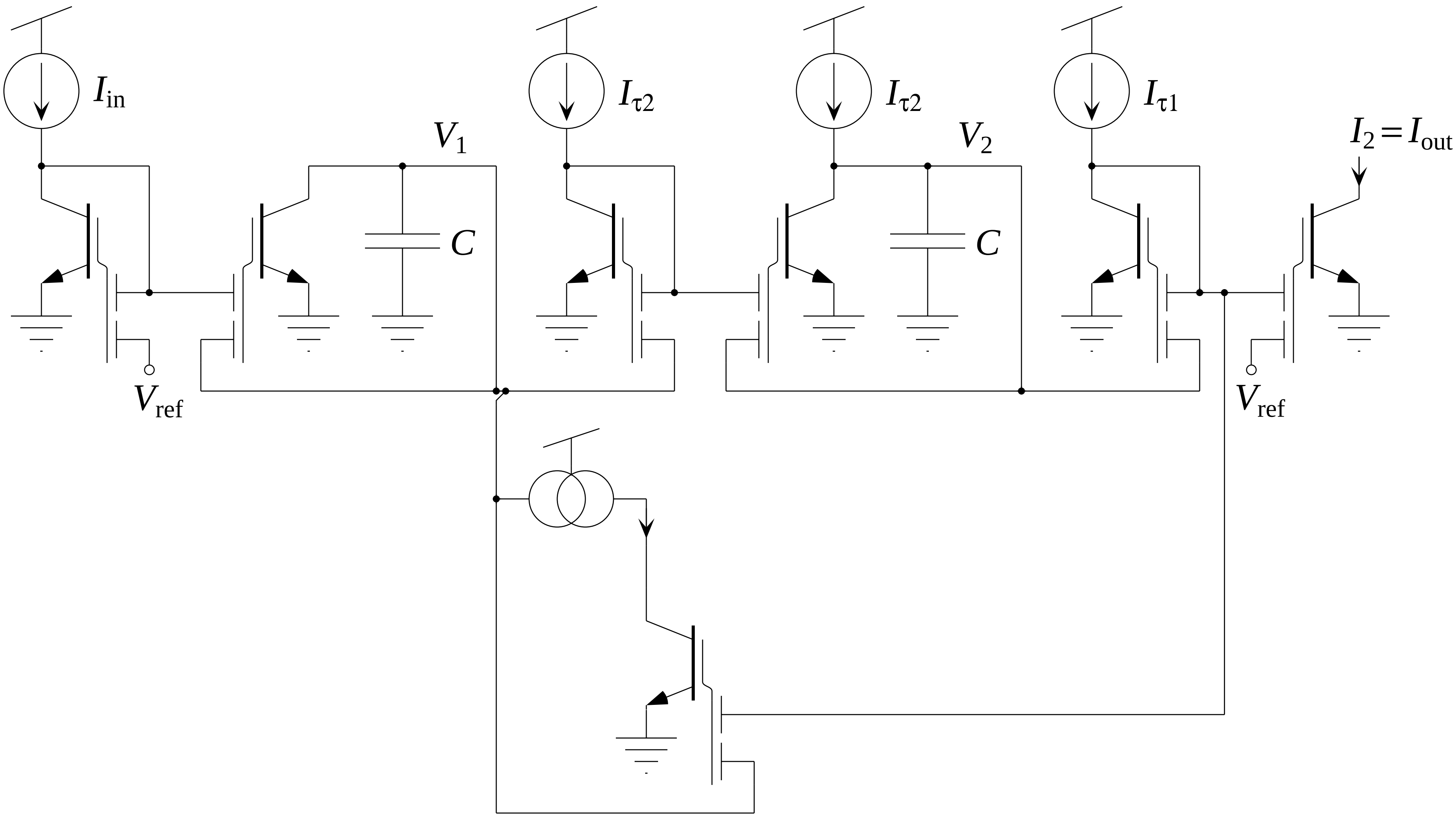


MITE Log-Domain Filter Synthesis: Second-Order Low-Pass Filter

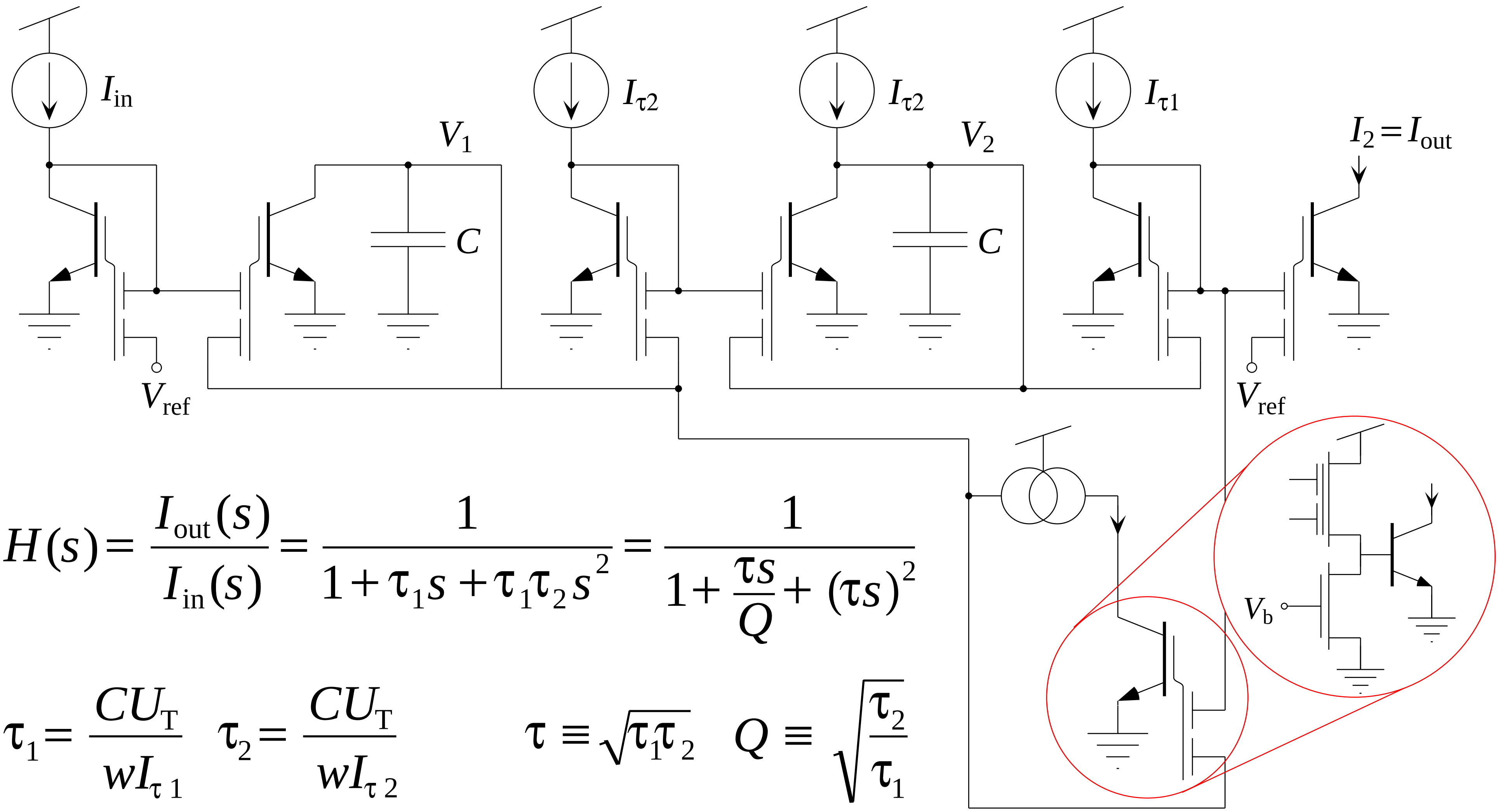


MITE Log-Domain Filter Synthesis: Second-Order Low-Pass Filter

⑥



MITE Log-Domain Filters :
Second-Order Low-Pass Filter



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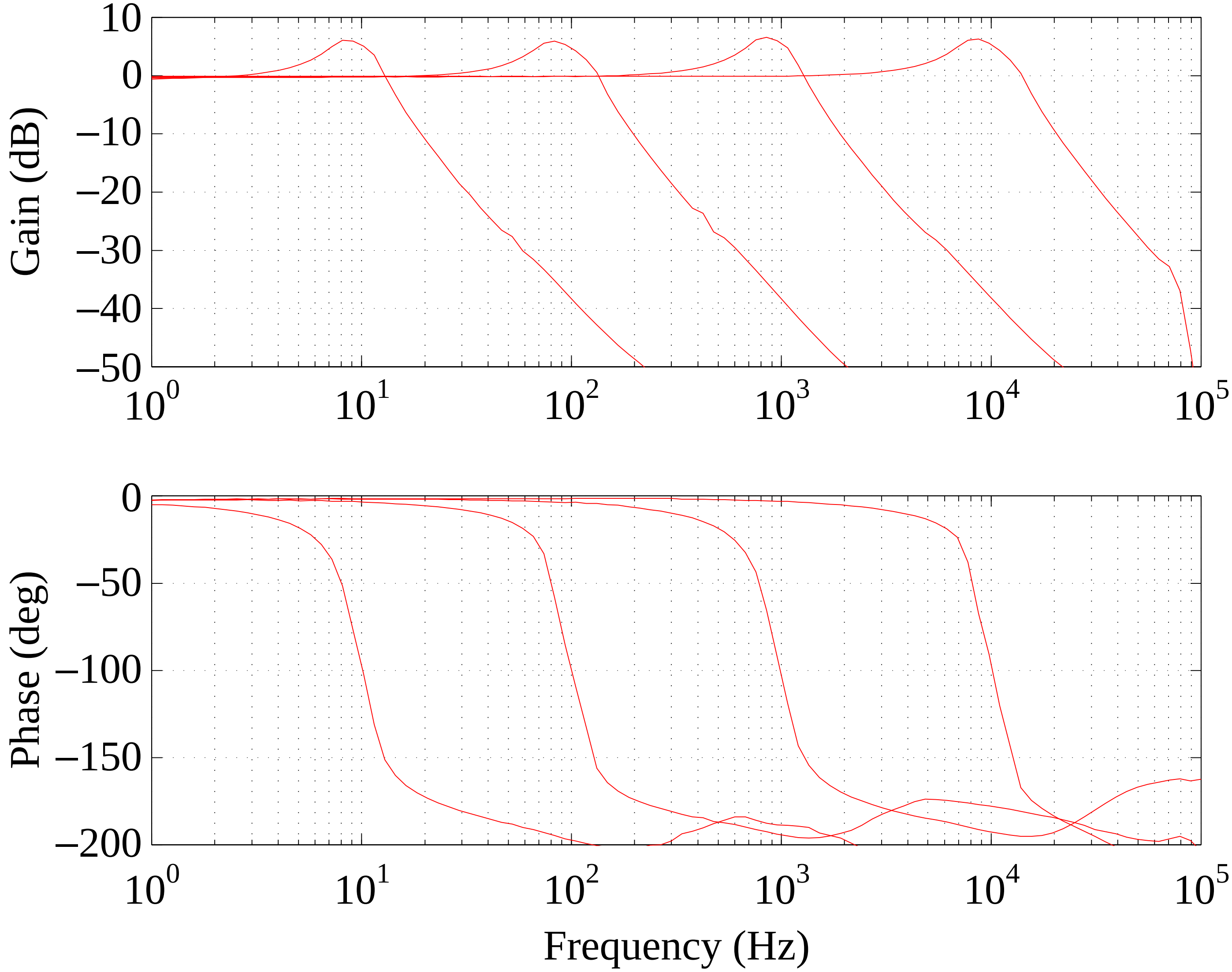
$$\tau_1 = \frac{CU_T}{wI_{\tau 1}} \quad \tau_2 = \frac{CU_T}{wI_{\tau 2}} \qquad \tau \equiv \sqrt{\tau_1 \tau_2} \quad Q \equiv \sqrt{\frac{\tau_2}{\tau_1}}$$

MITE Second-Order Low-Pass Log-Domain Filter: Tuning τ



MITE Second-Order Low-Pass Log-Domain Filter: Tuning *Q*