Static and Dynamic Translinear Circuits

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Translinear Devices and Circuits: What's In A Name?

A translinear element is one whose incremental transconductance gain is linear in its output current

$$I = \lambda I_{\rm s} e^{V/U_{\rm T}} \implies g_{\rm m} = \frac{\partial I}{\partial V} = \underbrace{\lambda I_{\rm s} e^{V/U_{\rm T}}}_{I} \cdot \frac{1}{U_{\rm T}} = \frac{I}{U_{\rm T}}$$

The word translinear is also meant to convey the notion of bridging the gap between the familiar world of linear circuit design and the largely uncharted territory of nonlinear circuit design.





The Translinear Principle

In a closed loop of junctions comprising an equal number of clockwise and counterclockwise elements, the product of the current densities flowing through the counterclockwise elements is equal to the product of the current densities flowing through the clockwise elements.

$$\prod_{n \in \mathrm{CW}} \frac{I_n}{\lambda_n} = \prod_{n \in \mathrm{CCW}} \frac{I_n}{\lambda_n}$$







The Ideal Multiple-Input Translinear Element

The ideal multiple-input translinear element (MITE) produces an output current given by

$$I = \lambda I_{\rm s} e^{(w_1 V_1 + \dots + w_K V_K)/U_{\rm T}}$$

where

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- $I_{\rm s}$ pre-exponential scaling current
- λ dimensionless constant scaling $I_{\rm s}$ proportionally
- V_k kth control-gate voltage
- w_k dimensionless positive weight scaling V_k
- $U_{\rm T}$ thermal voltage, kT/q.





Practical Floating-Gate MITE Implementations







Basic MITE Circuit Stages

$$\begin{array}{c|c} w_{ni} \\ V_i \circ & \\ V_k \circ & \\ \end{array} \end{array} \begin{array}{c|c} & & \\ & & \\ \hline \end{array} \begin{array}{c} & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ \hline & & \\ & & \\ & & \\ \hline & & \\ & & \\ & & \\ \hline \end{array}$$

 $I_n \propto e^{w_{ni}V_i/U_{\rm T}}e^{w_{nk}V_k/U_{\rm T}}$





Basic MITE Circuit Stages



$$I_n \propto e^{w_{ni}V_i/U_{\rm T}} e^{w_{nk}V_k/U_{\rm T}}$$

$$V_i = \frac{U_{\rm T}}{w_{ii}} \log I_i - \cdots$$





Basic MITE Circuit Stages







Elementary MITE Networks

$$I_n \propto e^{w_{ni}V_i/U_{\rm T}} e^{w_{nk}V_k/U_{\rm T}}$$

$$\implies I_n \propto \exp\left(\frac{w_{ni}}{U_{\rm T}} \left(\frac{U_{\rm T}}{w_{ii}}\log I_i - \cdots\right)\right)\right)$$

$$\times \exp\left(\frac{w_{nk}}{U_{\rm T}} \left(\frac{U_{\rm T}}{w_{kk}}\log I_k - \cdots\right)\right)\right)$$

$$\implies I_n \propto e^{(w_{ni}/w_{ii})\log I_i} e^{(w_{nk}/w_{kk})\log I_k}$$

$$\implies I_n \propto I_i^{w_{ni}/w_{ii}} \times I_k^{w_{nk}/w_{kk}}$$







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$$\implies I_n \propto \exp\left(\frac{w_{ni}}{U_{\rm T}} \left(\frac{U_{\rm T}}{w_{ii}} \log I_i\right) - \frac{w_{ij}}{w_{ii}} \left(\frac{U_{\rm T}}{w_{jj}} \log I_j - \cdots\right) - \cdots\right)\right)$$

$$\implies I_n \propto e^{(w_{ni}/w_{ii}) \log I_i} e^{-(w_{ni}w_{ij}/w_{ii}w_{jj}) \log I_j}$$

$$\implies I_n \propto \frac{I_i^{w_{ni}/w_{ii}}}{I_j^{w_{ni}w_{ij}/w_{ii}w_{jj}}}$$







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 $z = \sqrt{x}$, where x > 0.





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$$\frac{I_z}{I_1} = \sqrt{\frac{I_x}{I_1}} \implies I_z = \sqrt{I_x I_1} \implies I_z^2 = I_x I_1.$$





Static MITE Network Synthesis: Square-Root Circuit TLP: $I_z^2 = I_x I_1$





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TLP: $I_z^2 = I_x I_1$



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Experimental Measurements: Square-Root Circuit







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Experimental Measurements: Squaring Circuit







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 $r = \sqrt{x^2 + y^2}$, where x > 0 and y > 0.





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TLP:
$$I_{r1}I_r = I_x^2$$
 KCL: $I_r = I_{r1} + I_{r2}$
 $I_{r2}I_r = I_y^2$





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Dynamic MITE Network Synthesis: Output Structures





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Dynamic MITE Network Synthesis: First-Order LPF

Synthesize a first-order low-pass filter described by

$$\tau \frac{dy}{dt} + y = x$$
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Substituting these into the ODE, we obtain

$$\tau \frac{d}{dt} \left(\frac{I_y}{I_1} \right) + \frac{I_y}{I_1} = \frac{I_x}{I_1}$$





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$$\tau \frac{d}{dt} \left(\frac{I_y}{I_1} \right) + \frac{I_y}{I_1} = \frac{I_x}{I_1} \quad \Longrightarrow \quad \tau \frac{dI_y}{dt} + I_y = I_x.$$





$$\tau \frac{\partial I_y}{\partial V_y} \cdot \frac{dV_y}{dt} + I_y = I_x$$





$$\tau \frac{\partial I_y}{\partial V_y} \cdot \frac{dV_y}{dt} + I_y = I_x \quad \Longrightarrow \quad \tau \left(-\frac{w}{U_T} I_y \right) \frac{dV_y}{dt} + I_y = I_x$$





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$$\implies -\frac{w\tau}{U_{\rm T}} \cdot \frac{dV_y}{dt} + 1 = \frac{I_x}{I_y}$$





To implement the time derivative, we introduce a log-compressed voltage state variable, V_y . Using the chain rule, we can express the preceding equation as

$$\tau \frac{\partial I_y}{\partial V_y} \cdot \frac{dV_y}{dt} + I_y = I_x \quad \Longrightarrow \quad \tau \left(-\frac{w}{U_T} I_y \right) \frac{dV_y}{dt} + I_y = I_x$$

$$\implies -\frac{w\tau}{U_{\rm T}} \cdot \frac{dV_y}{dt} + 1 = \frac{I_x}{I_y} \implies -\frac{w\tau}{CU_{\rm T}} \cdot C\frac{dV_y}{dt} + 1 = \frac{I_x}{I_y}$$

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Synthesize a second-order low-pass filter described by

$$\tau^2 \frac{d^2 y}{dt^2} + \frac{\tau}{Q} \cdot \frac{dy}{dt} + y = x, \text{ where } x > 0.$$





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We can decompose this second-order ODE into two coupled first-order ODEs as

$$\tau \frac{d}{dt} \left(\tau \frac{dy}{dt} + \frac{y}{Q} \right) + y = x$$





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We can decompose this second-order ODE into two coupled first-order ODEs as

$$\tau \frac{d}{dt} \left(\underbrace{\tau \frac{dy}{dt} + \frac{y}{Q}}_{z} \right) + y = x \implies \begin{cases} \tau \frac{dz}{dt} = x - y \\ \tau \frac{dy}{dt} = z - \frac{y}{Q}. \end{cases}$$





We represent each signal as a ratio of a signal current to the unit current:

$$x \equiv \frac{I_x}{I_1}, \quad y \equiv \frac{I_y}{I_1}, \quad \text{and} \quad z \equiv \frac{I_z}{I_1}.$$





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Substituting these into the pair of ODEs, we obtain

$$\begin{cases} \tau \frac{d}{dt} \left(\frac{I_z}{I_1} \right) &= \frac{I_x}{I_1} - \frac{I_y}{I_1} \\ \tau \frac{d}{dt} \left(\frac{I_y}{I_1} \right) &= \frac{I_z}{I_1} - \frac{1}{Q} \cdot \frac{I_y}{I_1} \end{cases}$$





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To implement the time derivatives, we introduce log-compressed voltage state variables, V_z and V_y . Using the chain rule, we can express the preceding pair of equations as

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By introducing

$$I_{\tau} \equiv \frac{CU_{\rm T}}{w\tau}, \quad I_{cz} \equiv C\frac{dV_z}{dt}, \quad \text{and} \quad I_{cy} \equiv C\frac{dV_y}{dt},$$

we can express this pair of equations as

$$\begin{cases} \frac{I_{cz}}{I_{\tau}} &= \frac{I_y}{I_z} - \frac{I_x}{I_z} \\ \frac{I_{cy}}{I_{\tau}} &= \frac{1}{Q} - \frac{I_z}{I_y} \end{cases}$$





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where we have further introduced

$$I_w \equiv \frac{I_y I_\tau}{I_z}, \quad I_{pz} \equiv \frac{I_x I_\tau}{I_z}, \quad \text{and} \quad I_{py} \equiv \frac{I_z I_\tau}{I_y}.$$





TLP:
$$I_z I_{pz} = I_x I_\tau$$
 $I_y I_{py} = I_z I_\tau$ KCL: $I_{pz} + I_{cz} = I_w$
 $I_z I_w = I_y I_\tau$ $I_{py} + I_{cy} = I_\tau/Q$





TLP: $I_z I_{pz} = I_x I_\tau$ $I_y I_{py} = I_z I_\tau$ KCL: $I_{pz} + I_{cz} = I_w$ $I_z I_w = I_u I_\tau$



 $I_{py} + I_{cy} = I_{\tau}/Q$













 V_4°

ORNELL



 V_8°

























































































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