Translinear Analog Signal Processing: A Modular Aproach to Low-Voltage Analog Computation with Logarithms

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Translinear Circuits: What's in a Name?

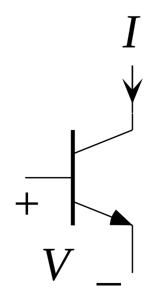
- Circuits made of devices whose *trans*conductances are *linear* in the currents flowing in them:

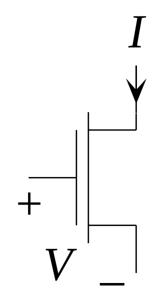
$$g_m \equiv \frac{\partial I}{\partial V} \propto I$$

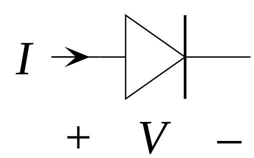
$$\Rightarrow \int \frac{\partial I}{I} \propto \int \partial V$$

$$\Rightarrow \log I \propto V$$

$$\Rightarrow I \propto e^V$$





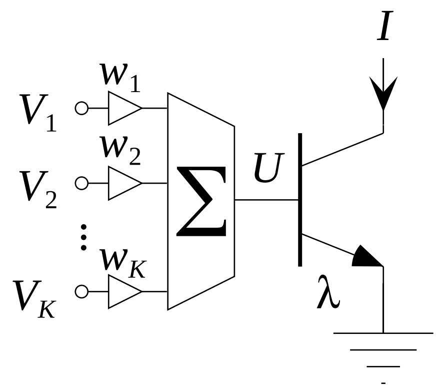


- Circuits whose behavior is conveniently expressed as a *trans*lation of *linear* relationships among voltages into product-of-power-law relationships among currents:

$$V_3 = w_1 V_1 + w_2 V_2 \xleftrightarrow{I \propto e^V} I_3 = I_1^{w_1} \times I_2^{w_2}$$

The Multiple-Input Translinear Element

$$I = \lambda I_s \exp\left[\sum_{k=1}^{K} \frac{w_k V_k}{U_T}\right]$$



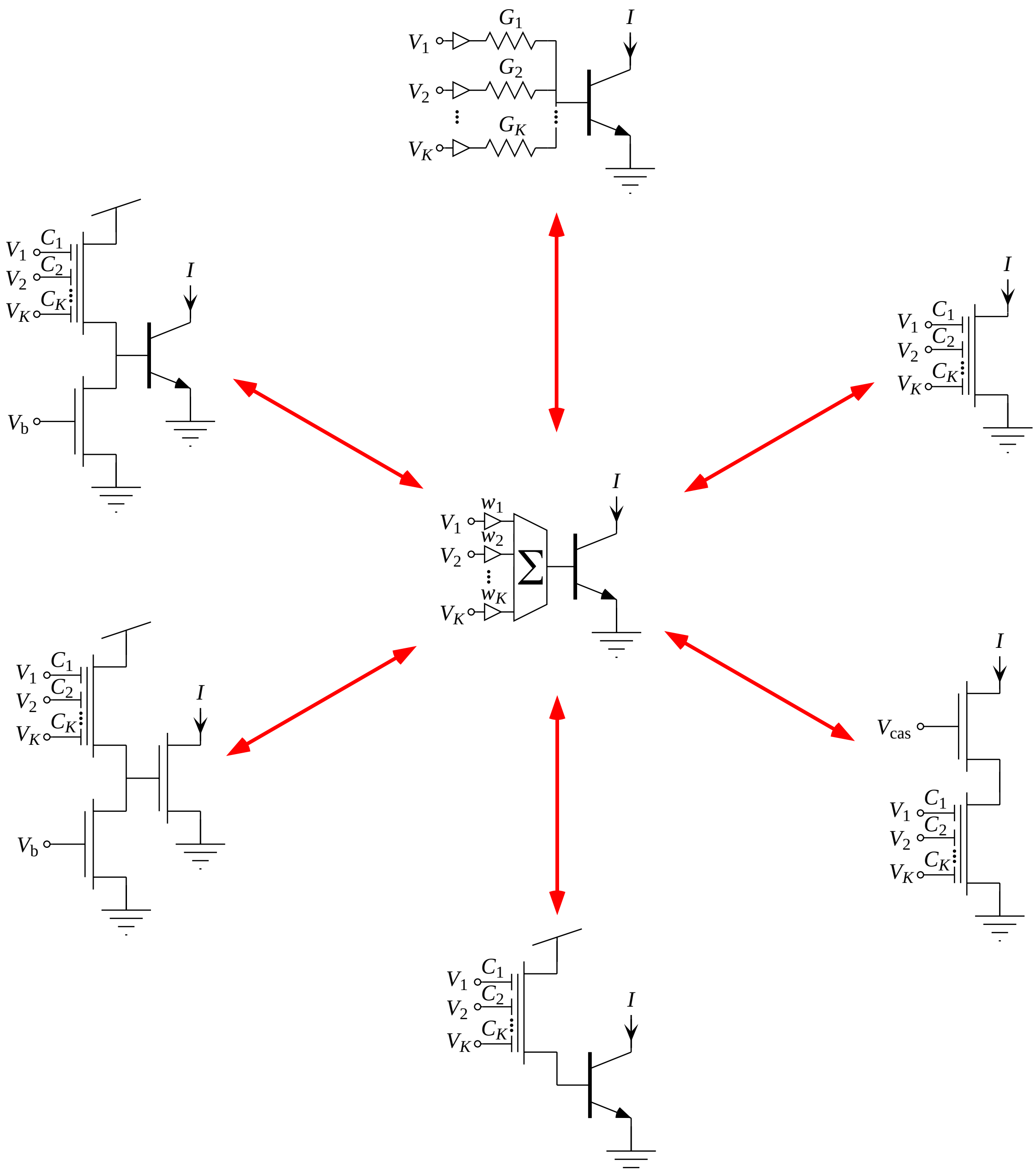
- The MITE has K *trans*conductances, each of which is *linear* in the output current, I:

$$g_k = \frac{\partial I}{\partial V_k}$$

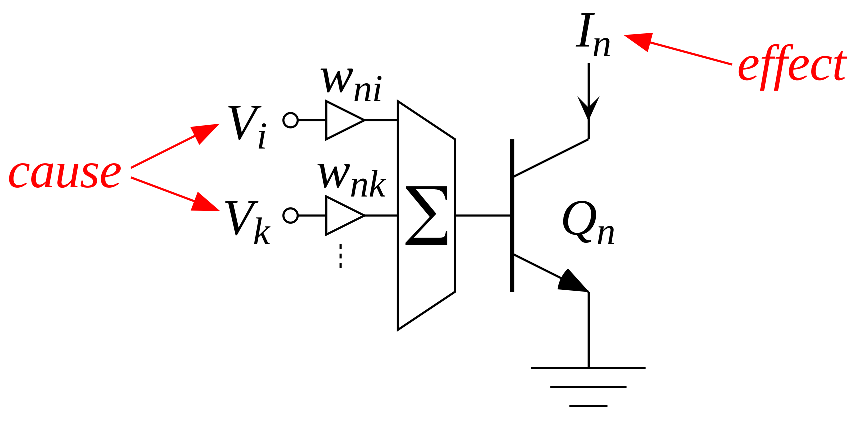
$$= \frac{w_k}{U_T} \lambda I_s \exp\left[\sum_{k=1}^{K} \frac{w_k V_k}{U_T}\right]$$

$$= \frac{w_k}{U_T} I$$

Multiple-Input Translinear Element Implementations



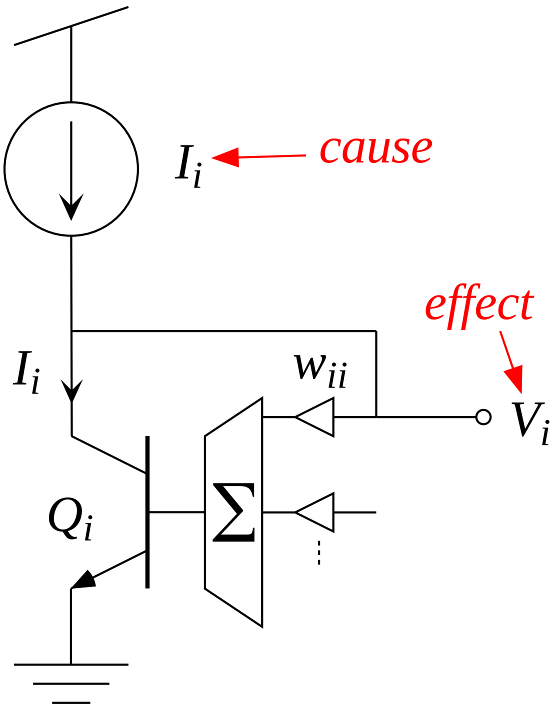
Basic MITE Configurations: Voltage-In, Current-Out



$$I_n \propto \exp\left[\frac{w_{ni}V_i + w_{nk}V_k + \ldots}{U_T}\right]$$

$$\Rightarrow \boxed{I_n \propto \exp\left[\frac{w_{ni}V_i}{U_T}\right]\exp\left[\frac{w_{nk}V_k}{U_T}\right]}$$

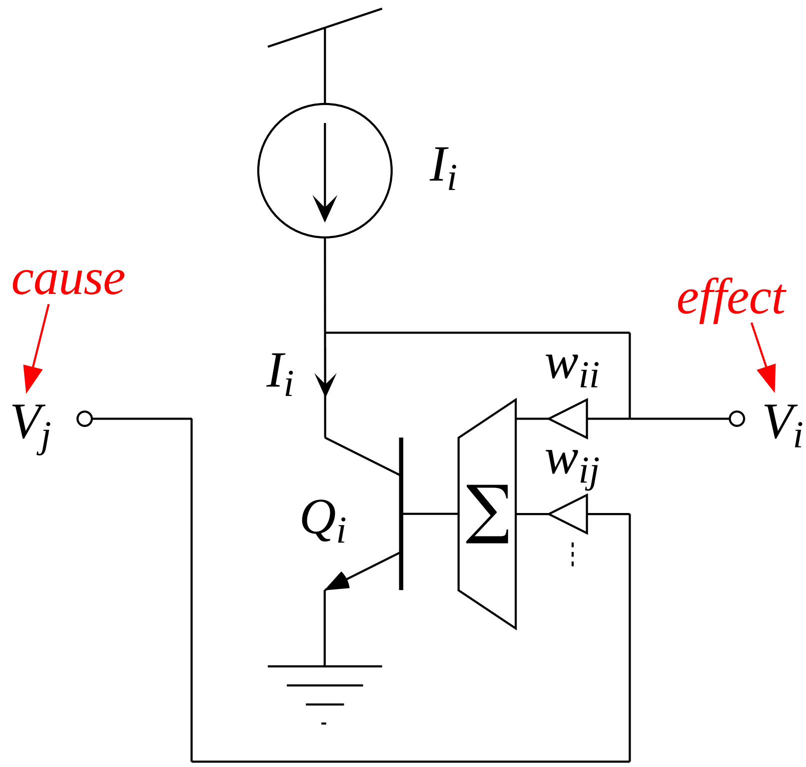
Basic MITE Configurations: Current-In, Voltage-Out



$$I_i \propto \exp\left[\frac{w_{ii} V_i + \ldots}{U_T}\right]$$

$$\Rightarrow \boxed{V_i = \frac{U_T}{w_{ii}} \log I_i - \ldots}$$

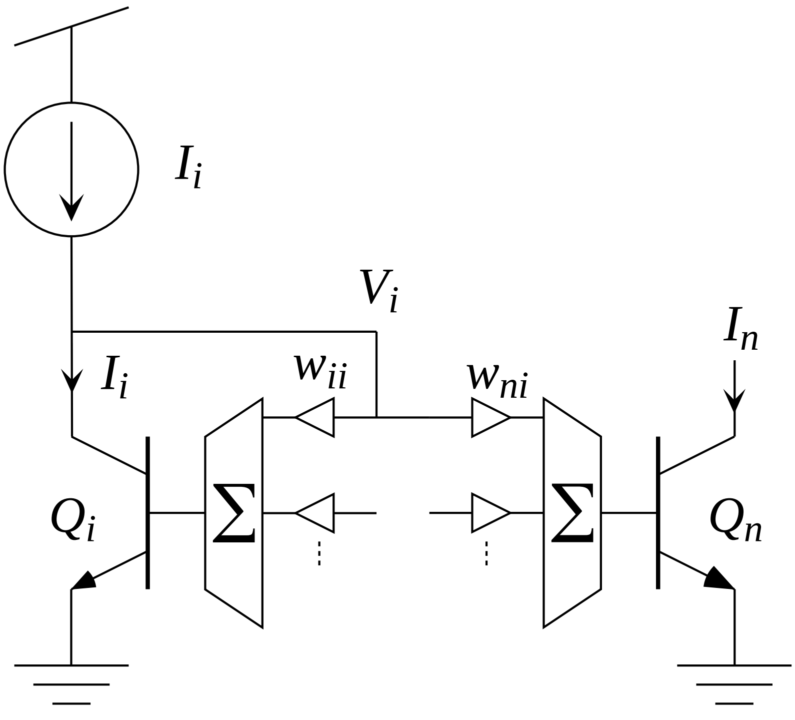
Basic MITE Configurations: Voltage-In, Voltage-Out



$$I_i \propto \exp\left[\frac{w_{ii} V_i + w_{ij} V_j + \ldots}{U_T}\right]$$

$$\Rightarrow \boxed{V_i = \frac{U_T}{w_{ii}} \log I_i - \frac{w_{ij}}{w_{ii}} V_j - \ldots}$$

MITE Networks: Current-In, Current-Out Power-Law Circuits



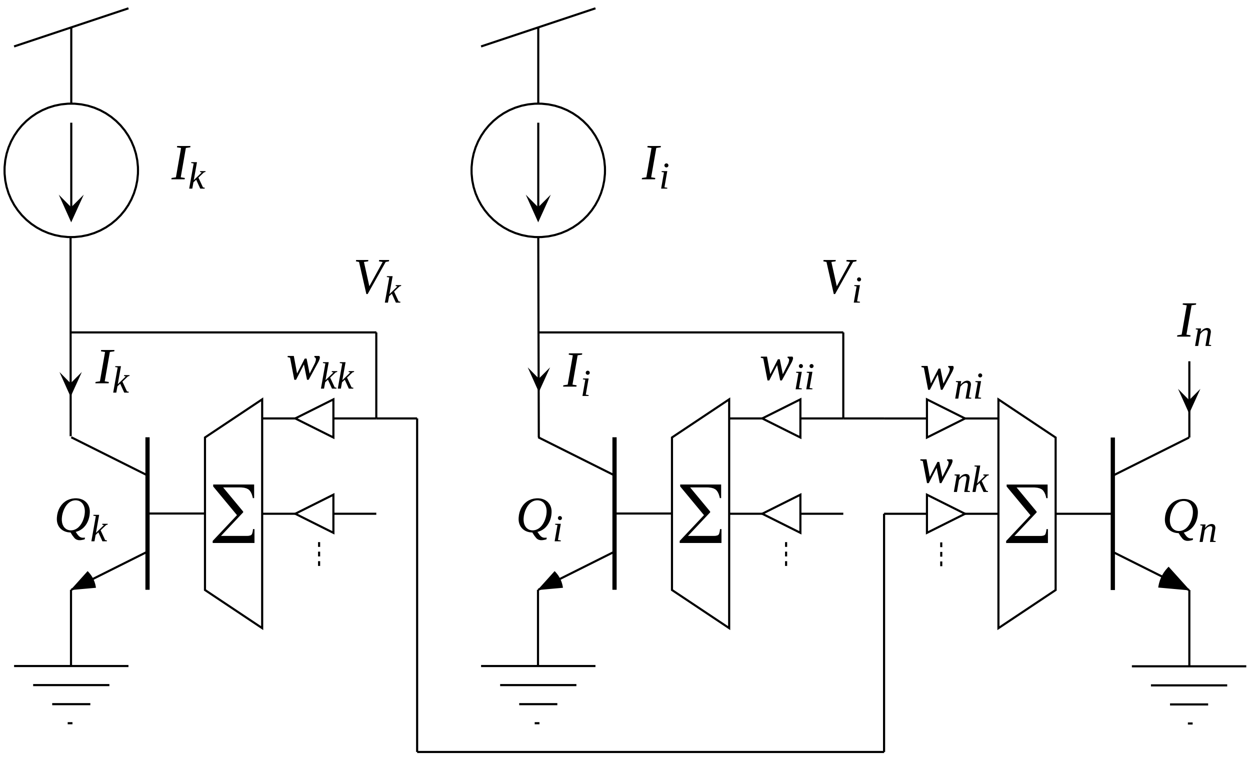
$$I_n \propto \exp\left[\frac{w_{ni}V_i + \ldots}{U_T}\right]$$

$$\Rightarrow I_n \propto \exp\left[\frac{w_{ni}}{U_T}\left(\frac{U_T}{w_{ii}}\log I_i - \ldots\right)\right]$$

$$\Rightarrow I_n \propto \exp\left[\frac{U_T}{U_T}\frac{w_{ni}}{w_{ii}}\log I_i\right]$$

$$\Rightarrow \boxed{I_n \propto I_i^{\frac{w_{ni}}{w_{ii}}}}$$

MITE Networks: Current-In, Current-Out Product-of-Power-Law Circuits



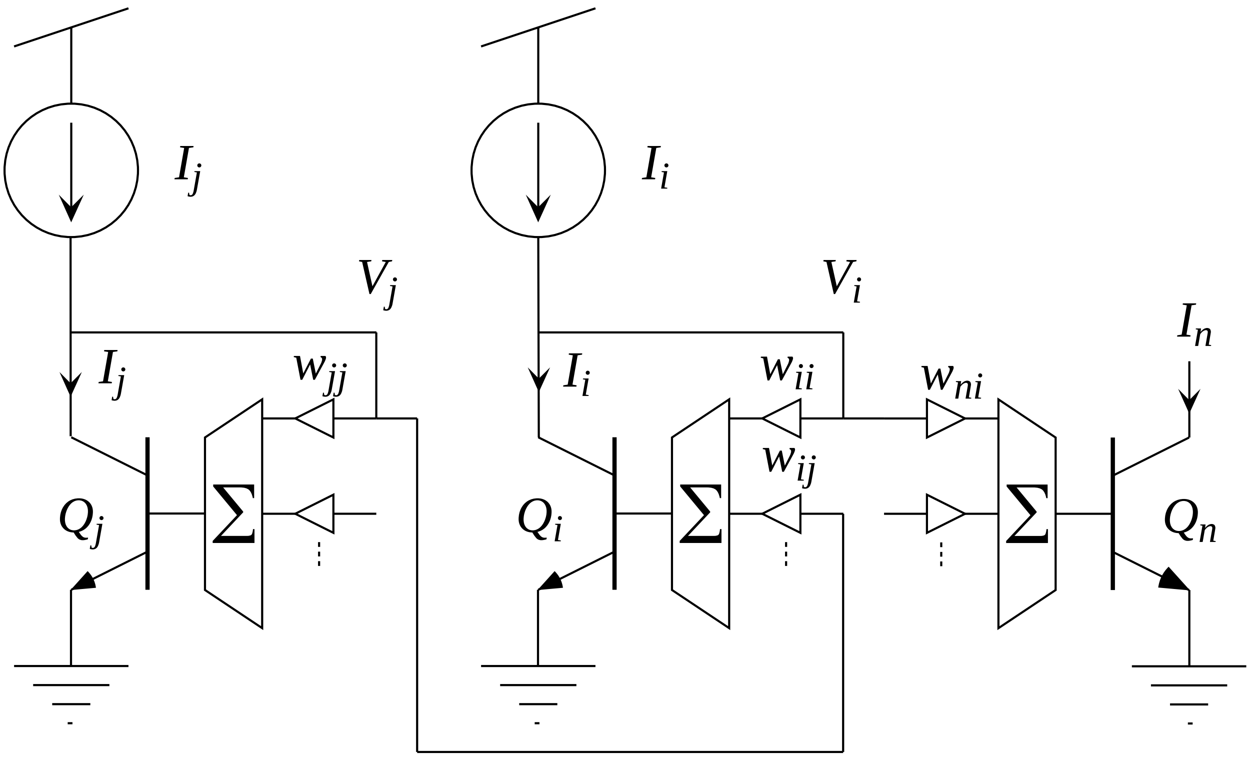
$$I_n \propto \exp\left[\frac{w_{ni} V_i}{U_T}\right] \exp\left[\frac{w_{nk} V_k}{U_T}\right]$$

$$\Rightarrow I_n \propto \exp\left[\frac{w_{ni}}{U_T}\left(\frac{U_T}{w_{ii}} \log I_i - \ldots\right)\right] \exp\left[\frac{w_{nk}}{U_T}\left(\frac{U_T}{w_{kk}} \log I_k - \ldots\right)\right]$$

$$\Rightarrow I_n \propto \exp\left[\frac{\cancel{U_T}}{\cancel{U_T}} \frac{w_{ni}}{w_{ii}} \log I_i\right] \exp\left[\frac{\cancel{U_T}}{\cancel{U_T}} \frac{w_{nk}}{w_{kk}} \log I_k\right]$$

$$\Rightarrow \boxed{I_n \propto I_i^{\frac{w_{ni}}{w_{ii}}} \times I_k^{\frac{w_{nk}}{w_{kk}}}}$$

MITE Networks: Current-In, Current-Out Quotient-of-Power-Law Circuits



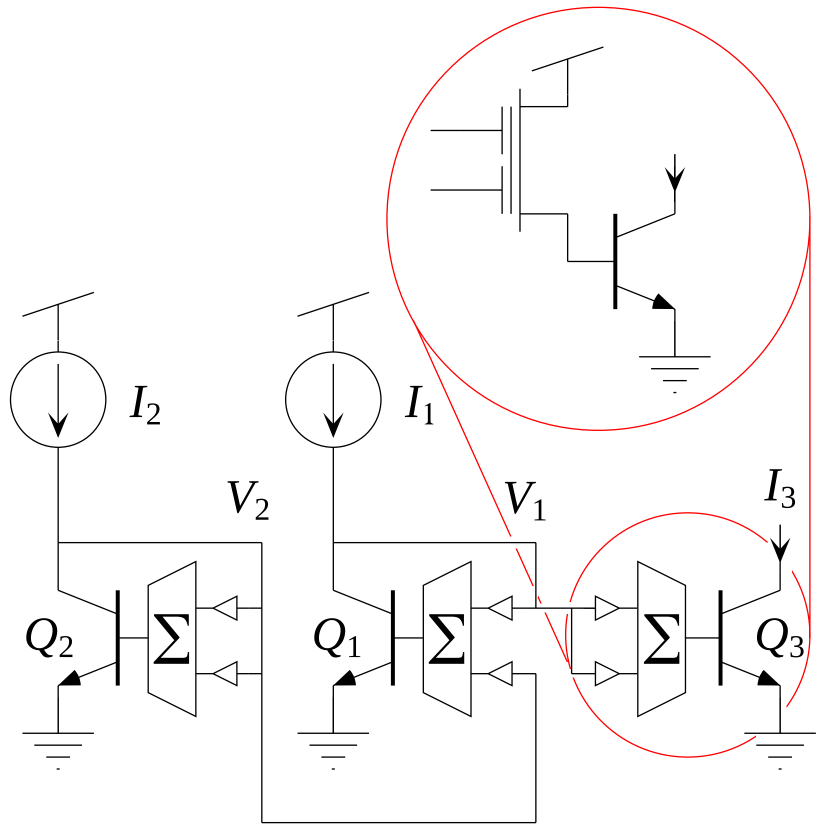
$$I_n \propto \exp\left[\frac{w_{ni}V_i + \ldots}{U_T}\right]$$

$$\Rightarrow I_n \propto \exp\left[\frac{w_{ni}}{U_T}\left(\frac{U_T}{w_{ii}}\log I_i - \frac{w_{ij}}{w_{ii}}\left(\frac{U_T}{w_{jj}}\log I_j - \ldots\right)\ldots\right)\right]$$

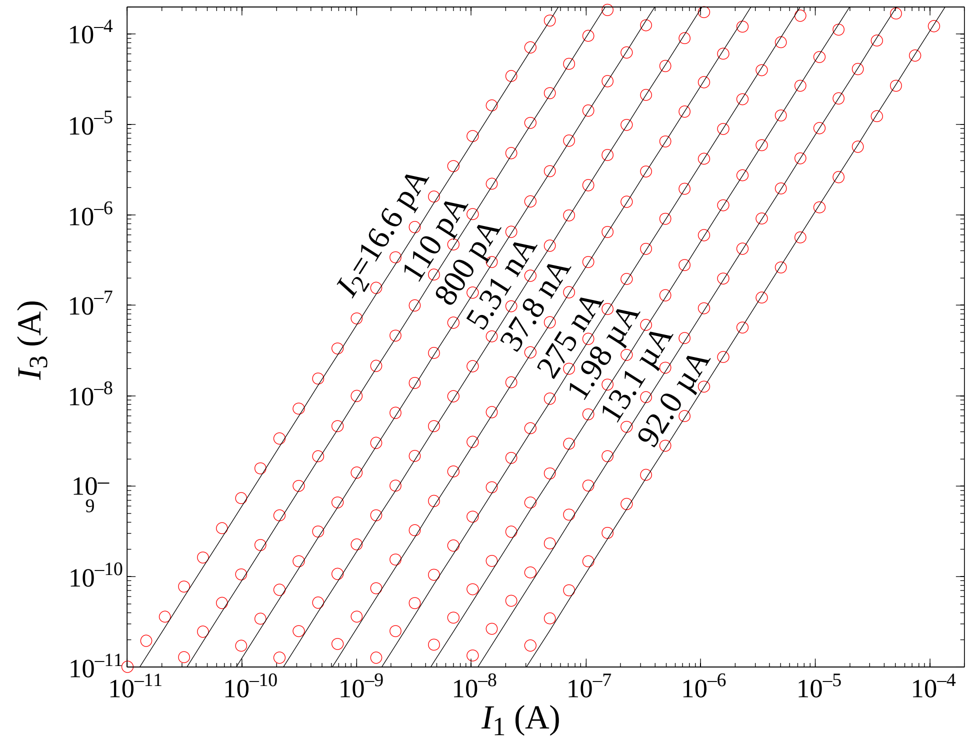
$$\Rightarrow I_n \propto \exp\left[\frac{\cancel{U_T}}{\cancel{U_T}}\frac{w_{ni}}{w_{ii}}\log I_i\right]\exp\left[-\frac{\cancel{U_T}}{\cancel{U_T}}\frac{w_{ni}}{w_{ii}}\frac{w_{ij}}{w_{jj}}\log I_j\right]$$

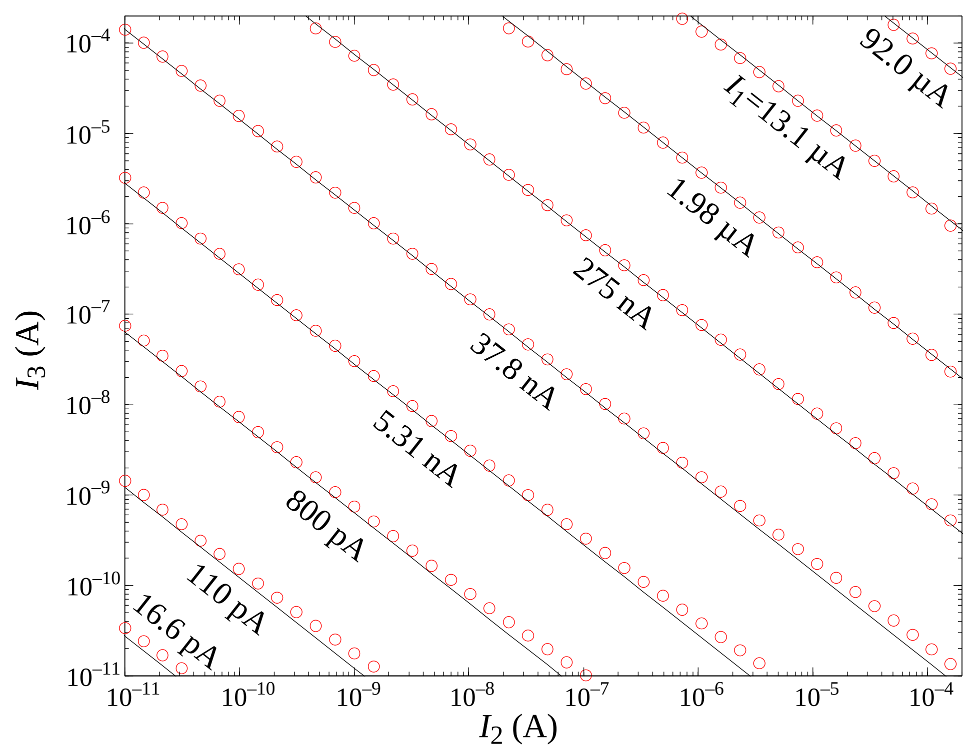
$$\Rightarrow I_n \propto I_i^{\frac{w_{ni}}{w_{ii}}} \times I_j^{-\frac{w_{ni}}{w_{ii}}\frac{w_{ij}}{w_{jj}}} \Rightarrow \boxed{I_n \propto I_i^{\frac{w_{ni}}{w_{ii}}} \div I_j^{\frac{w_{ni}}{w_{ii}}\frac{w_{ij}}{w_{jj}}}}$$

MITE Networks: Square/Reciprocal

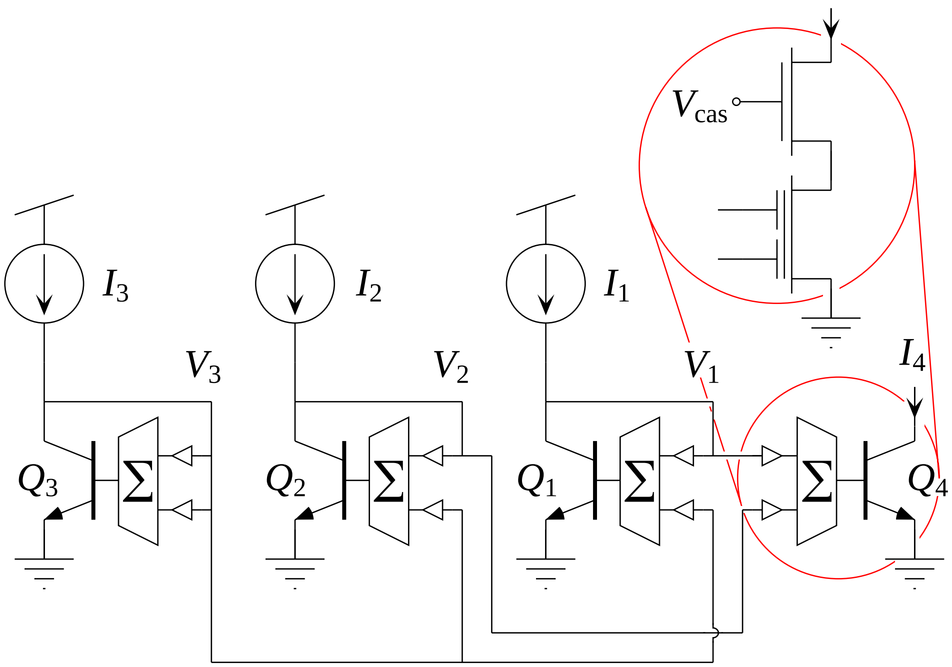


$$I_3 = \frac{I_1^2}{I_2}$$

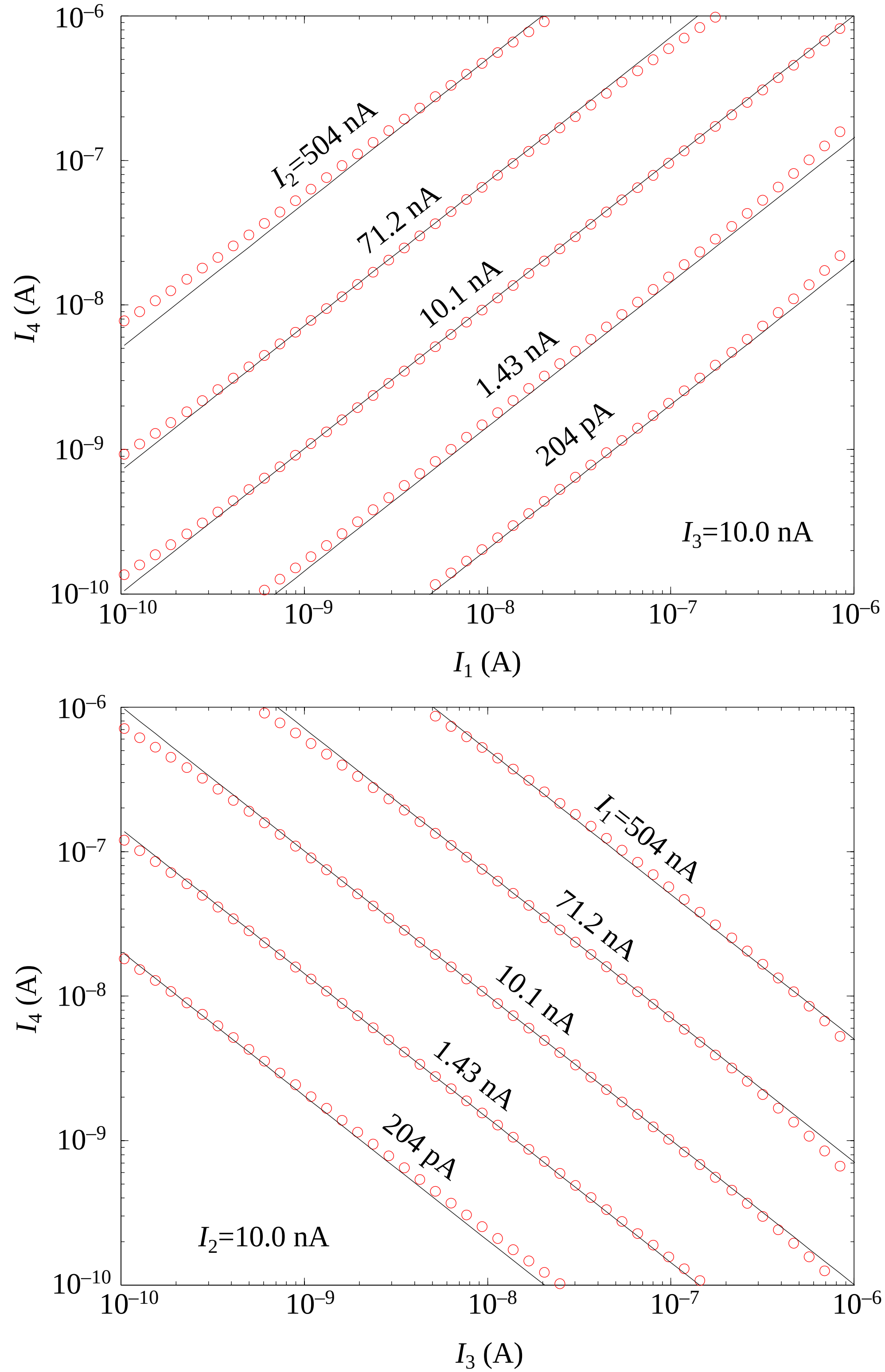




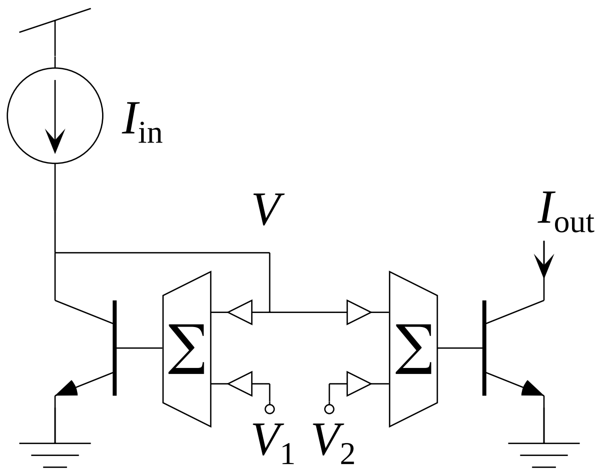
MITE Networks: Multiply/Reciprocal



$$I_4 = \frac{I_1 I_2}{I_3}$$



MITE Log-Domain Filter Building Block



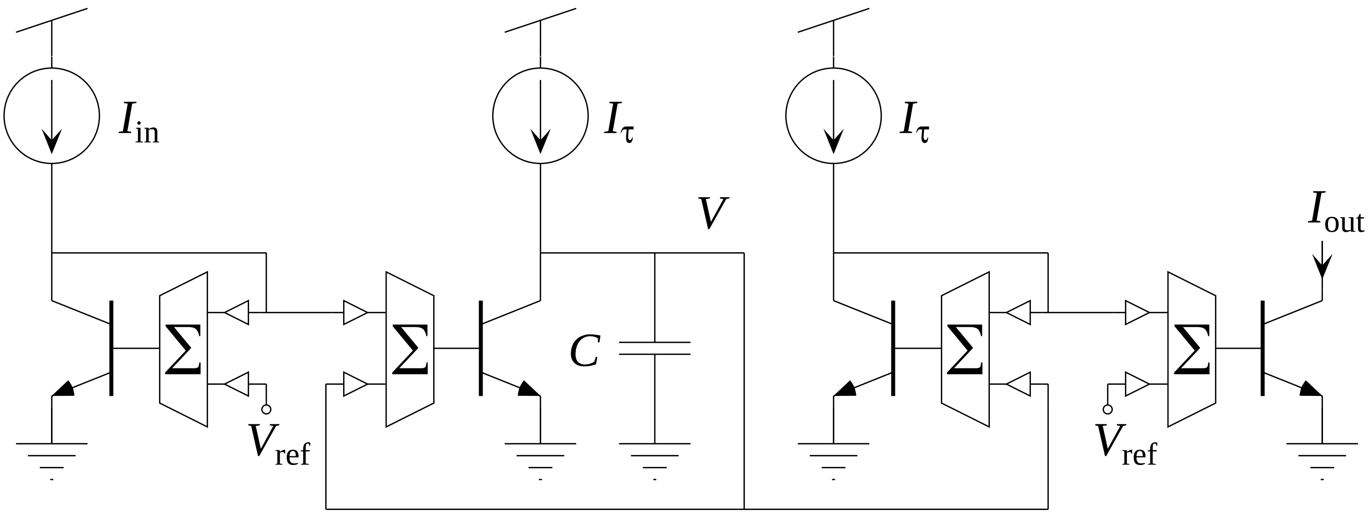
$$I_{in} = I_s e^{w(V+V_1)/U_T}$$

$$I_{out} = I_s e^{w(V+V_2)/U_T}$$

$$\Rightarrow \frac{I_{out}}{I_{in}} = \frac{I_s}{I_s} \frac{e^{wV/U_T}}{e^{wV/U_T}} \frac{e^{wV_2/U_T}}{e^{wV_1/U_T}}$$

$$\Rightarrow \boxed{I_{out} = I_{in} e^{w(V_2 - V_1)/U_T}}$$

MITE Log-Domain Filters



$$I_{out} = I_\tau e^{w(V_{ref} - V)/U_T}$$

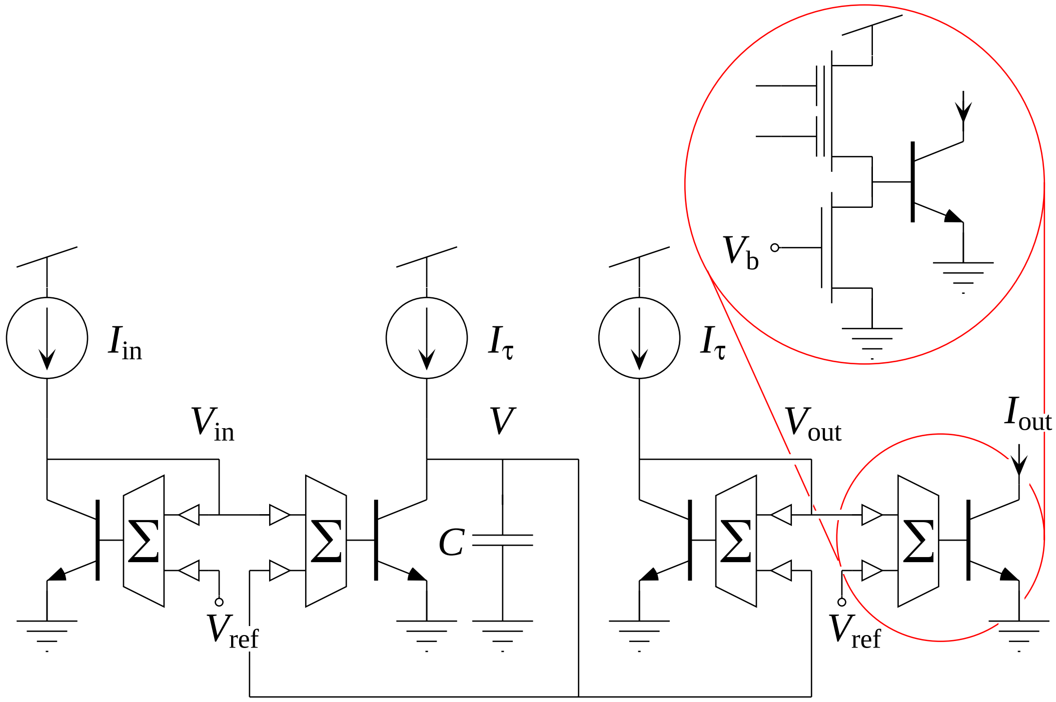
$$\Rightarrow \frac{dV}{dt} = -\frac{U_T}{w}\frac{1}{I_{out}}\frac{dI_{out}}{dt} \quad \text{and} \quad e^{w(V - V_{ref})/U_T} = \frac{I_\tau}{I_{out}}$$

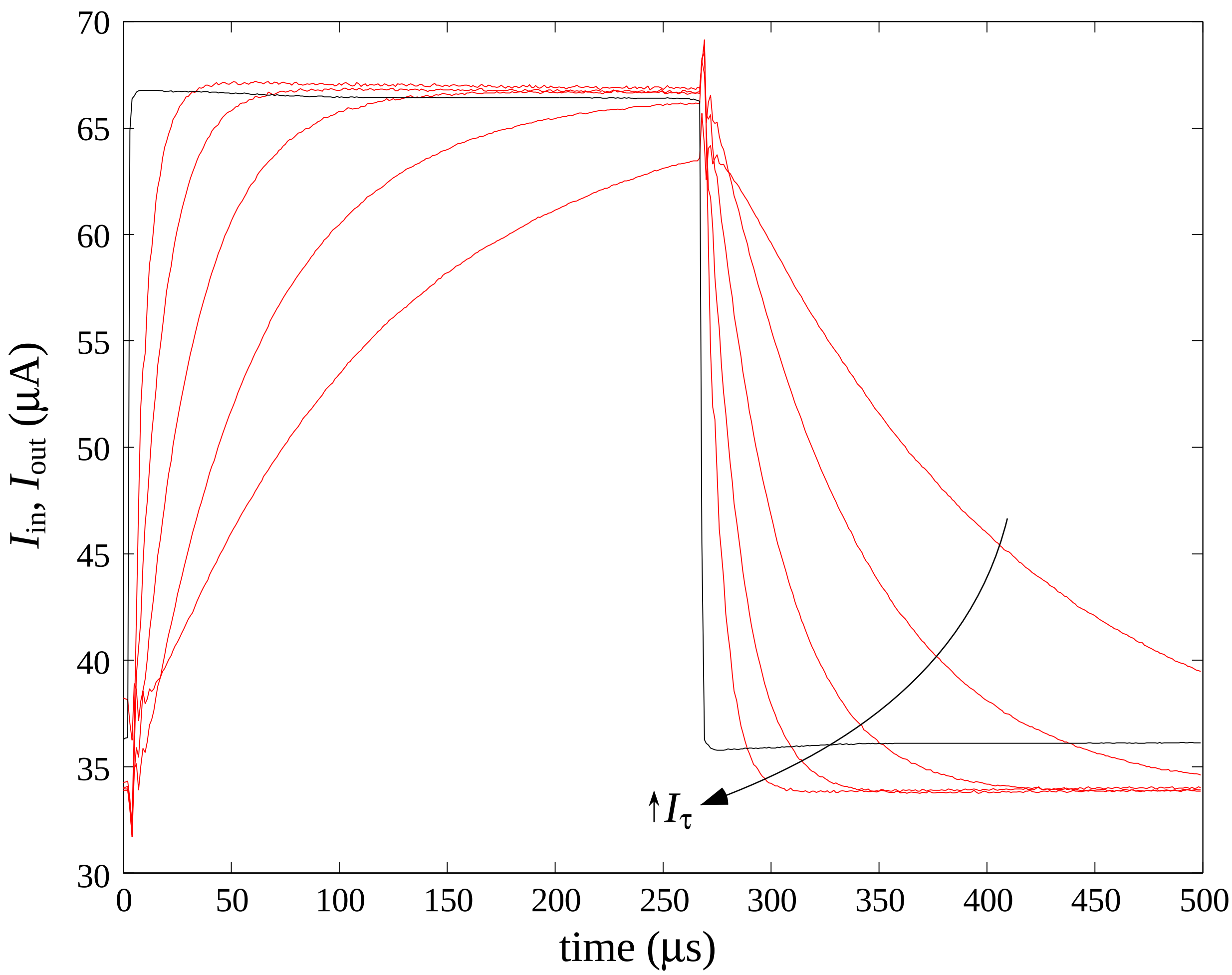
$$\text{KCL} \Rightarrow C\frac{dV}{dt} = I_\tau - I_{in} e^{w(V - V_{ref})/U_T}$$

$$\Rightarrow -\frac{CU_T}{w}\frac{1}{I_{out}}\frac{dI_{out}}{dt} = I_\tau - I_{in}\frac{I_\tau}{I_{out}}$$

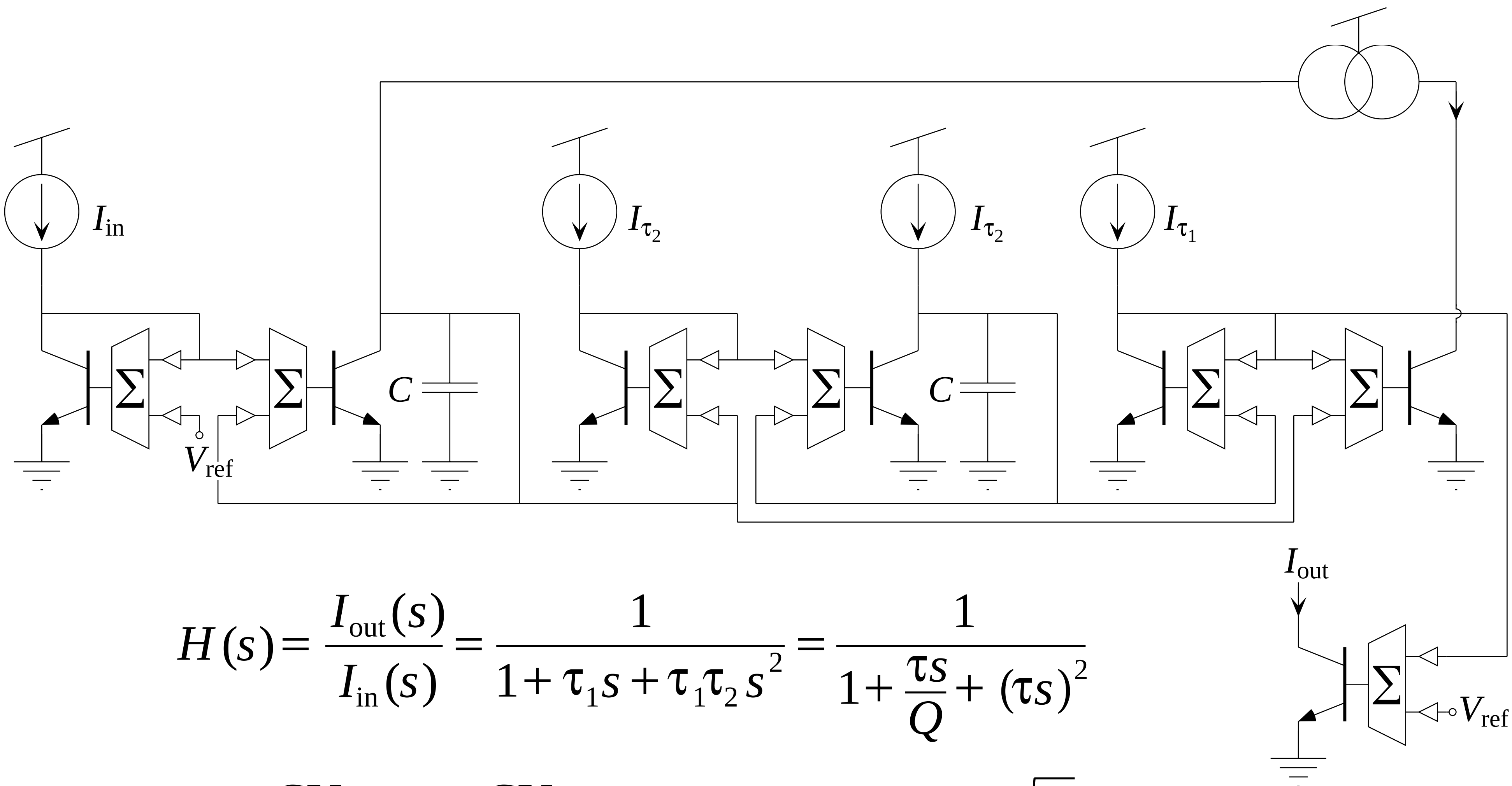
$$\tau \equiv \frac{CU_T}{wI_\tau} \Rightarrow \boxed{\tau\frac{dI_{out}}{dt} + I_{out} = I_{in}}$$

MITE Log-Domain First-Order Lowpass Filter



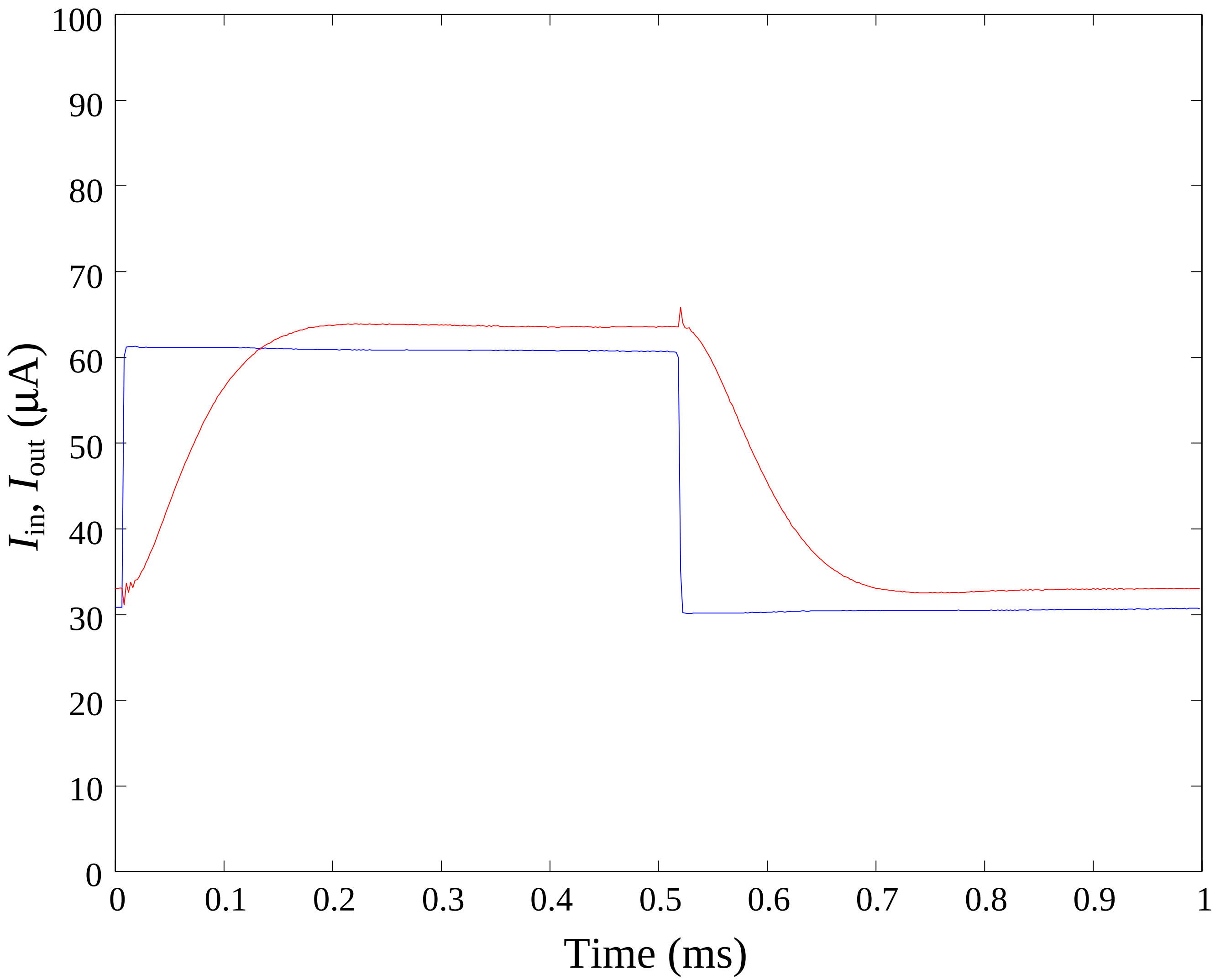


MITE Log-Domain Second-Order Lowpass Filter



$$H(s) = \frac{I_{out}(s)}{I_{in}(s)} = \frac{1}{1 + \tau_1 s + \tau_1 \tau_2 s^2} = \frac{1}{1 + \frac{\tau s}{Q} + (\tau s)^2}$$

$$\tau_1 = \frac{C U_T}{w I_{\tau_1}} \quad \tau_2 = \frac{C U_T}{w I_{\tau_2}} \qquad \tau \equiv \sqrt{\tau_1 \tau_2} \quad Q \equiv \sqrt{\frac{\tau_2}{\tau_1}}$$



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10
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I_{in}, I_{out} (μA)
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0.5
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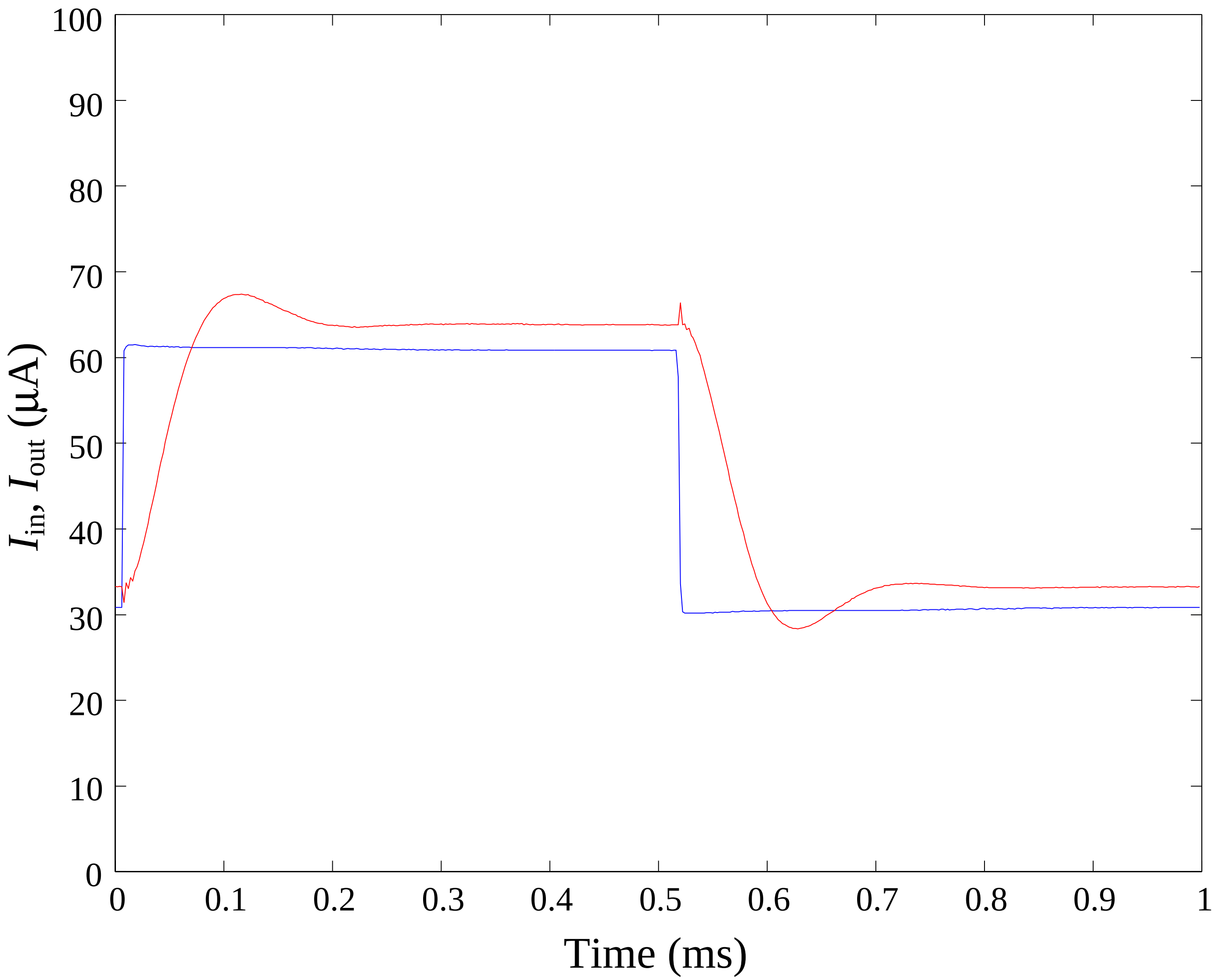
0.7
0.8
0.9
1
Time (ms)



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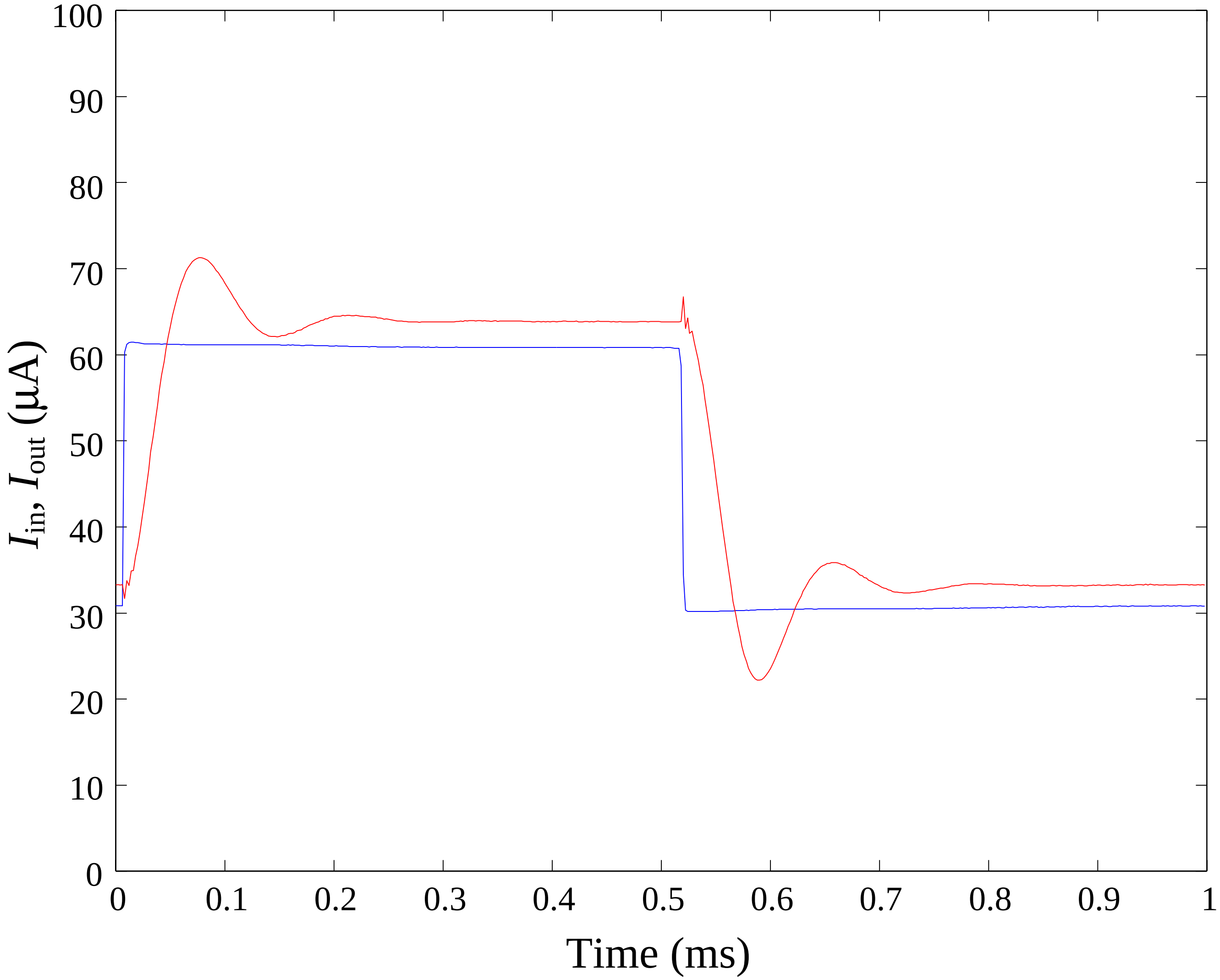
I_{in}, I_{out} (μA)
Time (ms)



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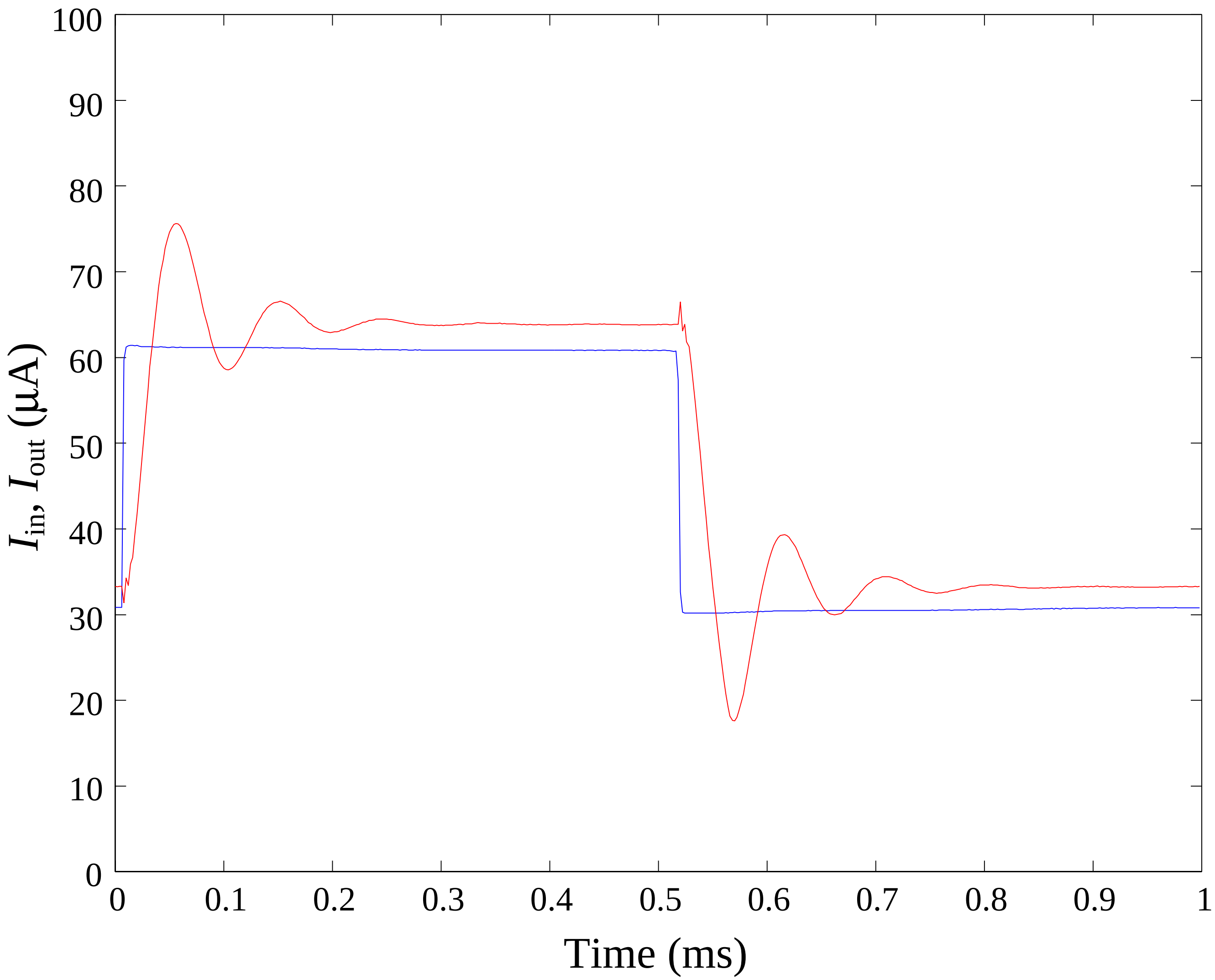
Time (ms)
I_{in}, I_{out} (μA)



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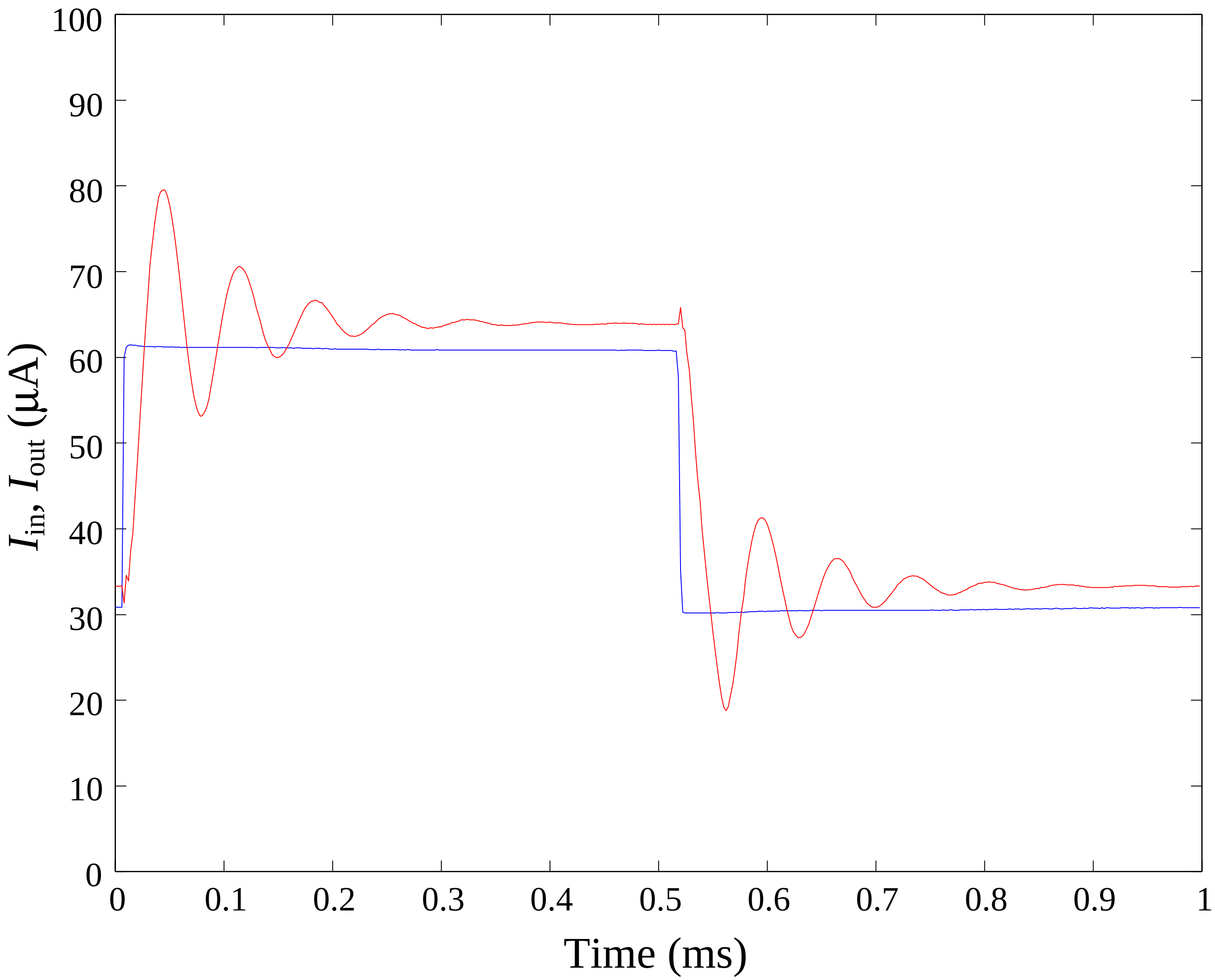
I_in, I_out (μA)
Time (ms)



I_{in}, I_{out} (μA)
Time (ms)
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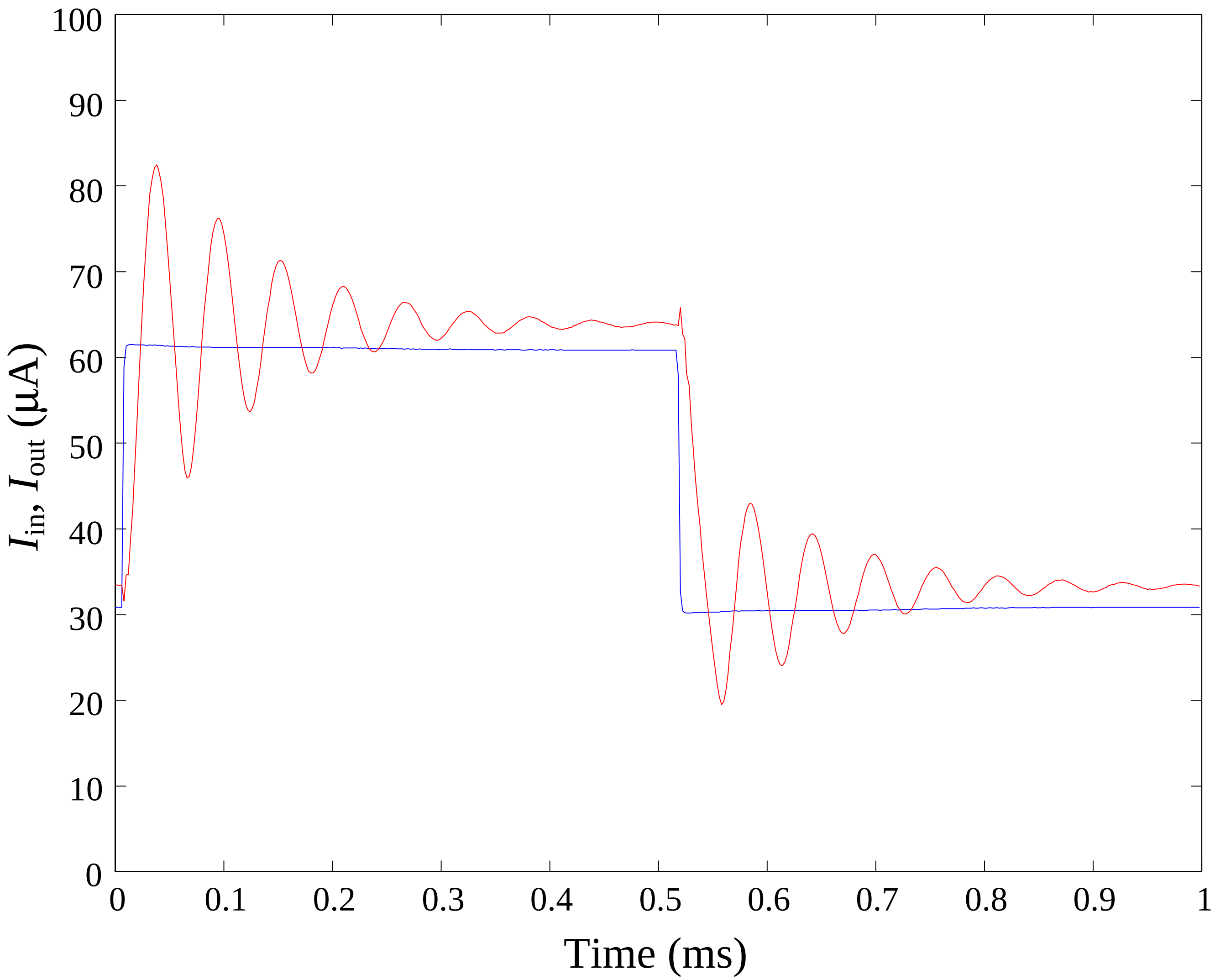
1



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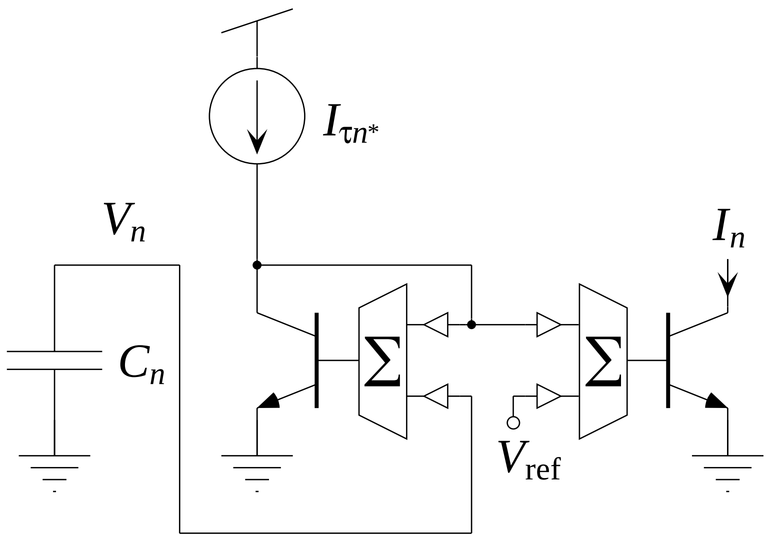
1
I_{in}, I_{out} (μA)
Time (ms)

MITE Log-Domain Filters: Output Structure



$$\tau_n \frac{dI_n}{dt} = \ldots$$

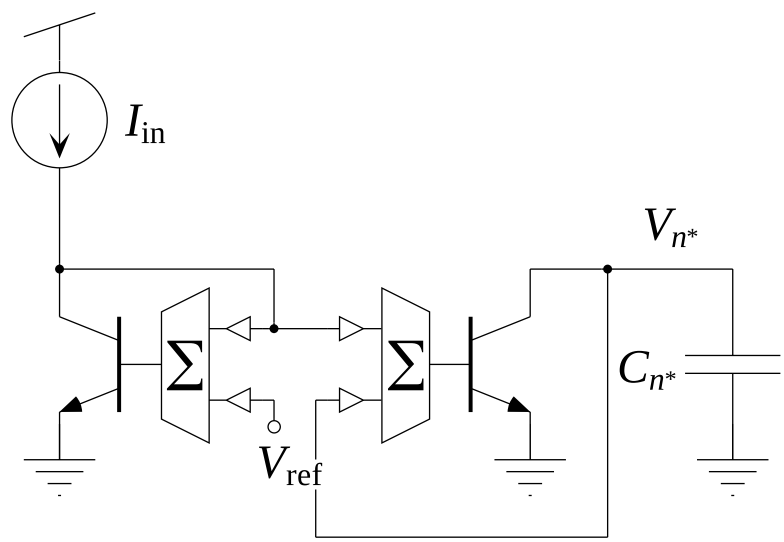
Note: n^* is the index of the state that is excited by the external input.

$$\tau_n = \frac{C_n U_T}{w I_{\tau n}}$$

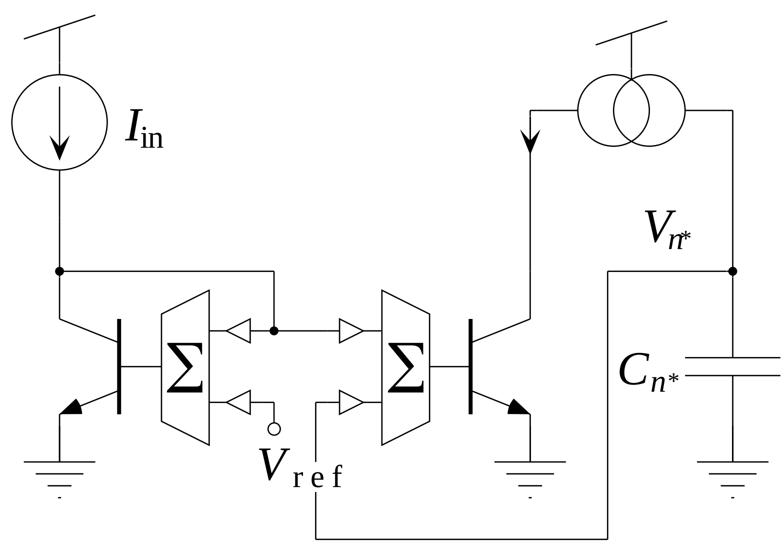
$$I_n = I_{\tau n^*} e^{w(V_{\text{ref}} - V_n)/U_T}$$

$$\frac{dV_n}{dt} = -\frac{U_T}{w} \frac{1}{I_n} \frac{dI_n}{dt}$$

MITE Log-Domain Filters: Input Structures

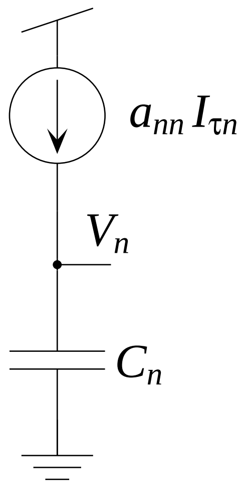


$$\tau_{n*}\frac{dI_{n*}}{dt} = \ldots + I_{\text{in}} - \ldots$$

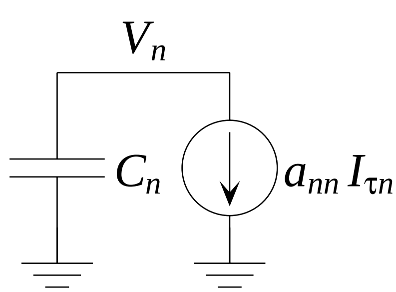


$$\tau_{n*}\frac{dI_{n*}}{dt} = \ldots - I_{\text{in}} - \ldots$$

MITE Log-Domain Filters: Diagonal Coupling Terms

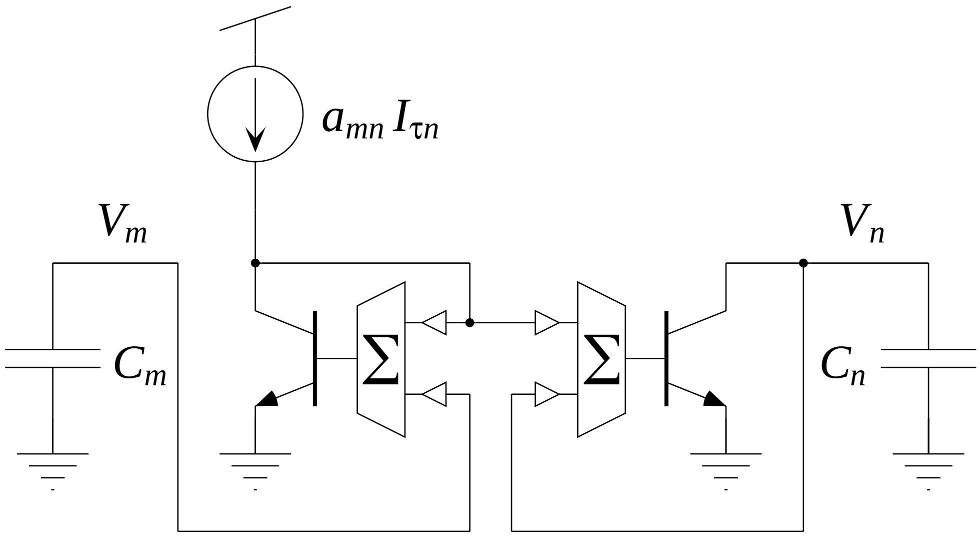


$$\tau_n \frac{dI_n}{dt} = \ldots - a_{nn} I_n - \ldots$$

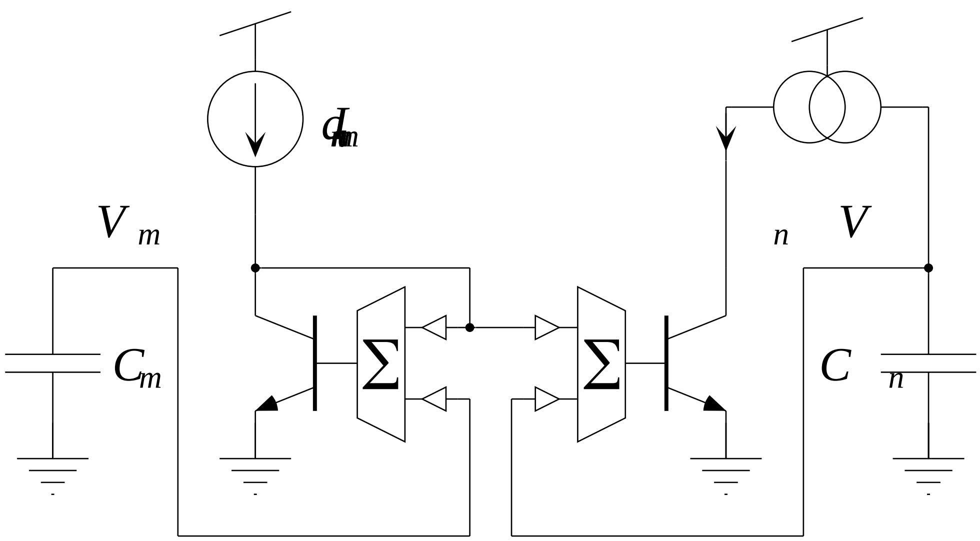


$$\tau_n \frac{dI_n}{dt} = \ldots + a_{nn} I_n - \ldots$$

MITE Log-Domain Filters: Off-Diagonal Coupling Terms

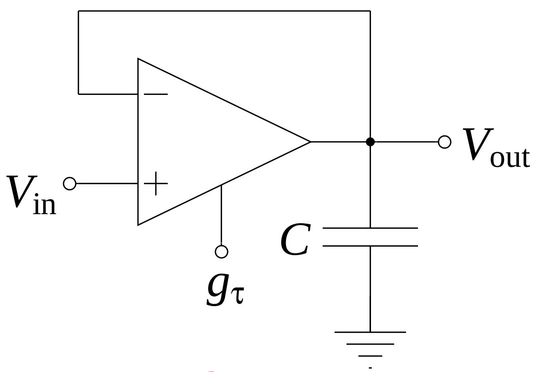


$$\tau_n \frac{dI_n}{dt} = \ldots + a_{nm} I_m - \ldots$$



$$\tau_n \frac{dI_n}{dt} = \ldots - a_{nm} I_m - \ldots$$

MITE Log-Domain Filters: Synthesis of a First-Order Low-Pass

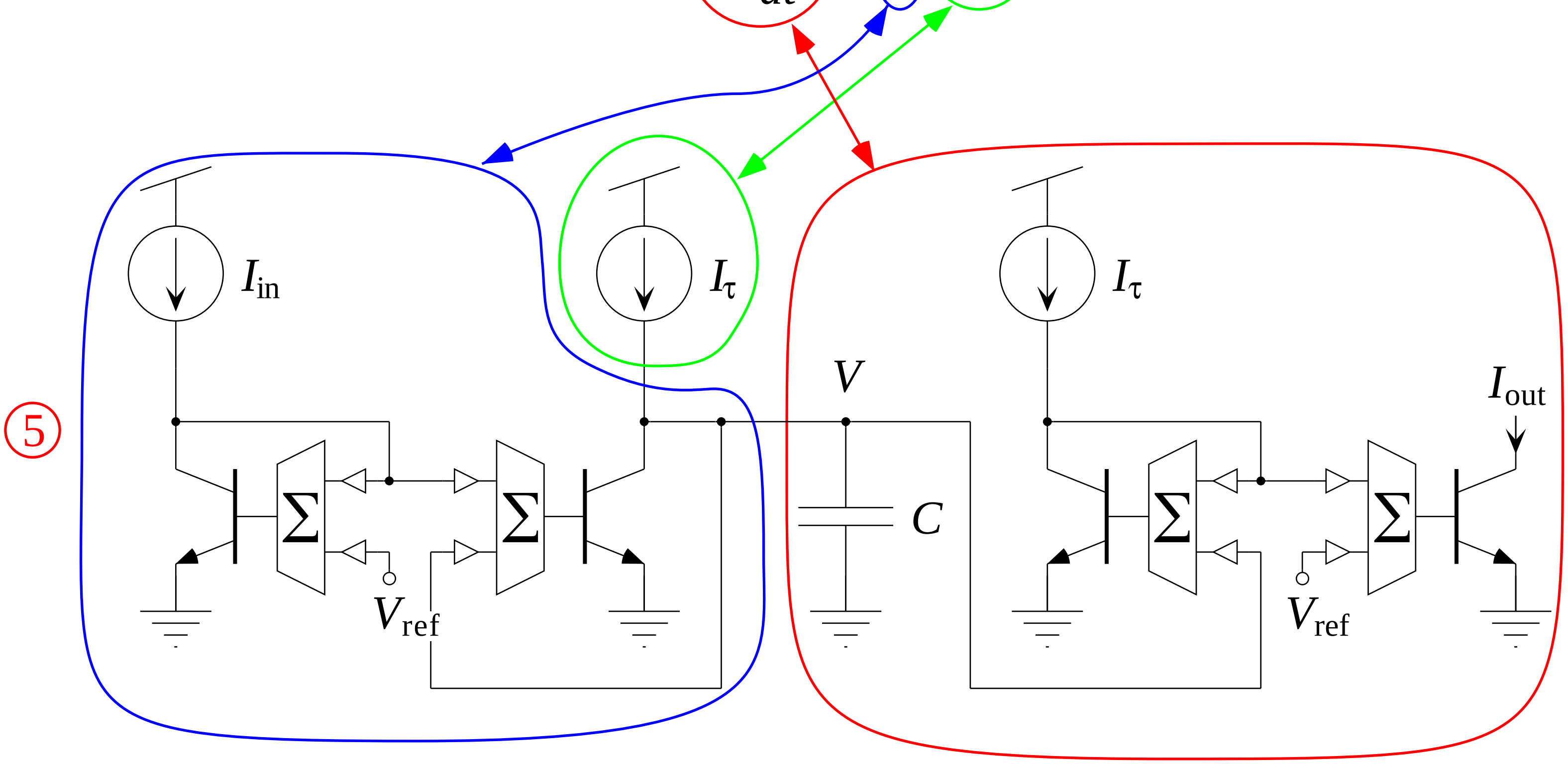


(1)

(2) $$C\frac{dV_{out}}{dt} = g_\tau(V_{in} - V_{out})$$

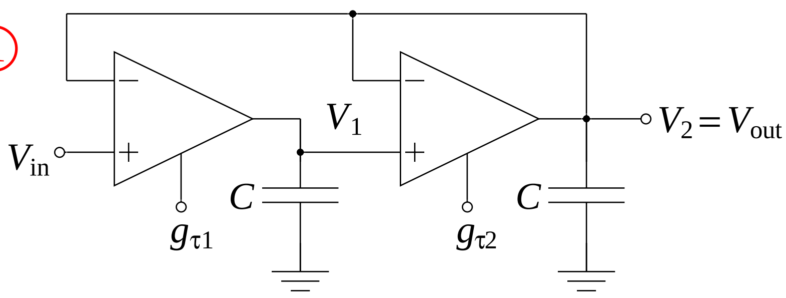
(3) $$\tau\frac{dV_{out}}{dt} = V_{in} - V_{out}$$

(4) $$\tau\frac{dI_{out}}{dt} = I_{in} - I_{out}$$



MITE Log-Domain Filters: Synthesis of a Second-Order Low-Pass

1

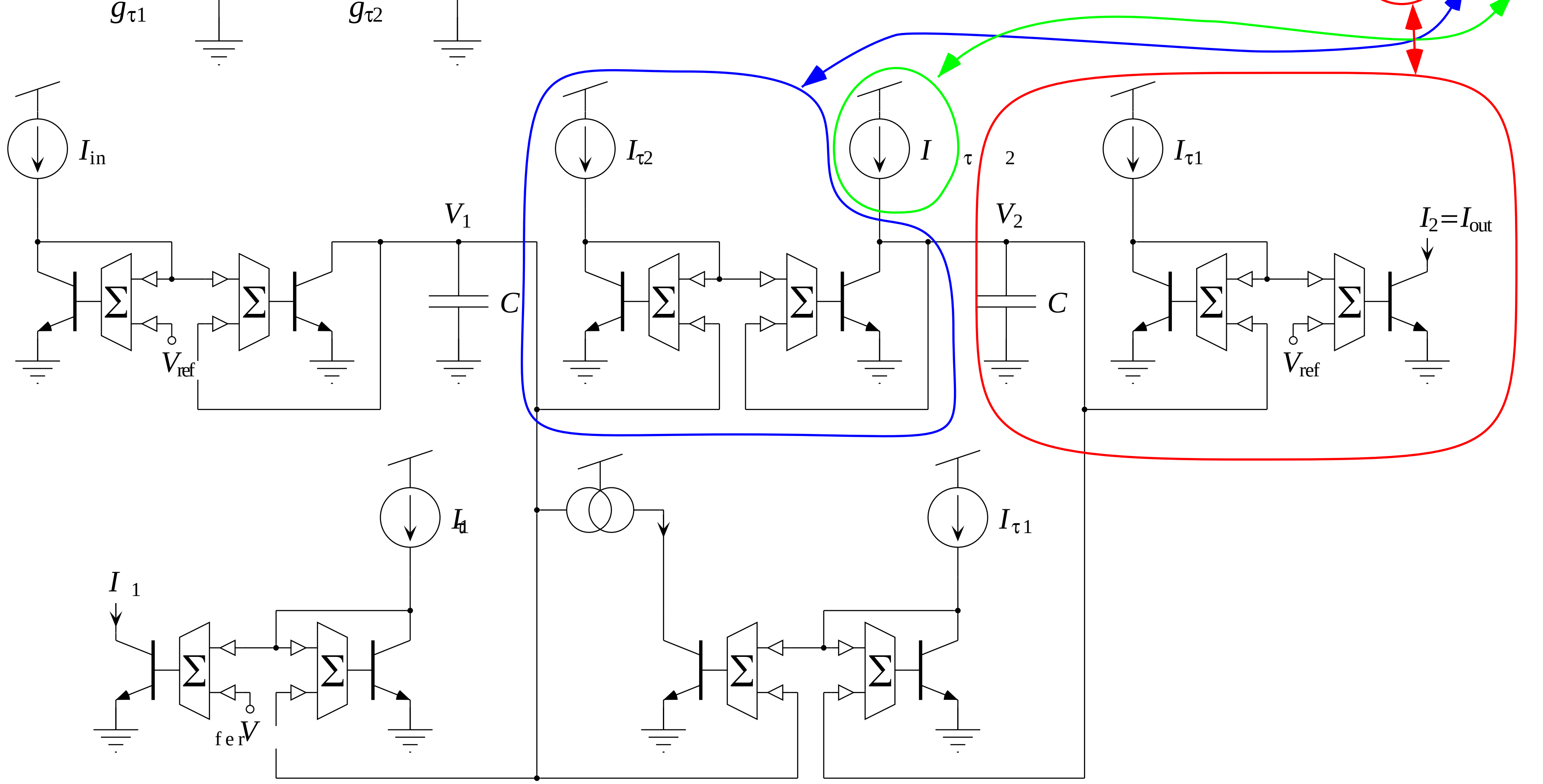


2
$$\begin{cases} C\dfrac{dV_1}{dt} = g_{\tau 1}(V_{in} - V_2) \\ C\dfrac{dV_2}{dt} = g_{\tau 2}(V_1 - V_2) \end{cases}$$

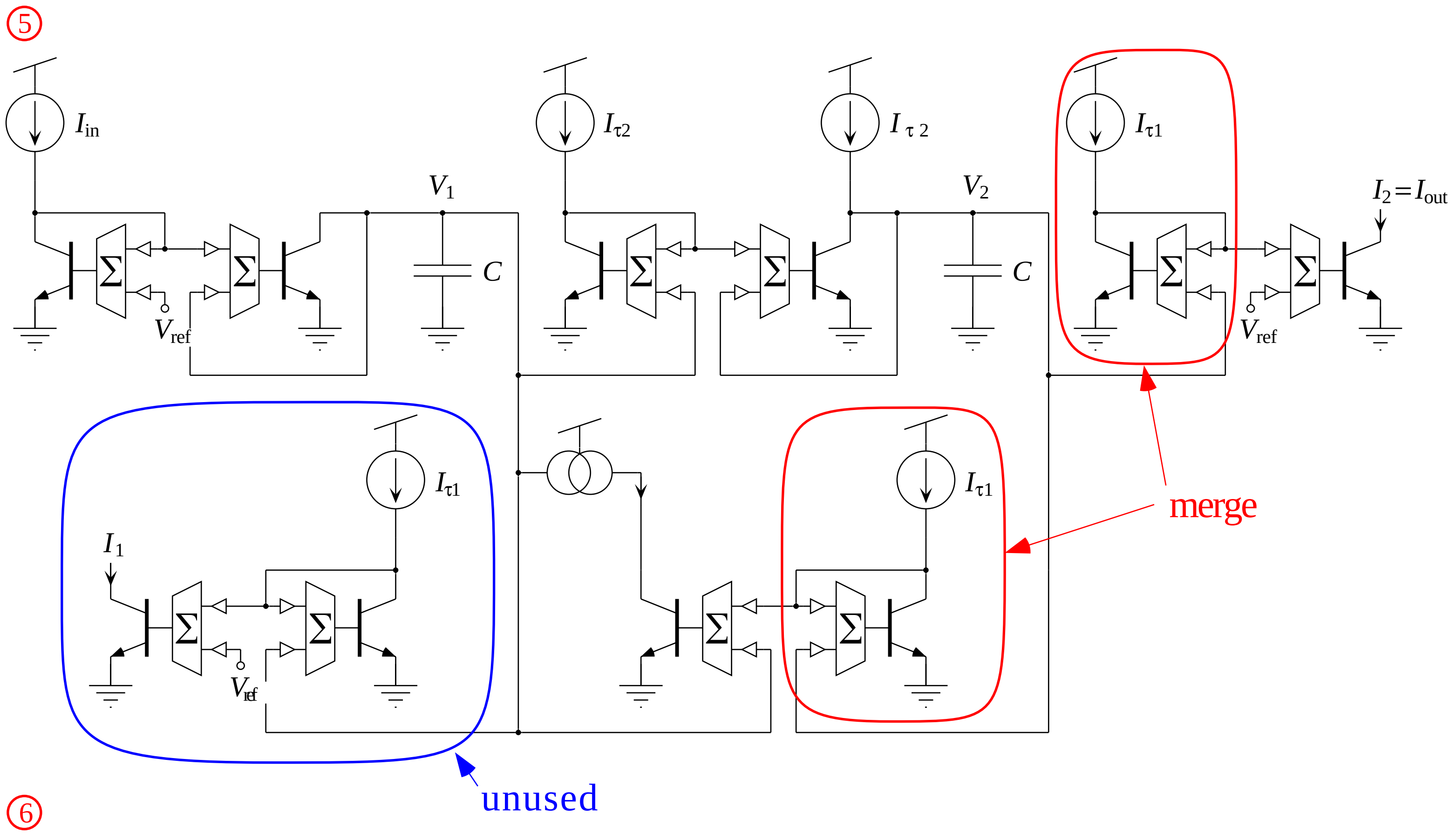
3
$$\begin{cases} \tau_1\dfrac{dV_1}{dt} = V_{in} - V_2 \\ \tau_2\dfrac{dV_2}{dt} = V_1 - V_2 \end{cases}$$

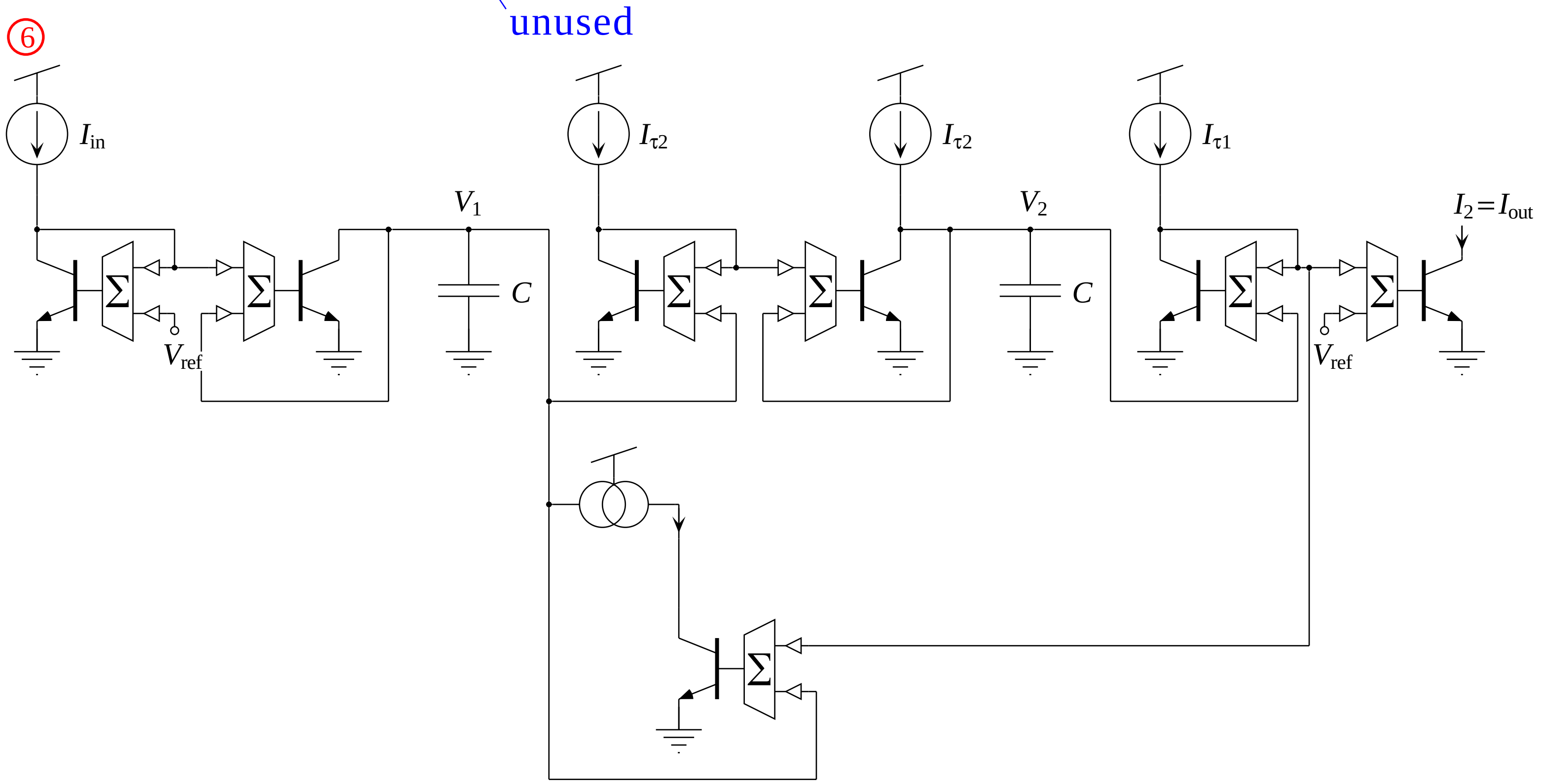
4
$$\begin{cases} \tau_1\dfrac{dI_1}{dt} = I_{in} - I_2 \\ \tau_2\dfrac{dI_2}{dt} = I_1 - I_2 \end{cases}$$

5



MITE Log-Domain Filters: Synthesis of a Second-Order Low-Pass





Translinear Analog Signal Processing

$$x(t) \equiv \frac{I_x(t)}{I_u} \qquad y(t) \equiv \frac{I_y(t)}{I_u}$$

$$z(t) \equiv \frac{I_z(t)}{I_u} = y(t)\int_0^{\infty} x(t-\Delta)e^{-\Delta/\tau}\,d\Delta$$