

# Signal Processing On A Shrinking Supply

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Mixed Analog-Digital VLSI Circuits and Systems Lab

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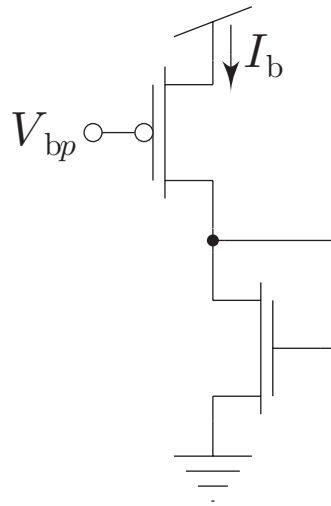
The logo consists of a solid red square with the word "CORNELL" written in white, serif, uppercase letters across the center.

CORNELL

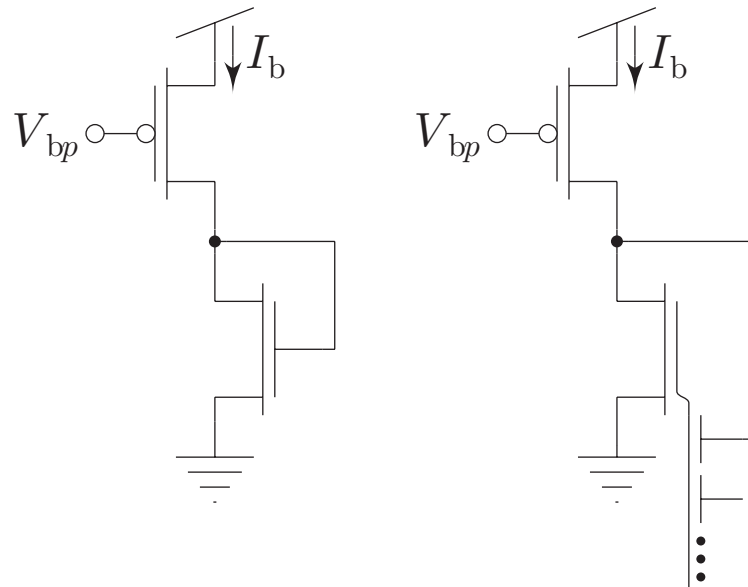
# Mixed Analog-Digital VLSI Circuits and Systems Lab

- **Research focus:** Low-voltage/low-power analog and mixed-signal circuit design
- **Current M.S./Ph.D. students:**  
Abhishek Kammula, Sunitha Bandla, Eric McDonald, Kofi Odame, Sheng-Yu Peng
- **Former M.S./Ph.D. students:**  
Karan Mathur, Mark Neidengard, Yuan Yang
- **Current projects:**
  - High-level synthesis of translinear and log-domain circuits and systems
  - Floating-gate MOS (FGMOS) circuit design
  - Double-gate MOS (DGMOS) modeling and circuit design
  - Chemical sensing with **chemoreceptive neuron MOS** (C $\nu$ MOS) transistors
  - **Electrochemical camera:** amperometric study of exocytosis

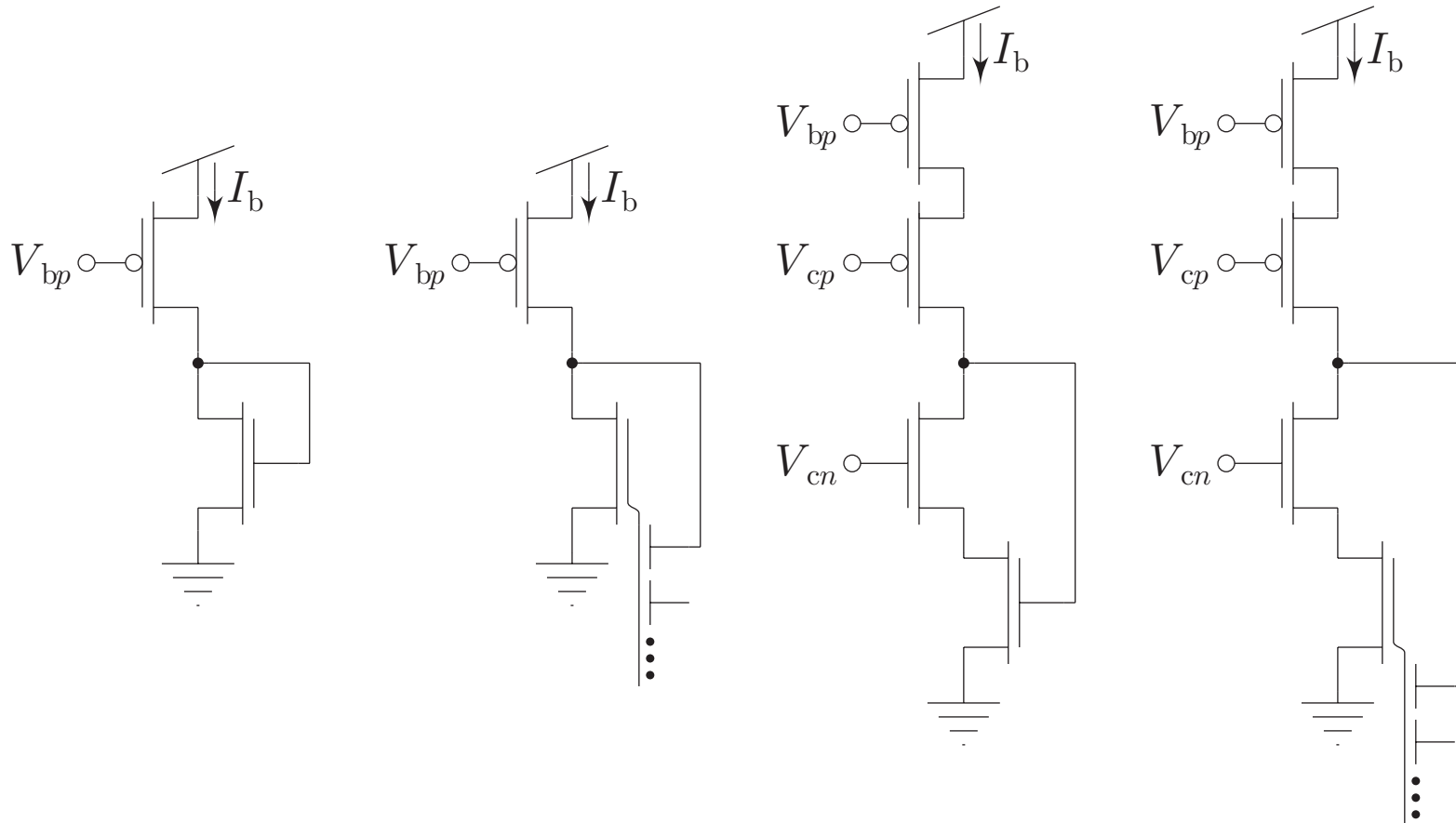
## Low Voltage: How Low Can We Go?



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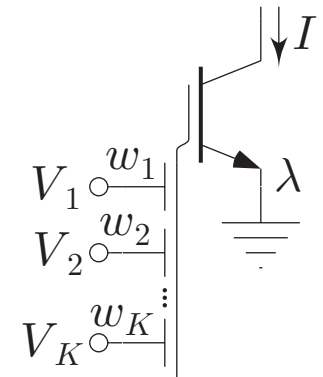
# The Ideal **M**ultiple-Input **T**ranslinear **E**lement

The ideal multiple-input translinear element (MITE) produces an output current given by

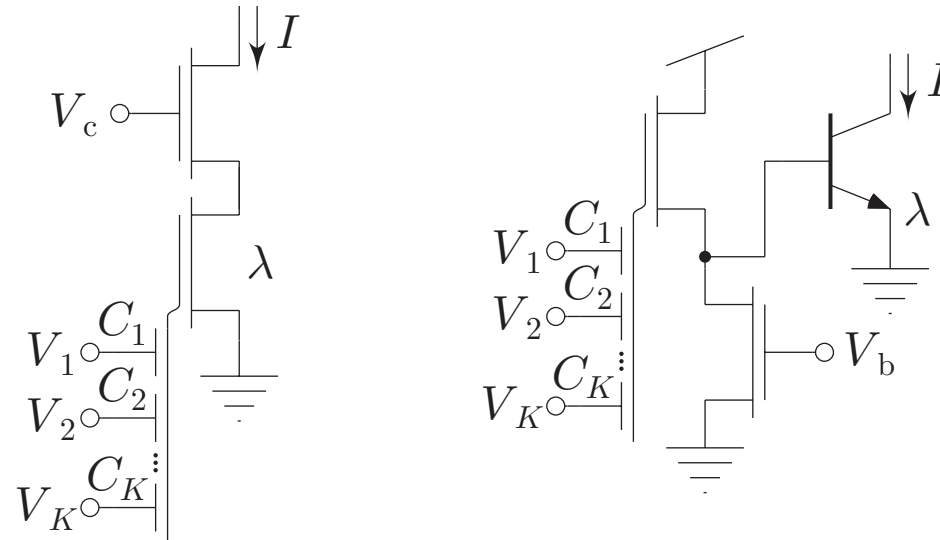
$$I = \lambda I_s e^{(w_1 V_1 + \dots + w_K V_K) / U_T}$$

where

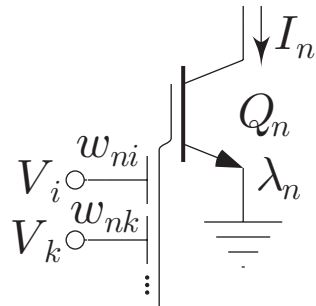
- $I_s$  pre-exponential scaling current
- $\lambda$  dimensionless constant scaling  $I_s$  proportionally
- $V_k$   $k$ th control-gate voltage
- $w_k$  dimensionless positive weight scaling  $V_k$
- $U_T$  thermal voltage,  $kT/q$ .



# Practical Floating-Gate MITE Implementations



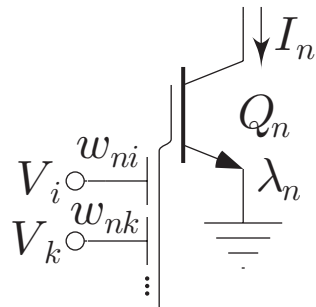
# Basic MITE Circuit Stages



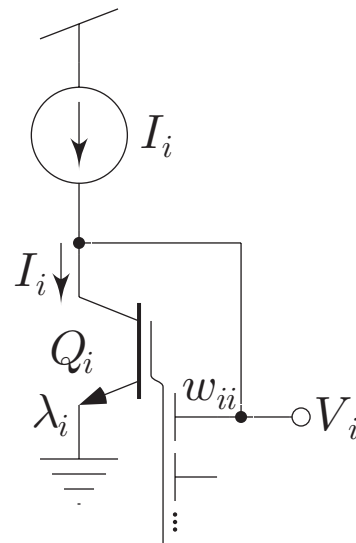
$$I_n \propto e^{w_{ni}V_i/U_T} e^{w_{nk}V_k/U_T}$$



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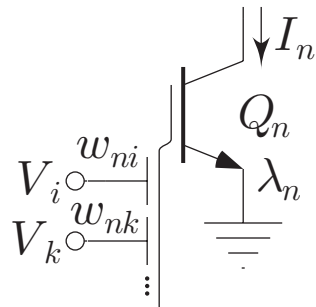


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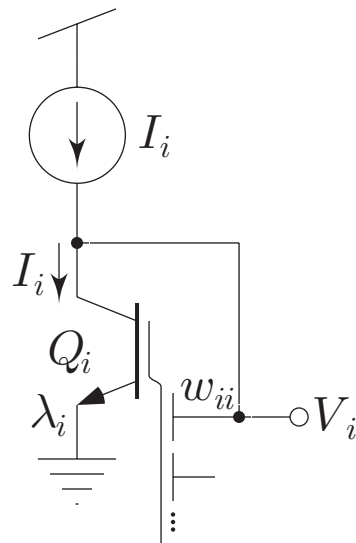


$$V_i = \frac{U_T}{w_{ii}} \log I_i - \dots$$

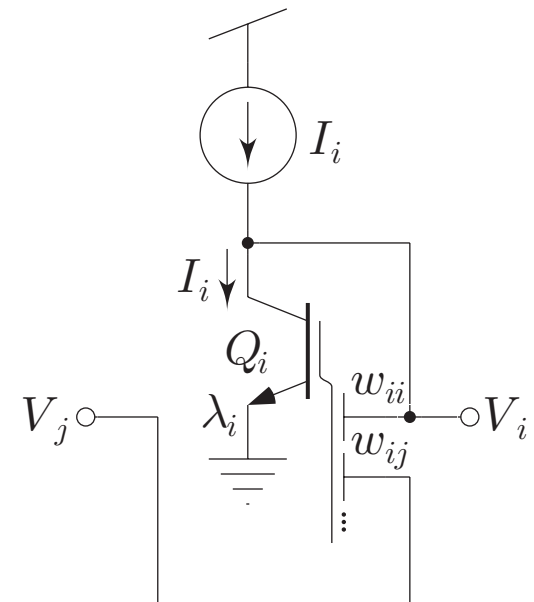
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$$V_i = \frac{U_T}{w_{ii}} \log I_i - \frac{w_{ij}}{w_{ii}} V_j - \dots$$

# Elementary MITE Networks

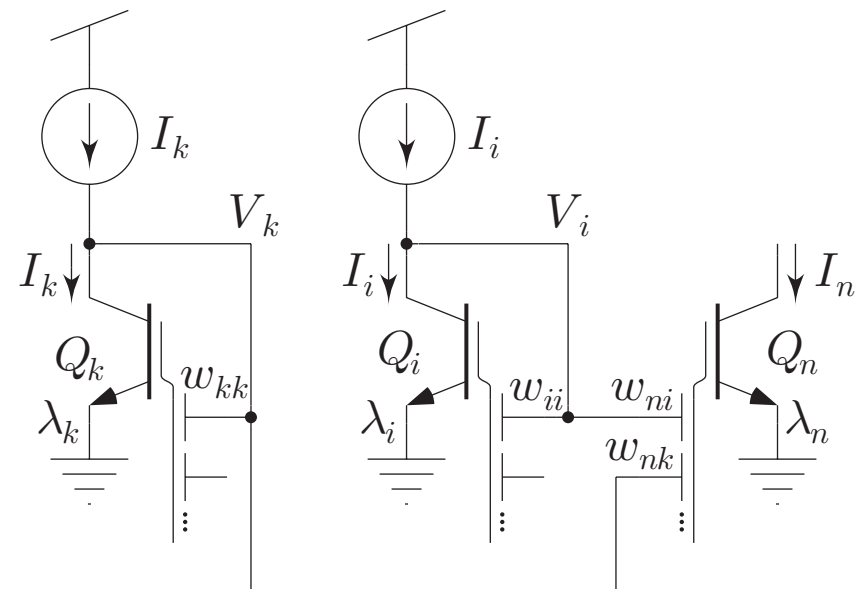
$$I_n \propto e^{w_{ni}V_i/U_T} e^{w_{nk}V_k/U_T}$$

$$\Rightarrow I_n \propto \exp\left(\frac{w_{ni}}{U_T} \left(\frac{U_T}{w_{ii}} \log I_i - \dots\right)\right)$$

$$\times \exp\left(\frac{w_{nk}}{U_T} \left(\frac{U_T}{w_{kk}} \log I_k - \dots\right)\right)$$

$$\Rightarrow I_n \propto e^{(w_{ni}/w_{ii}) \log I_i} e^{(w_{nk}/w_{kk}) \log I_k}$$

$$\Rightarrow I_n \propto I_i^{w_{ni}/w_{ii}} \times I_k^{w_{nk}/w_{kk}}$$



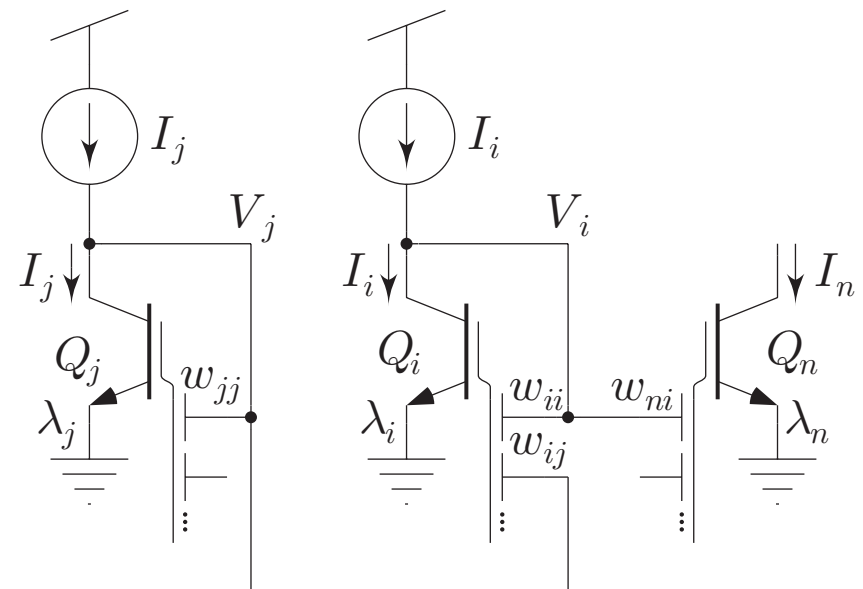
## Elementary MITE Networks

$$I_n \propto e^{w_{ni}V_i/U_T}$$

$$\Rightarrow I_n \propto \exp\left(\frac{w_{ni}}{U_T}\left(\frac{U_T}{w_{ii}}\log I_i - \frac{w_{ij}}{w_{ii}}\left(\frac{U_T}{w_{jj}}\log I_j - \dots\right) - \dots\right)\right)$$

$$\Rightarrow I_n \propto e^{(w_{ni}/w_{ii})\log I_i} e^{-\left(w_{ni}w_{ij}/w_{ii}w_{jj}\right)\log I_j}$$

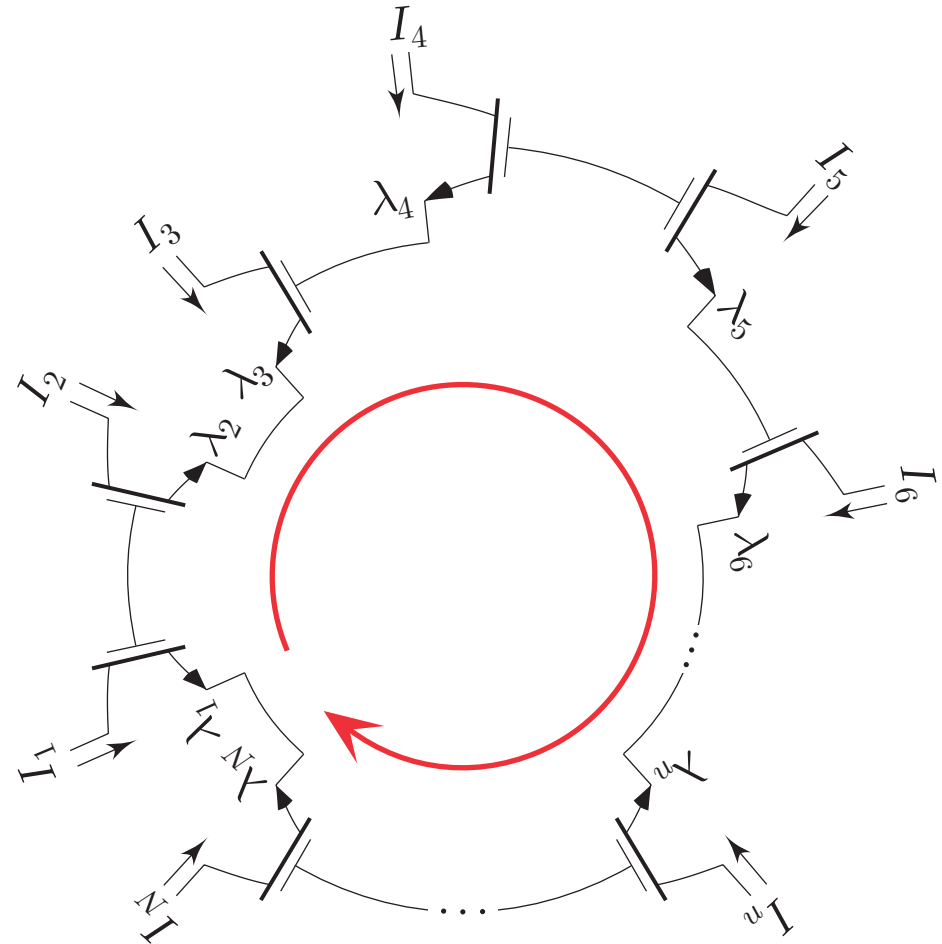
$$\Rightarrow I_n \propto \frac{I_i^{w_{ni}/w_{ii}}}{I_j^{w_{ni}w_{ij}/w_{ii}w_{jj}}}$$



# The Translinear Principle

In a closed loop of junctions comprising an equal number of clockwise and counterclockwise elements, the product of the current densities flowing through the counterclockwise elements is equal to the product of the current densities flowing through the clockwise elements.

$$\prod_{n \in \text{CW}} \frac{I_n}{\lambda_n} = \prod_{n \in \text{CCW}} \frac{I_n}{\lambda_n}$$



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# Static MITE Network Synthesis: **Square-Root Circuit**

Synthesize a square-root circuit described by

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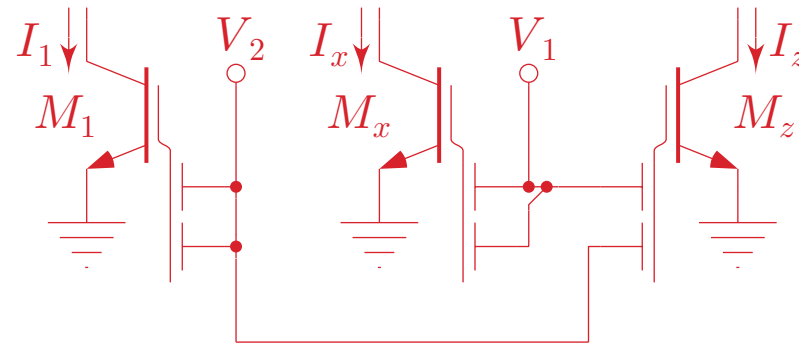
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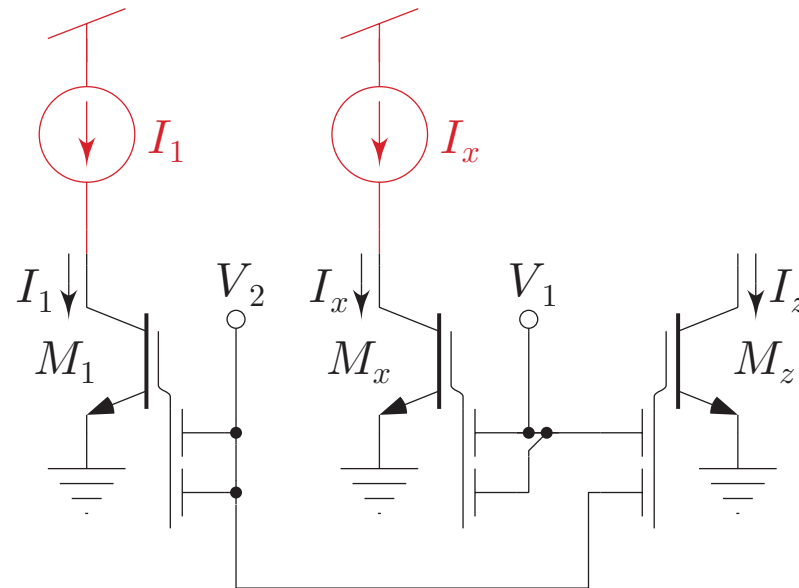
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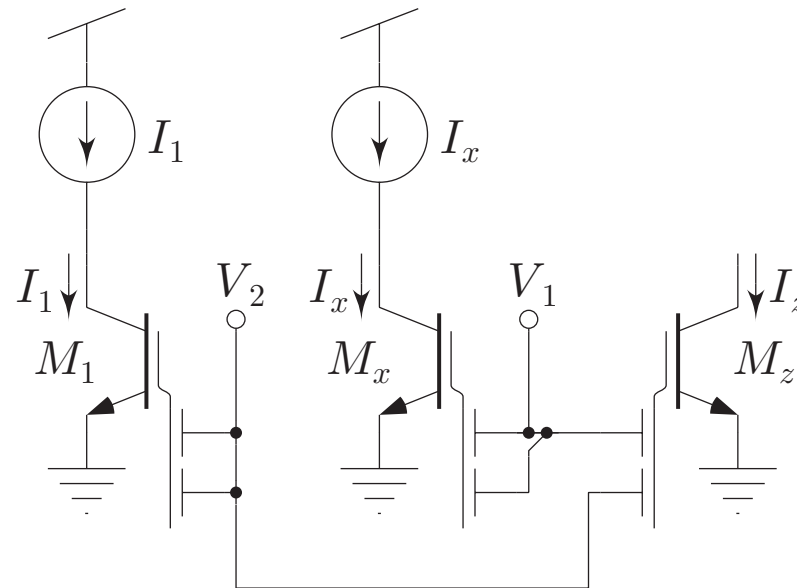
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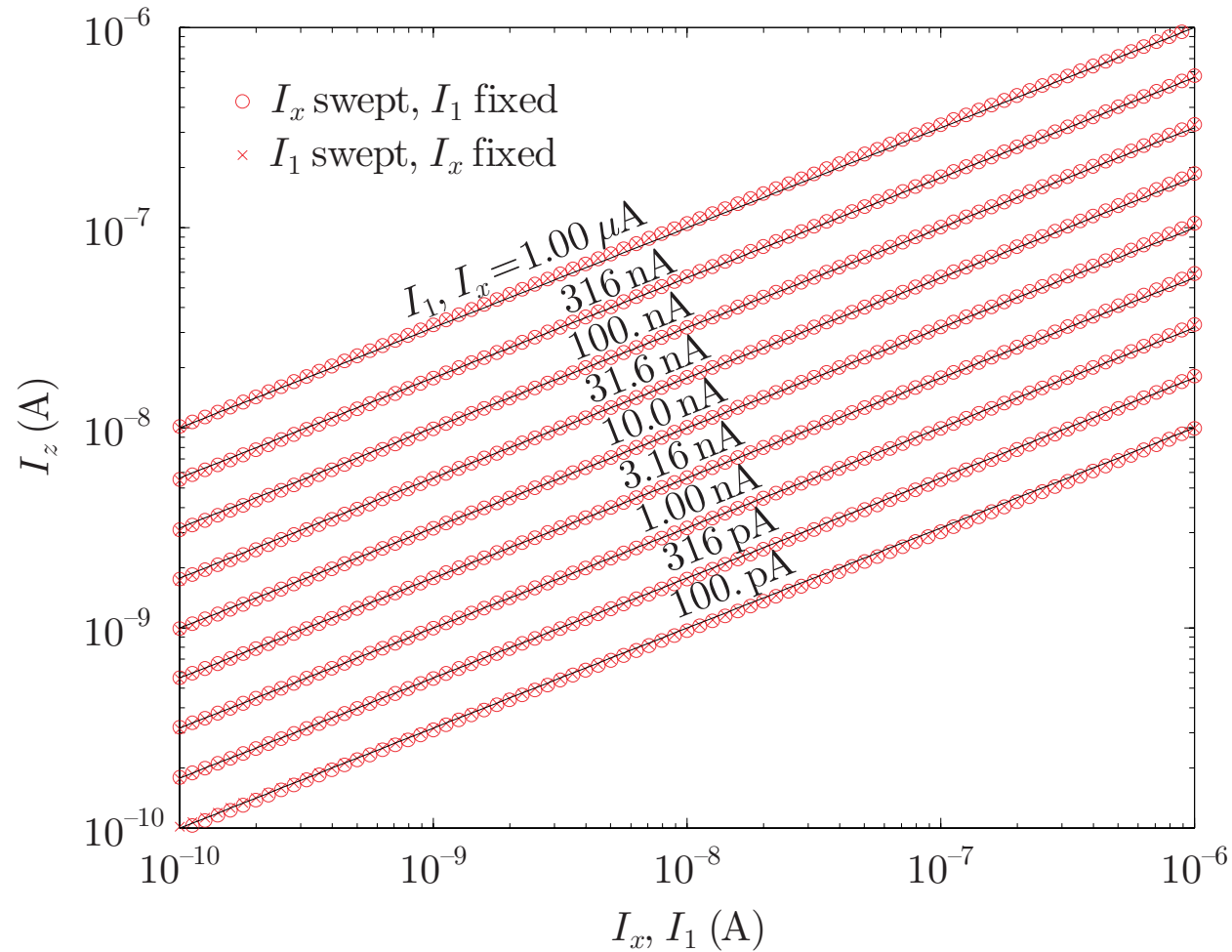








# Experimental Measurements: Square-Root Circuit



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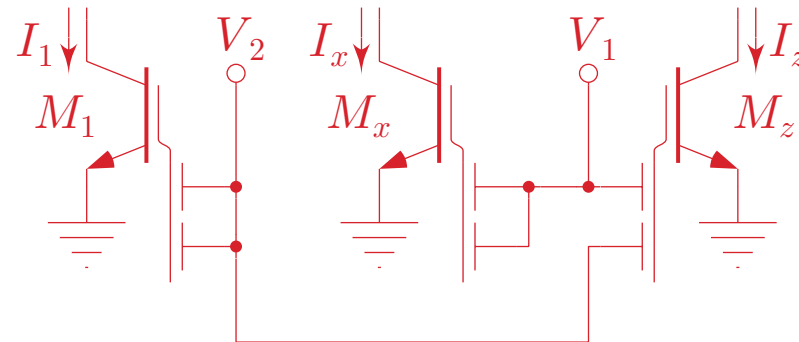
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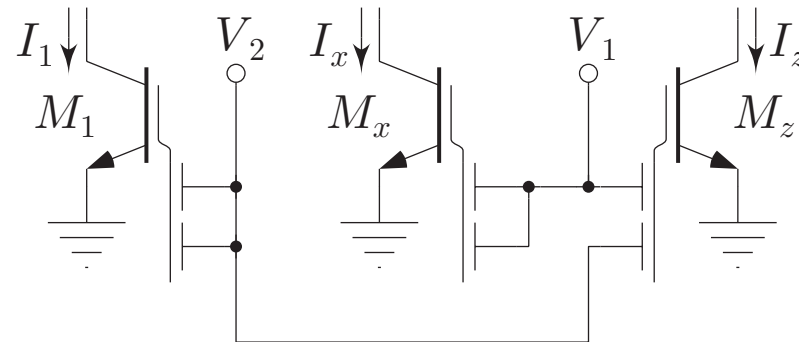
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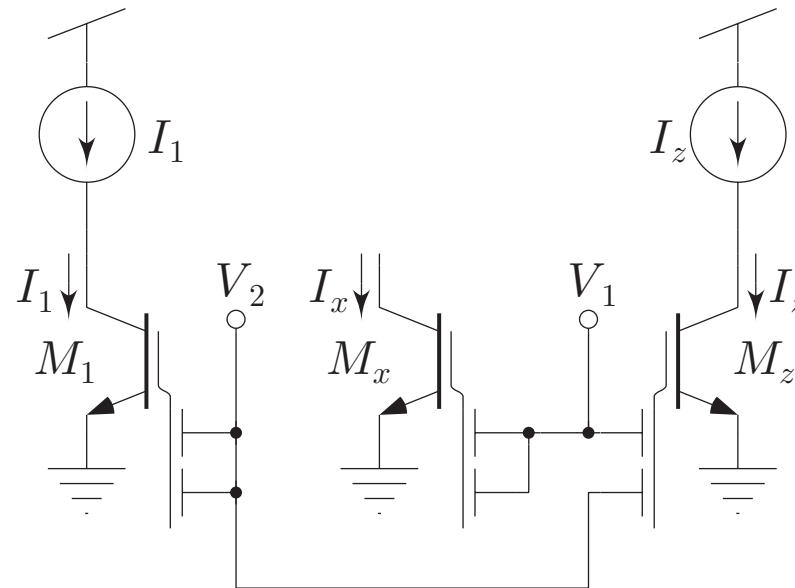
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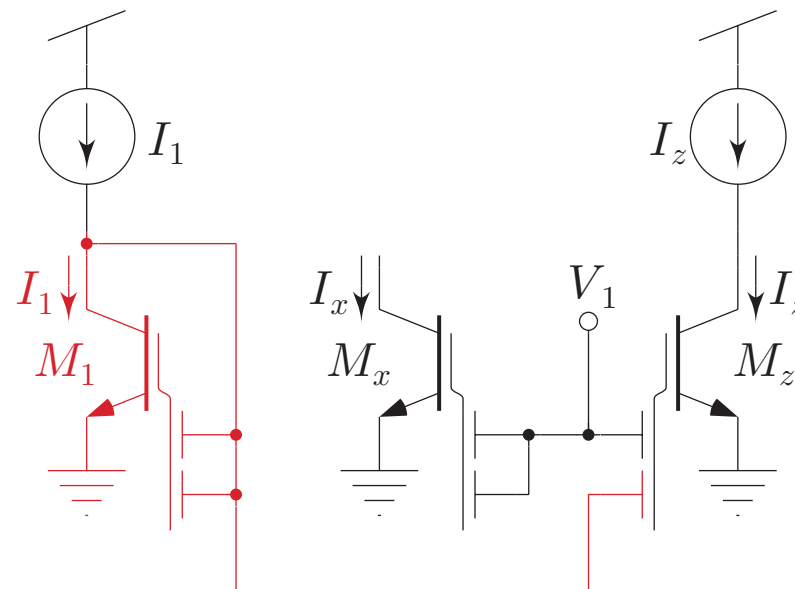
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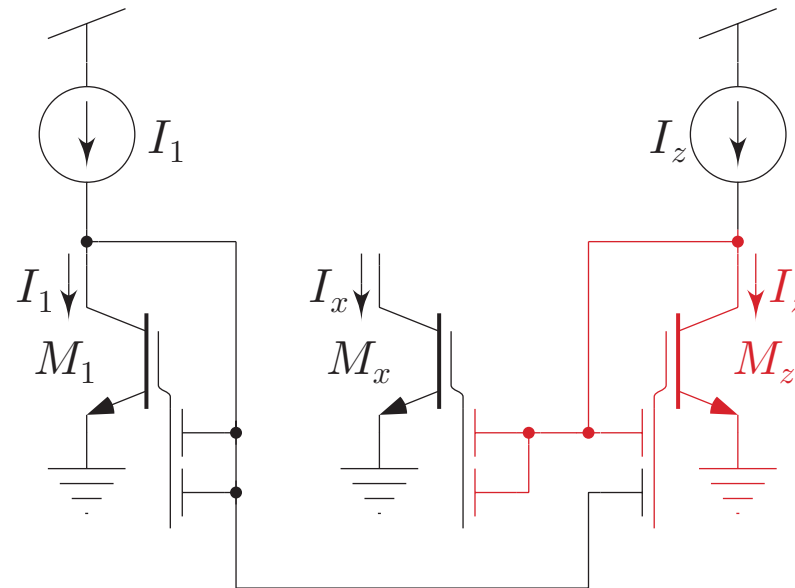
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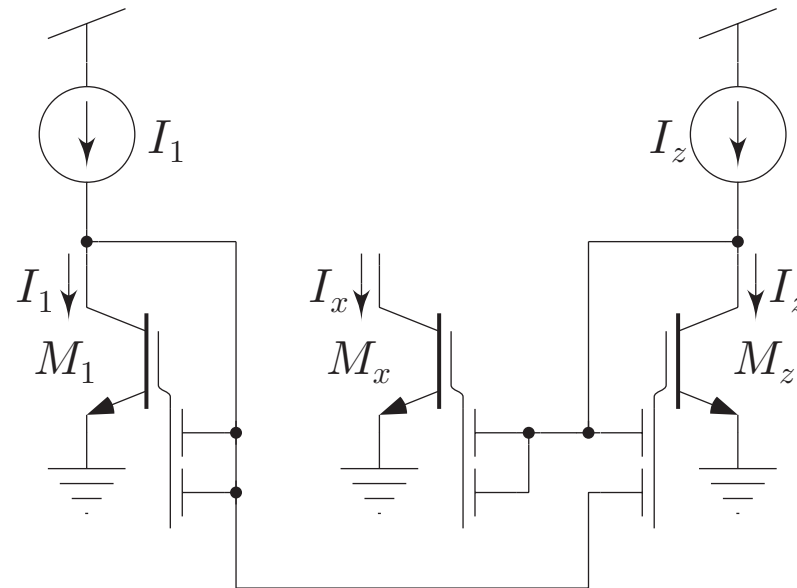
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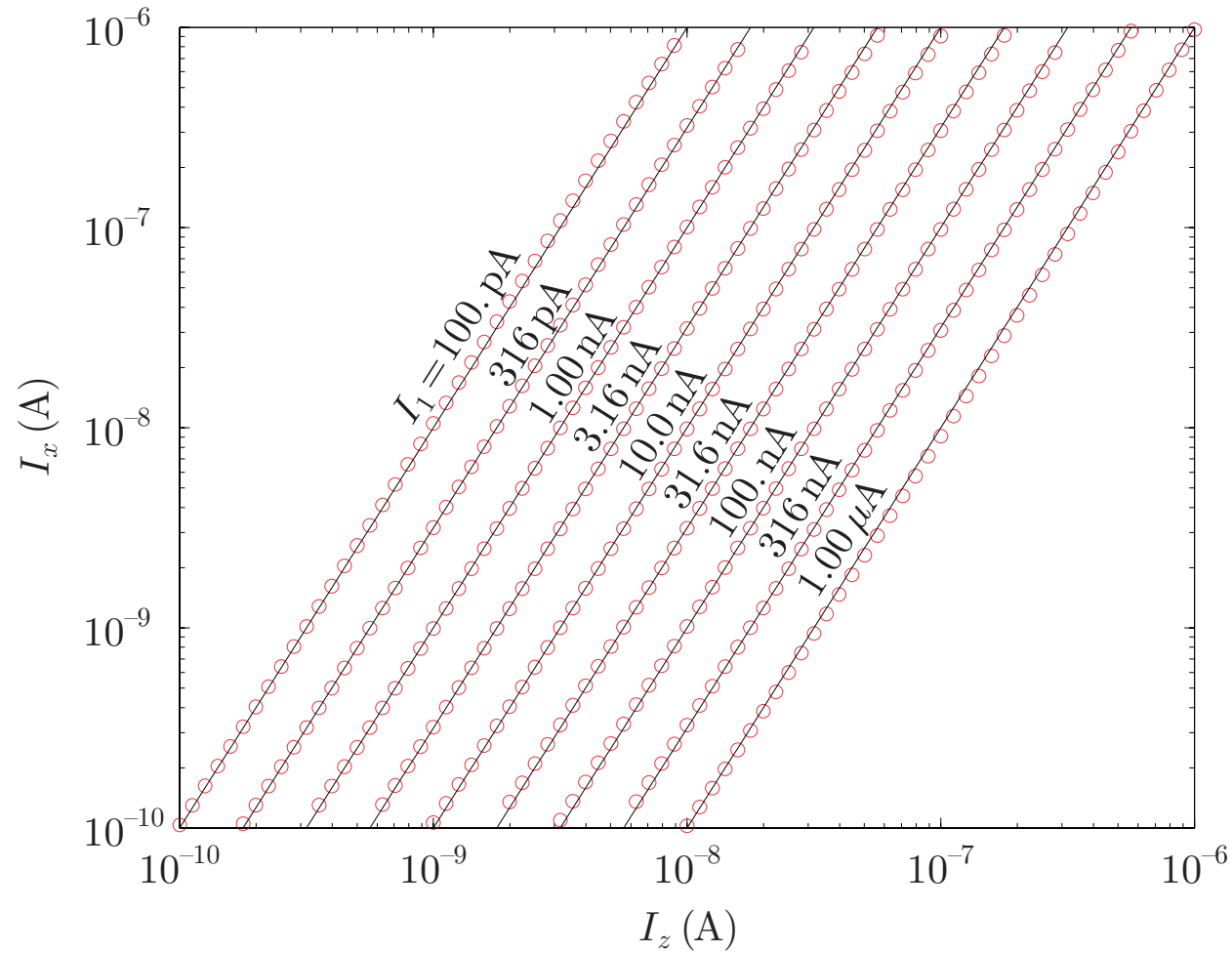


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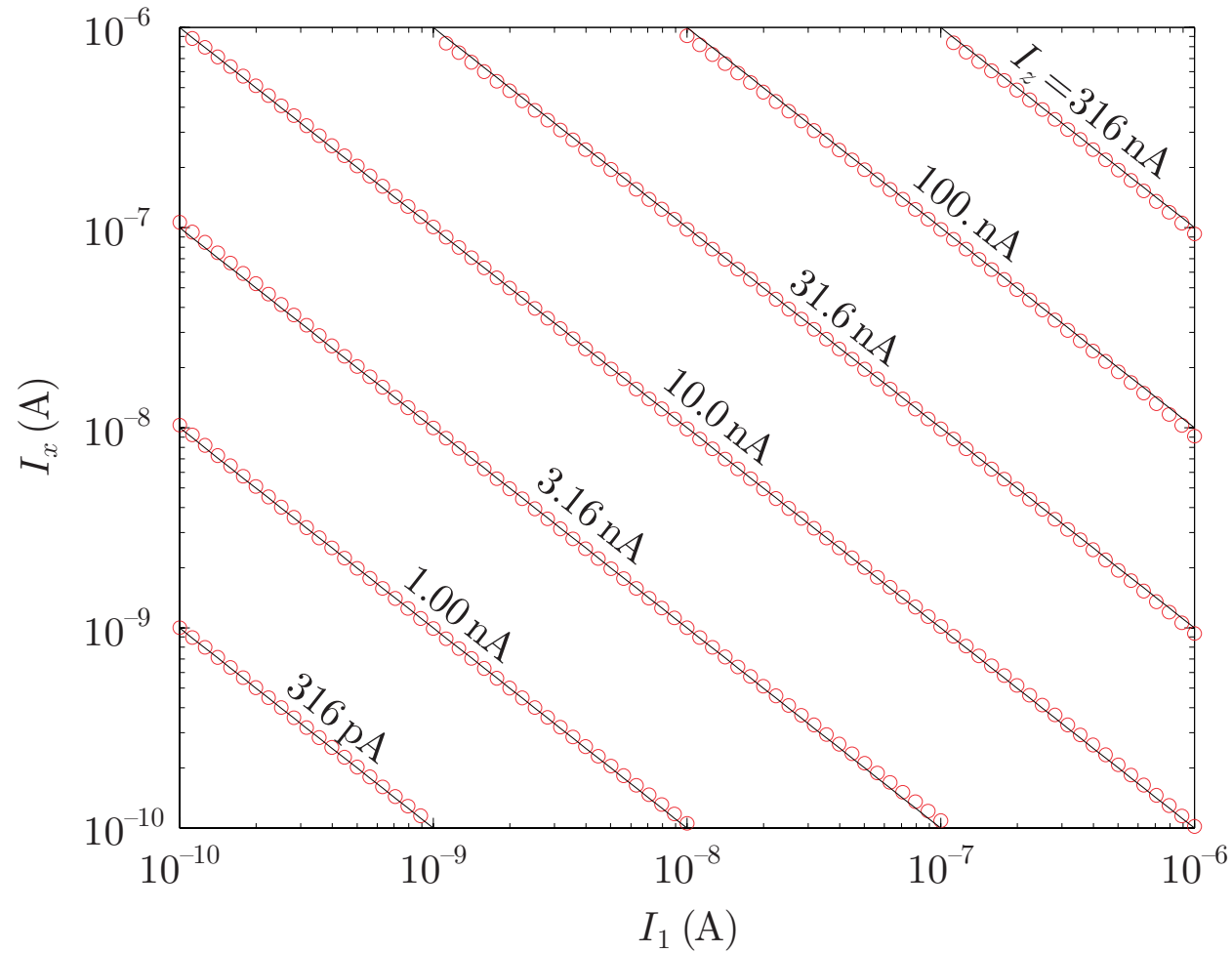


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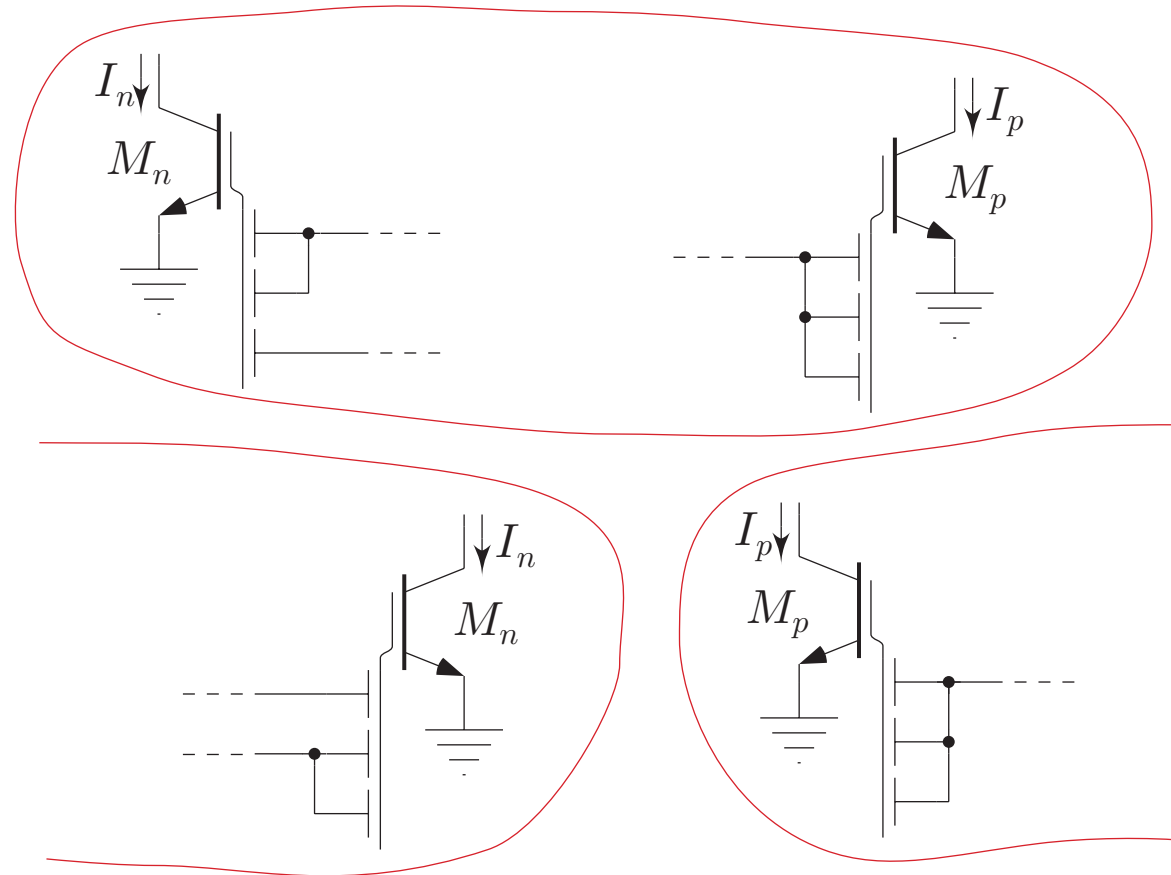




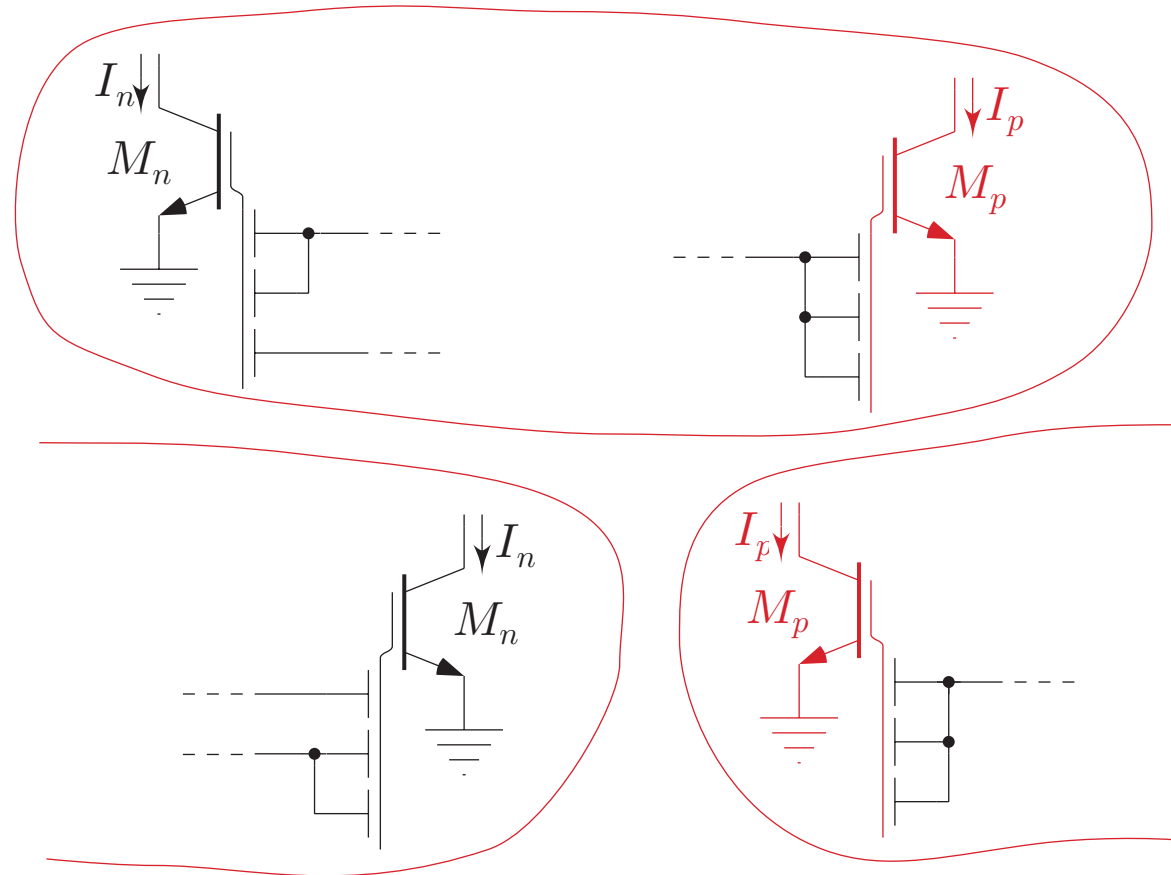
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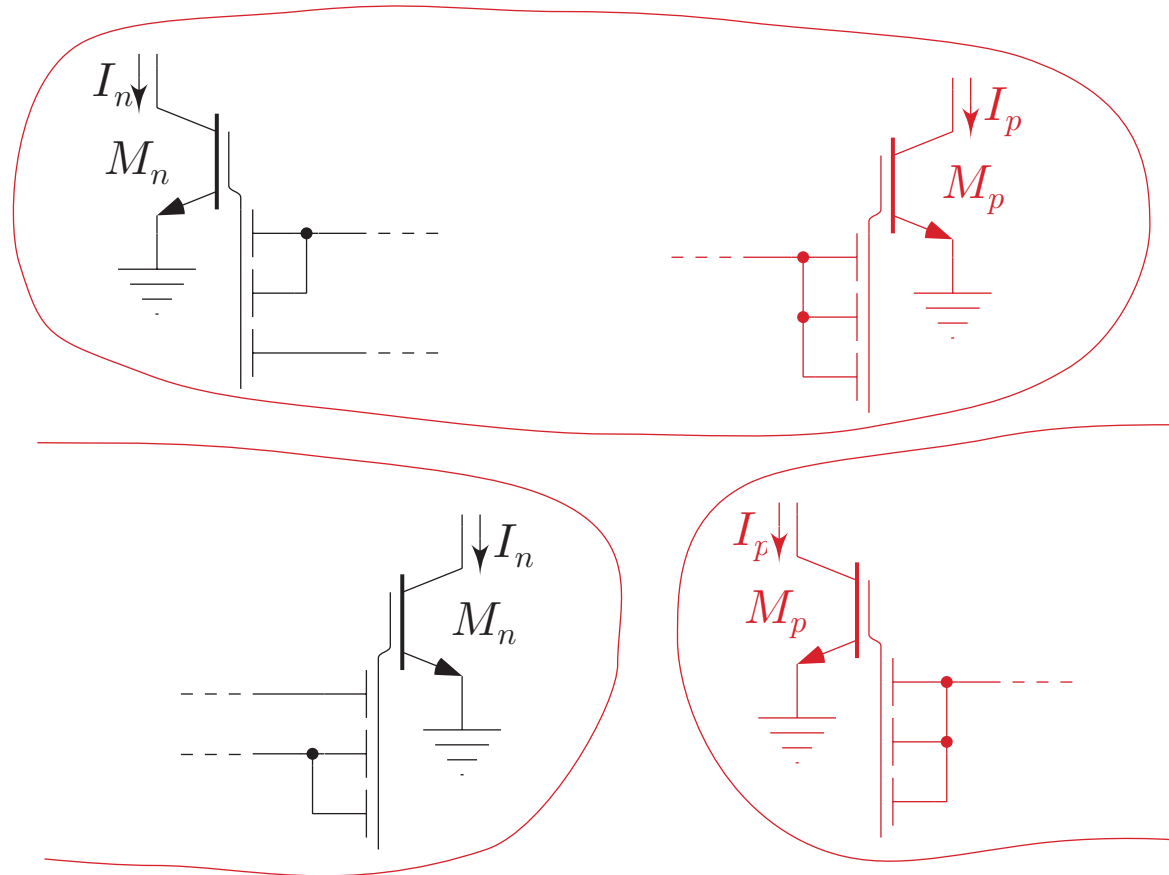
# Static MITE Network Synthesis: Consolidation



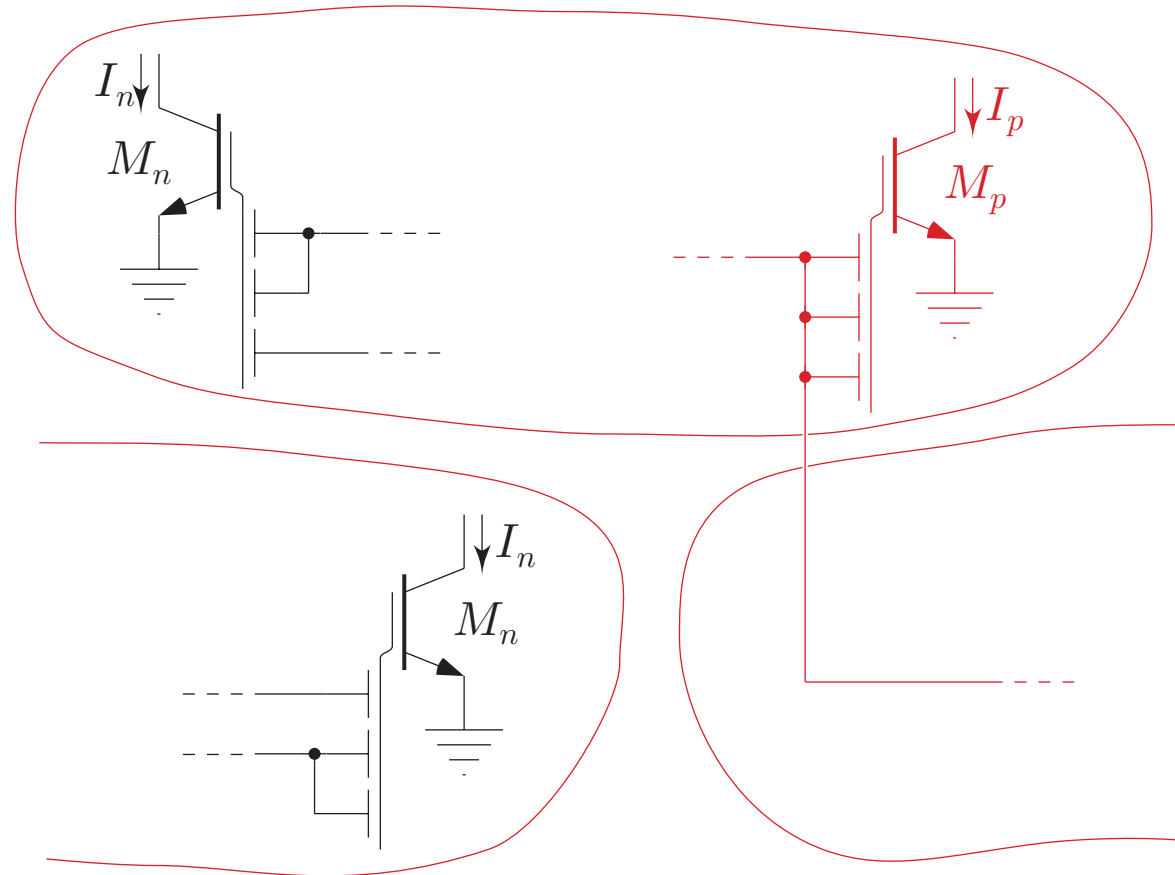
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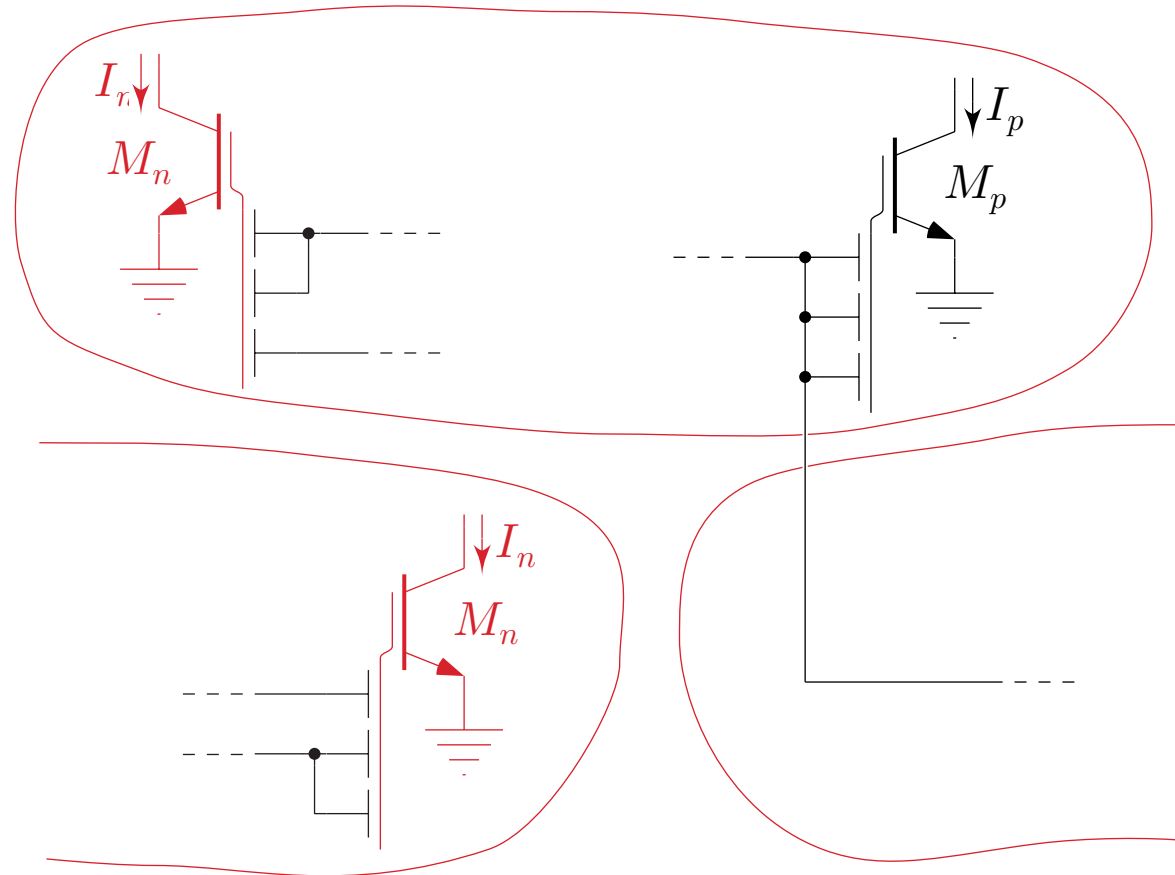
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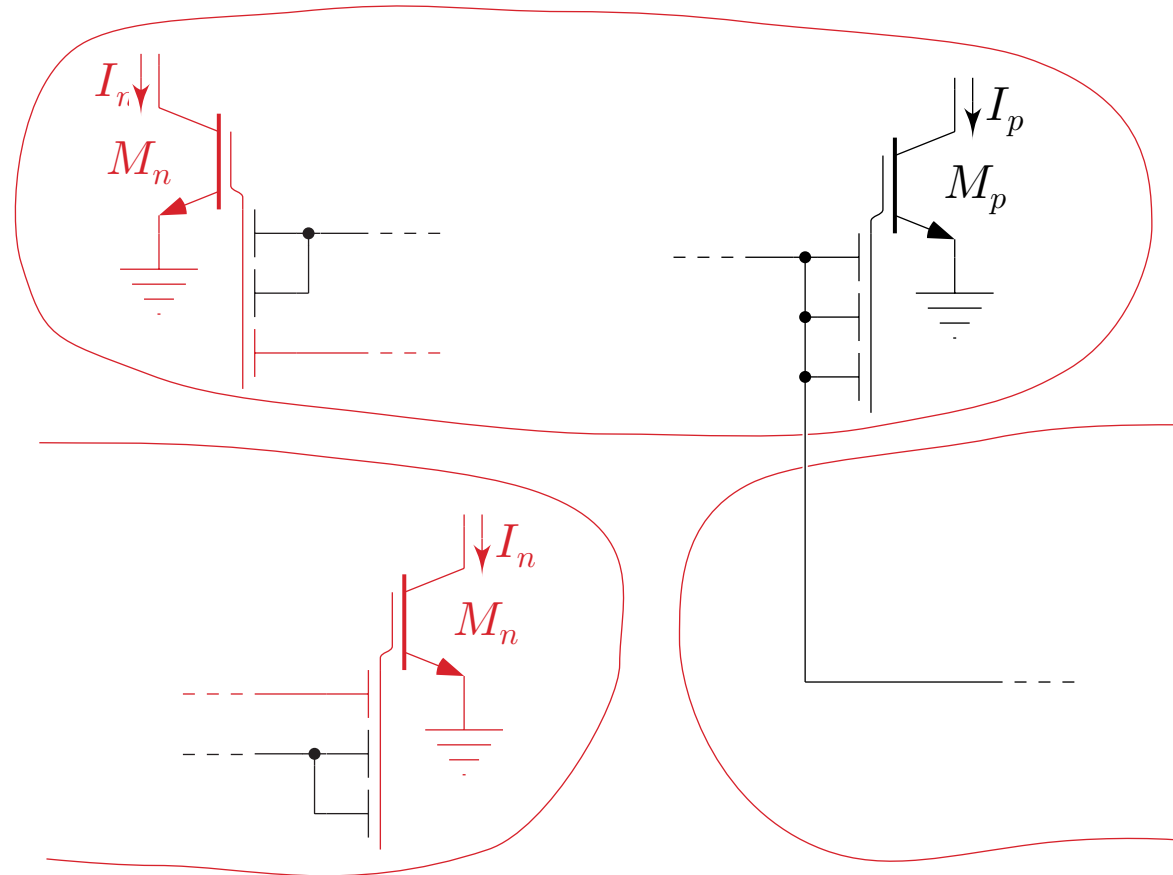
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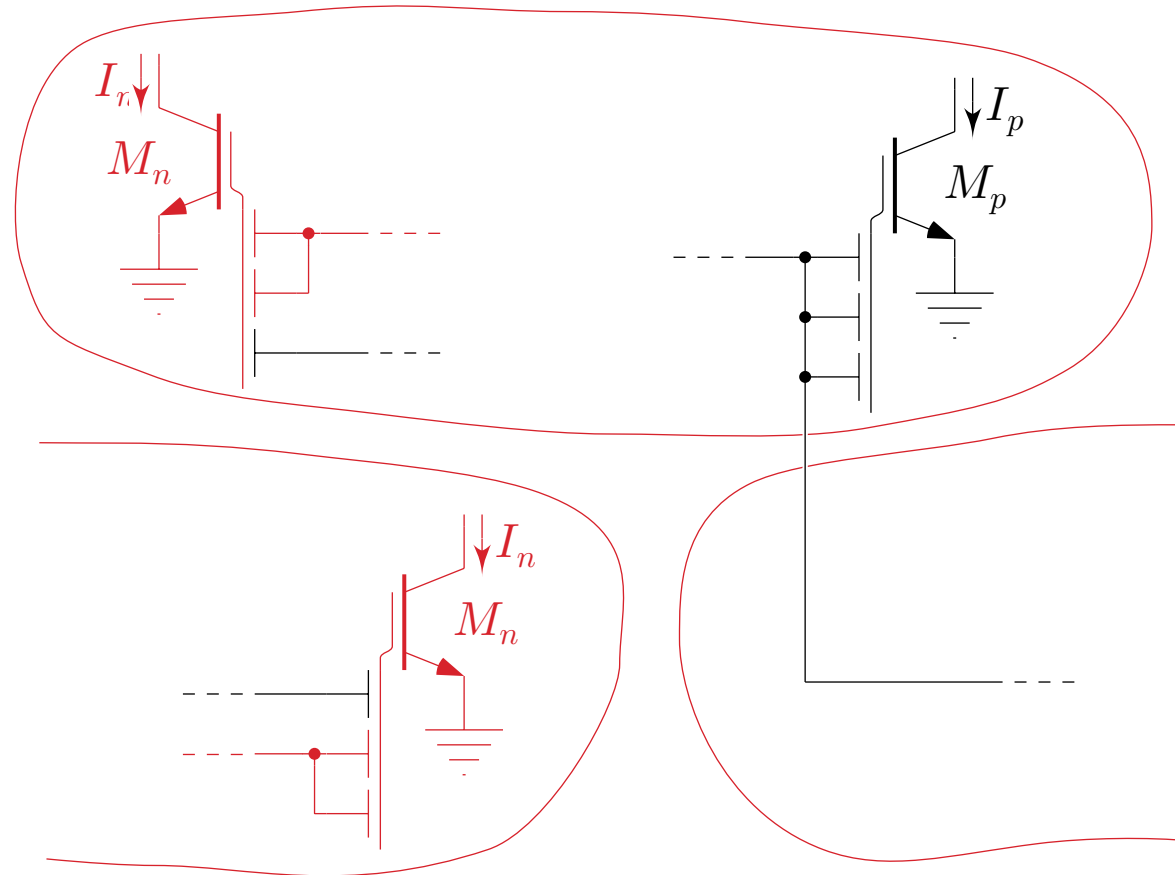
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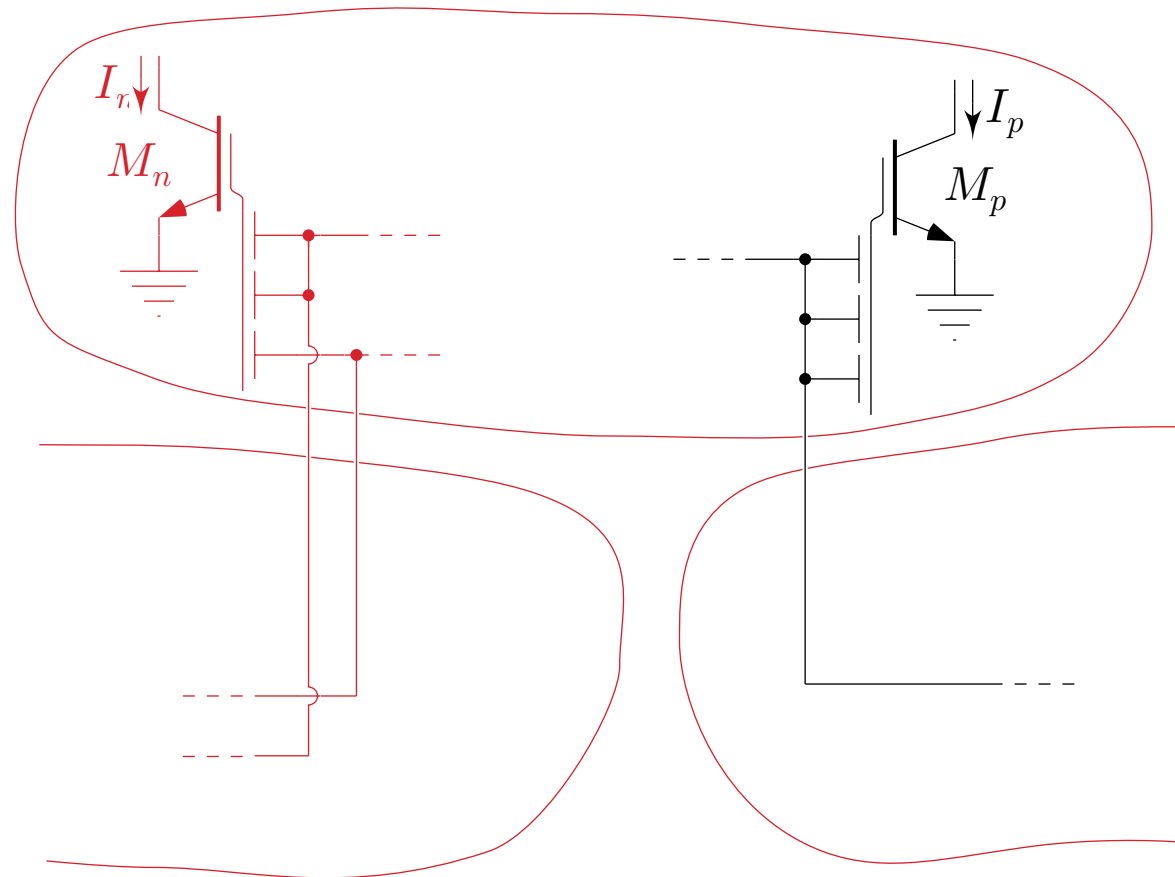


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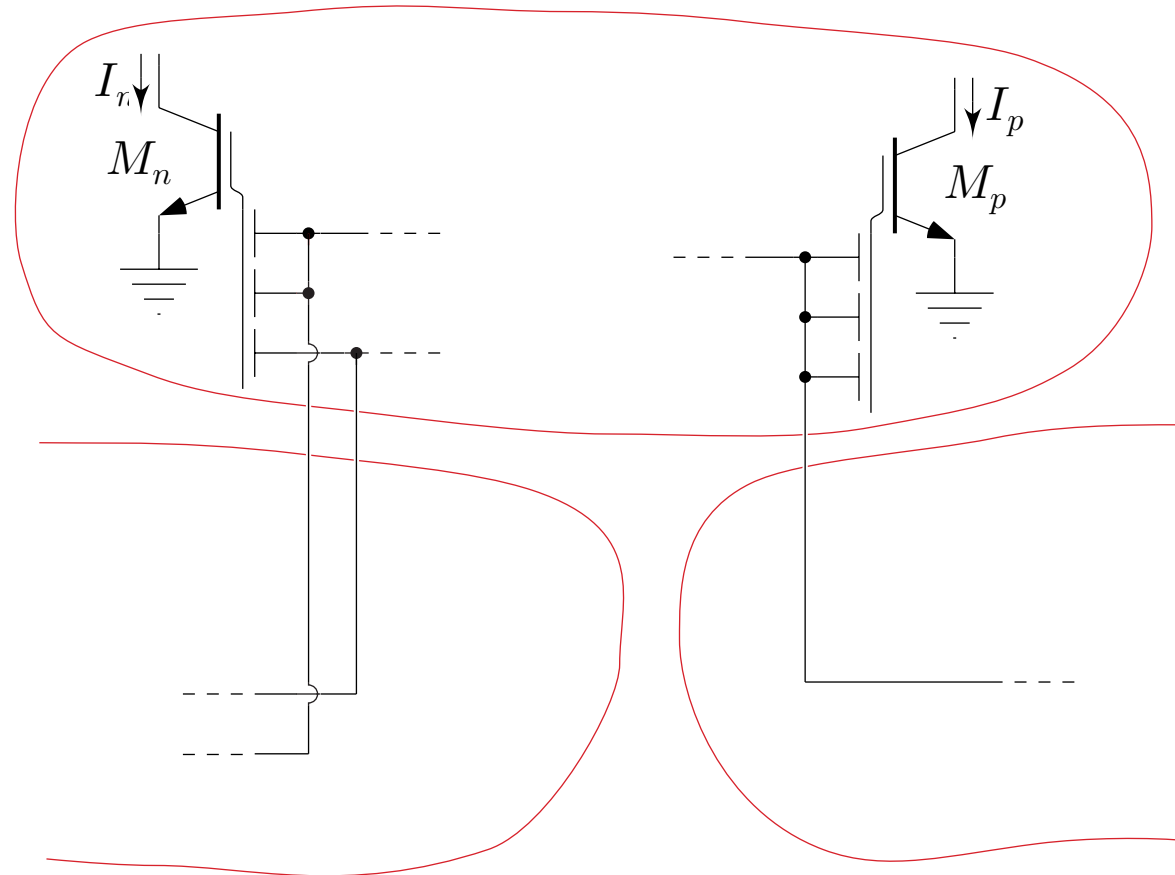




# Static MITE Network Synthesis: Consolidation



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## Static MITE Network Synthesis: **Vector Magnitude**

Synthesize a two-dimensional vector-magnitude circuit implementing

$$r = \sqrt{x^2 + y^2}, \quad \text{where } x > 0 \quad \text{and} \quad y > 0.$$

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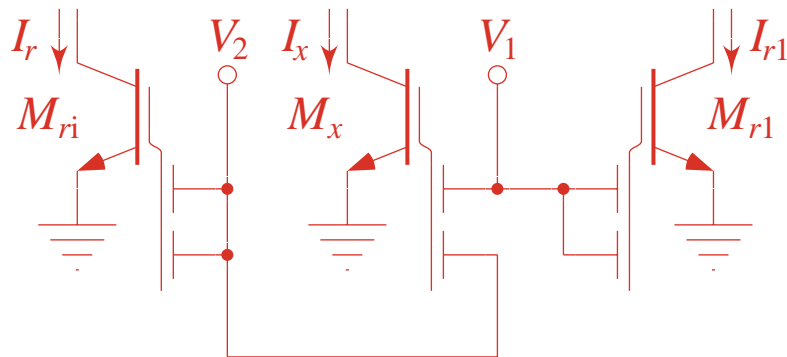
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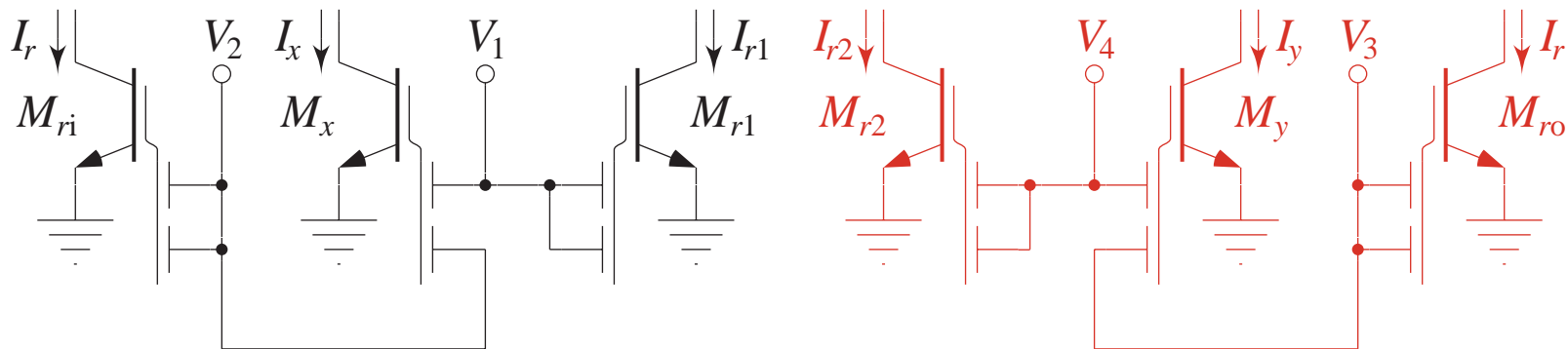
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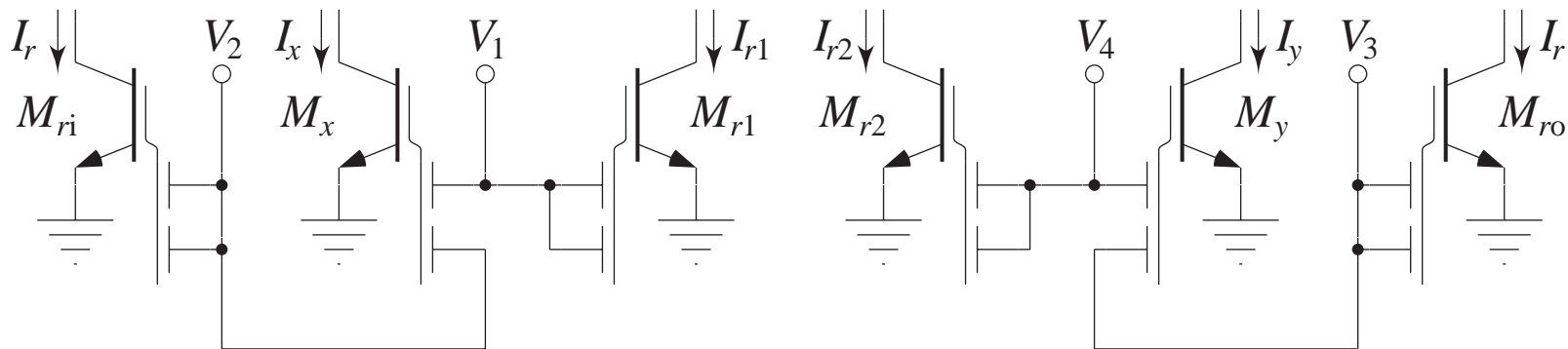
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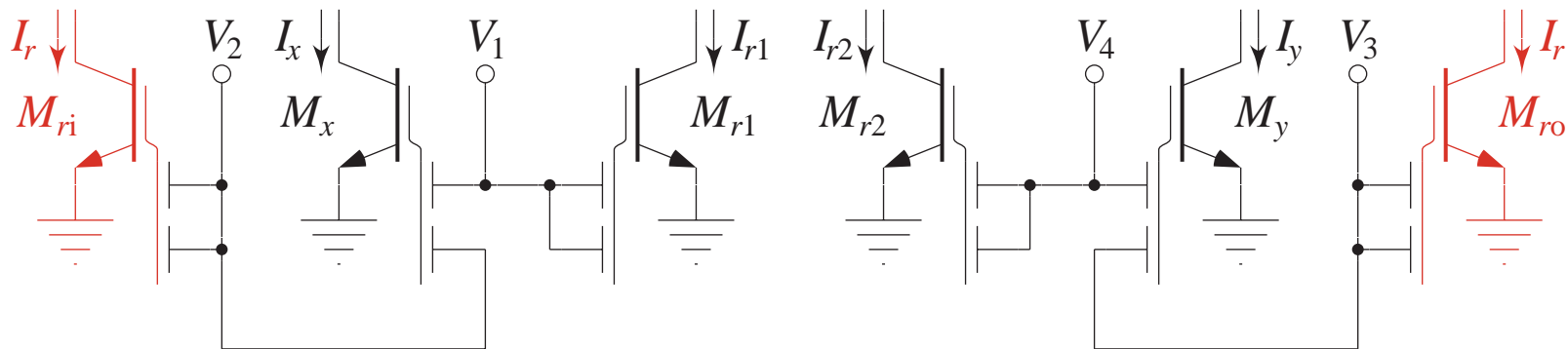
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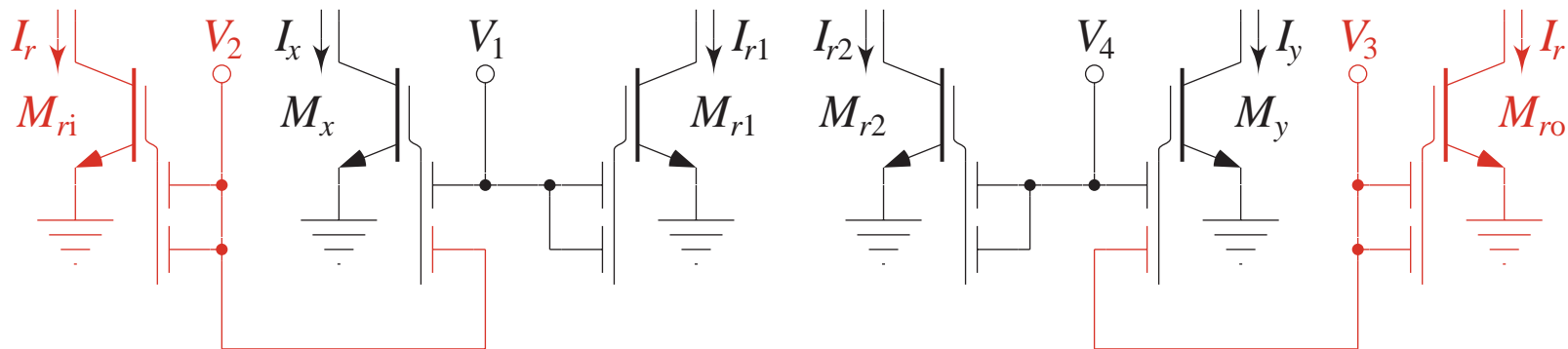
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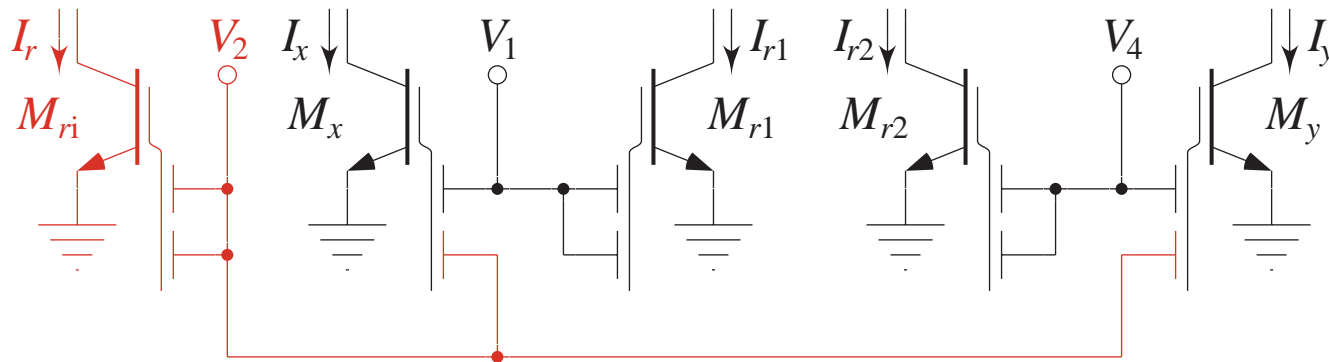
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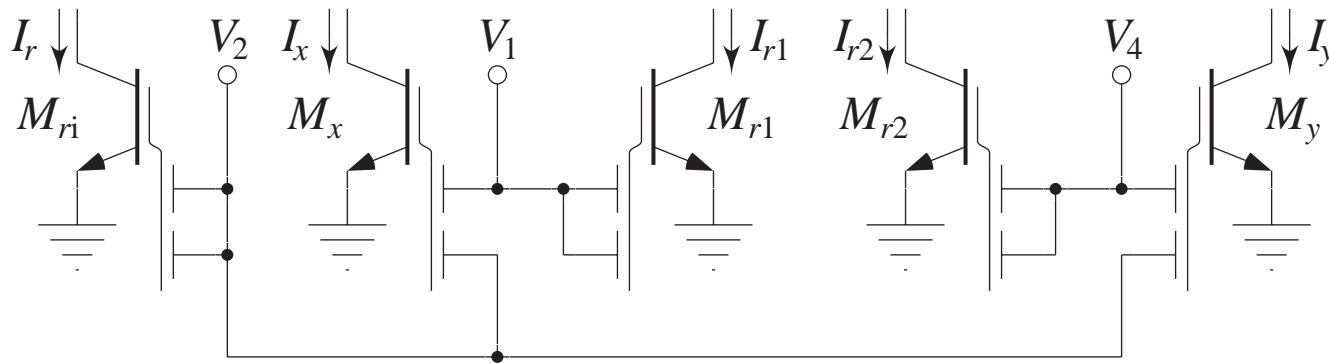
$$\text{KCL: } I_r = I_{r1} + I_{r2}$$



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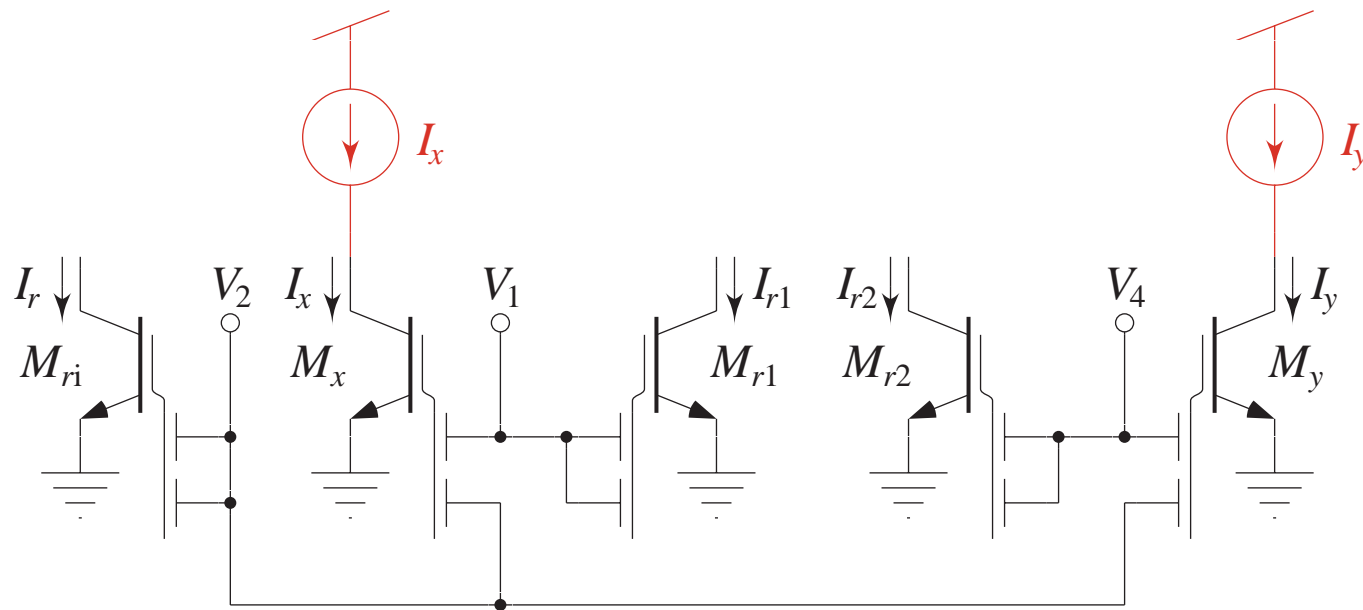




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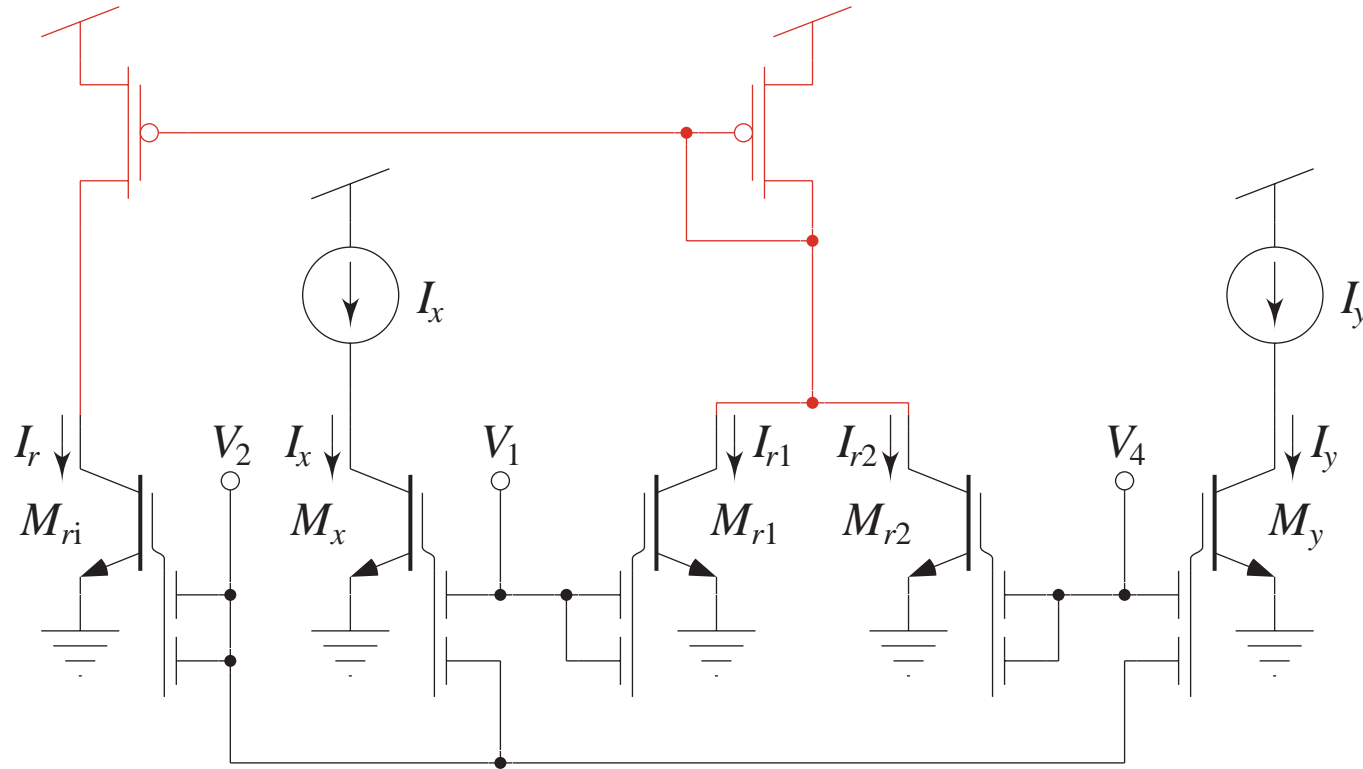
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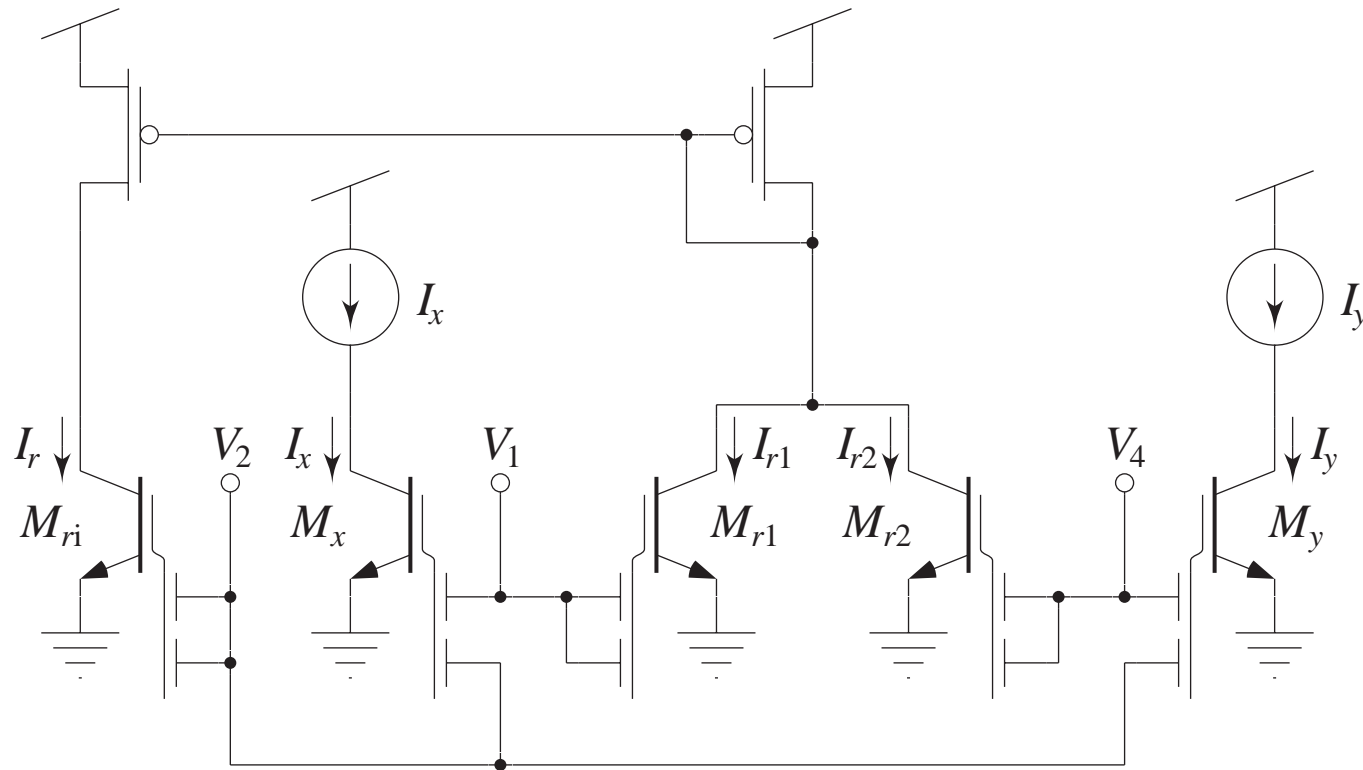
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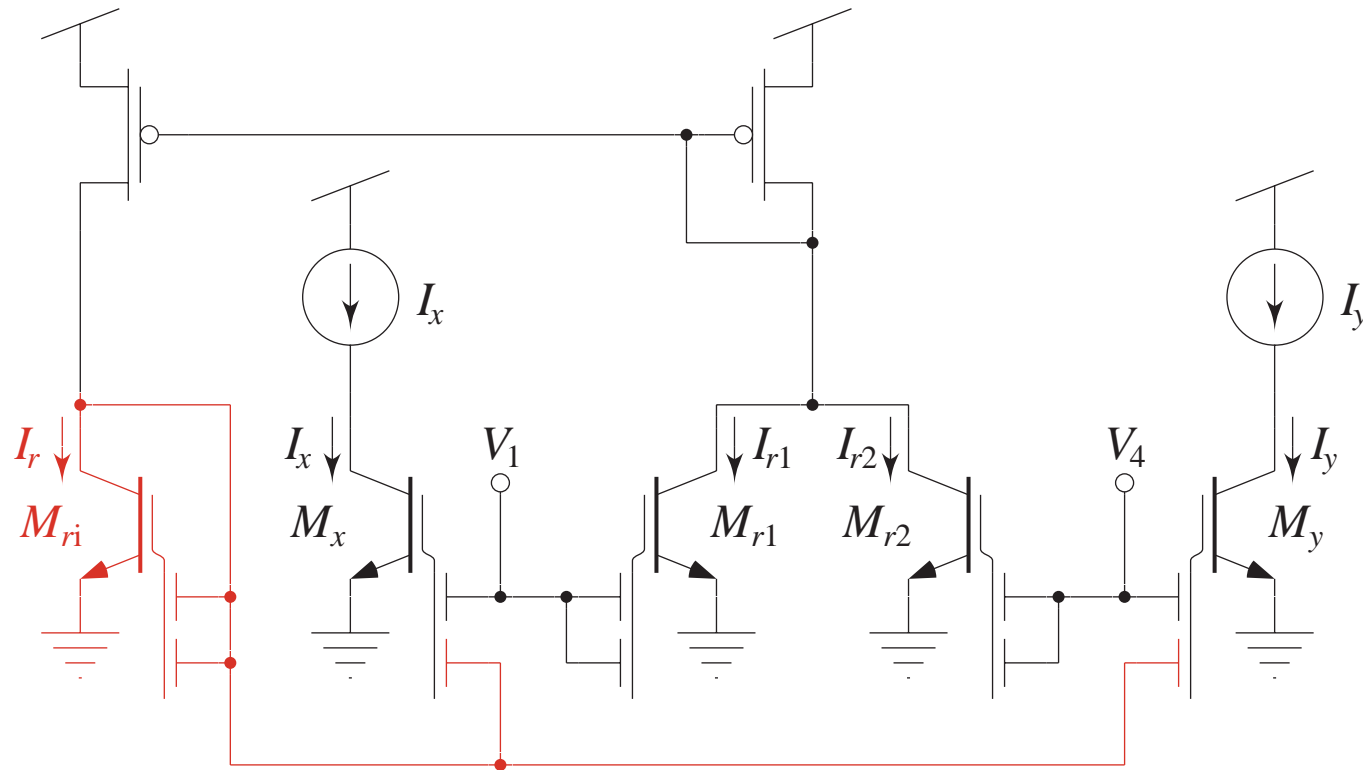
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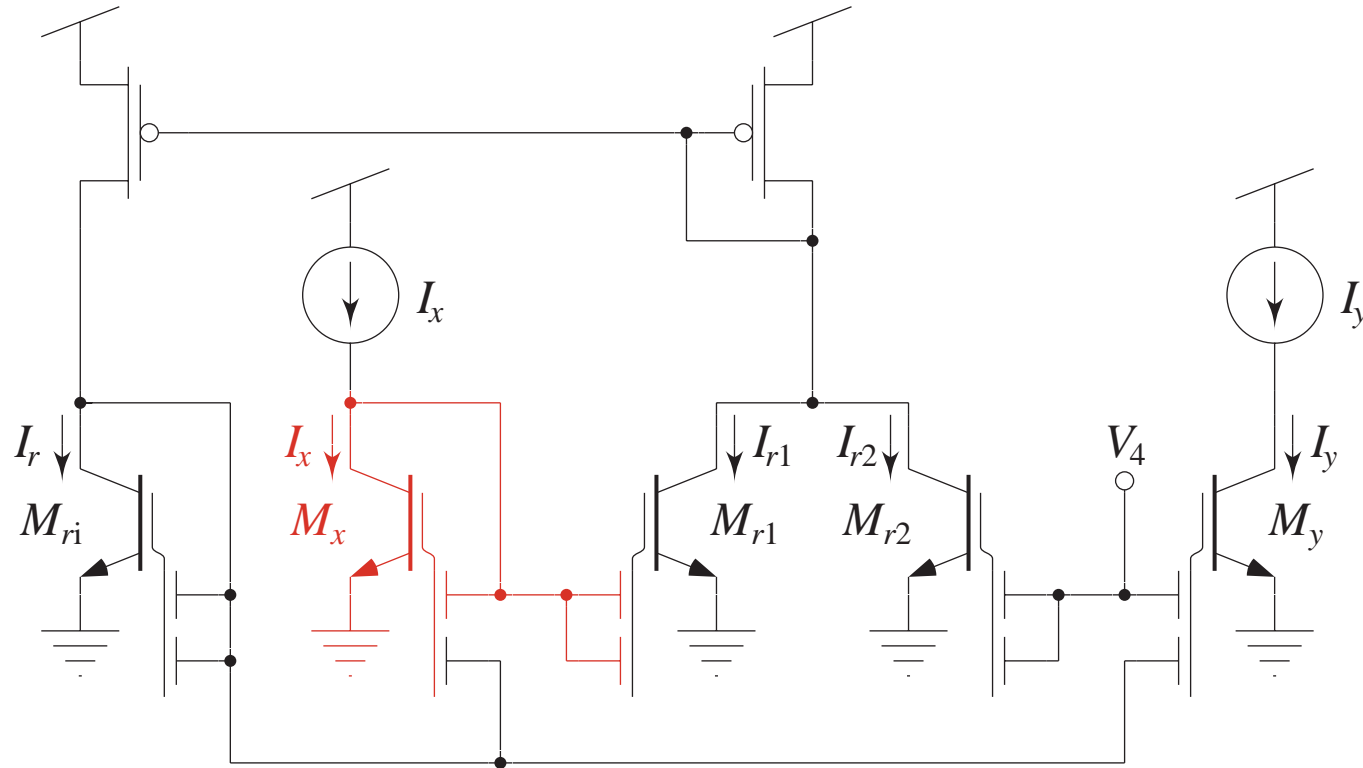
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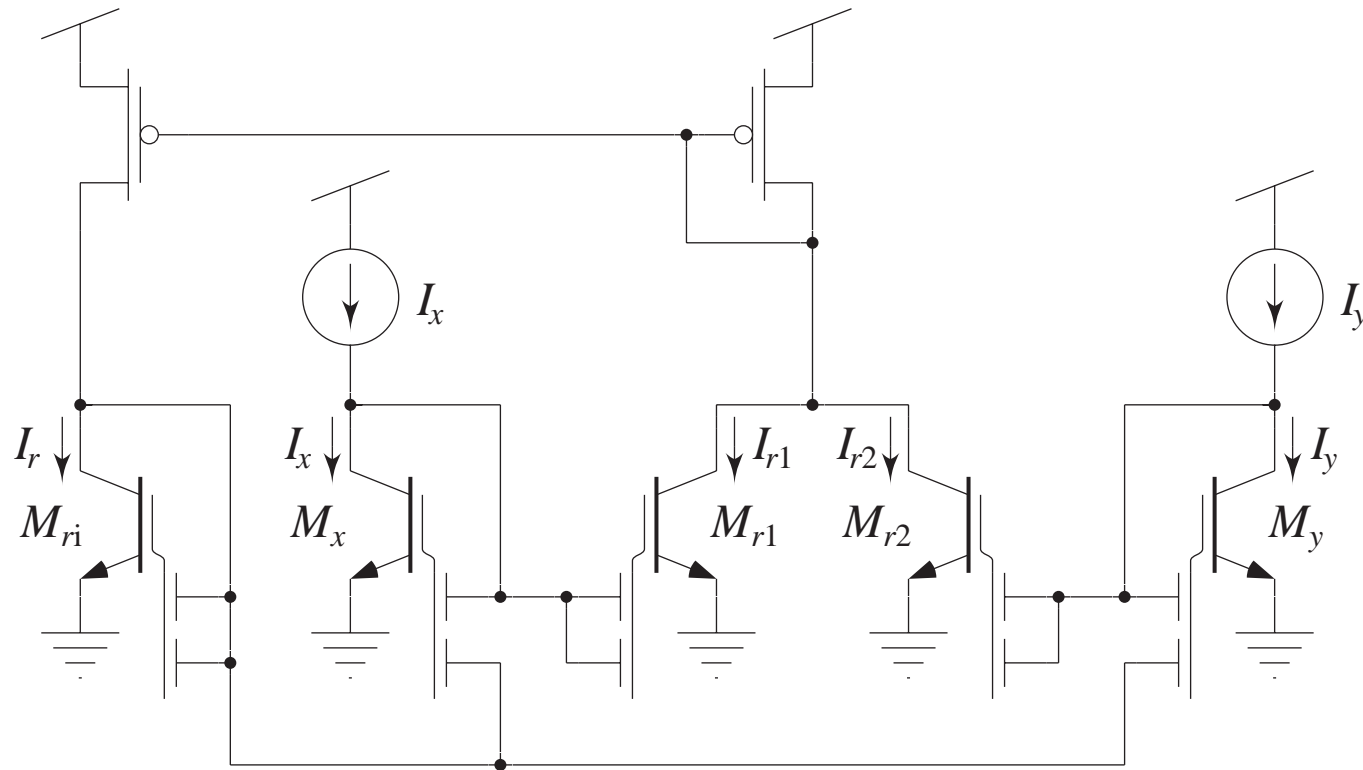




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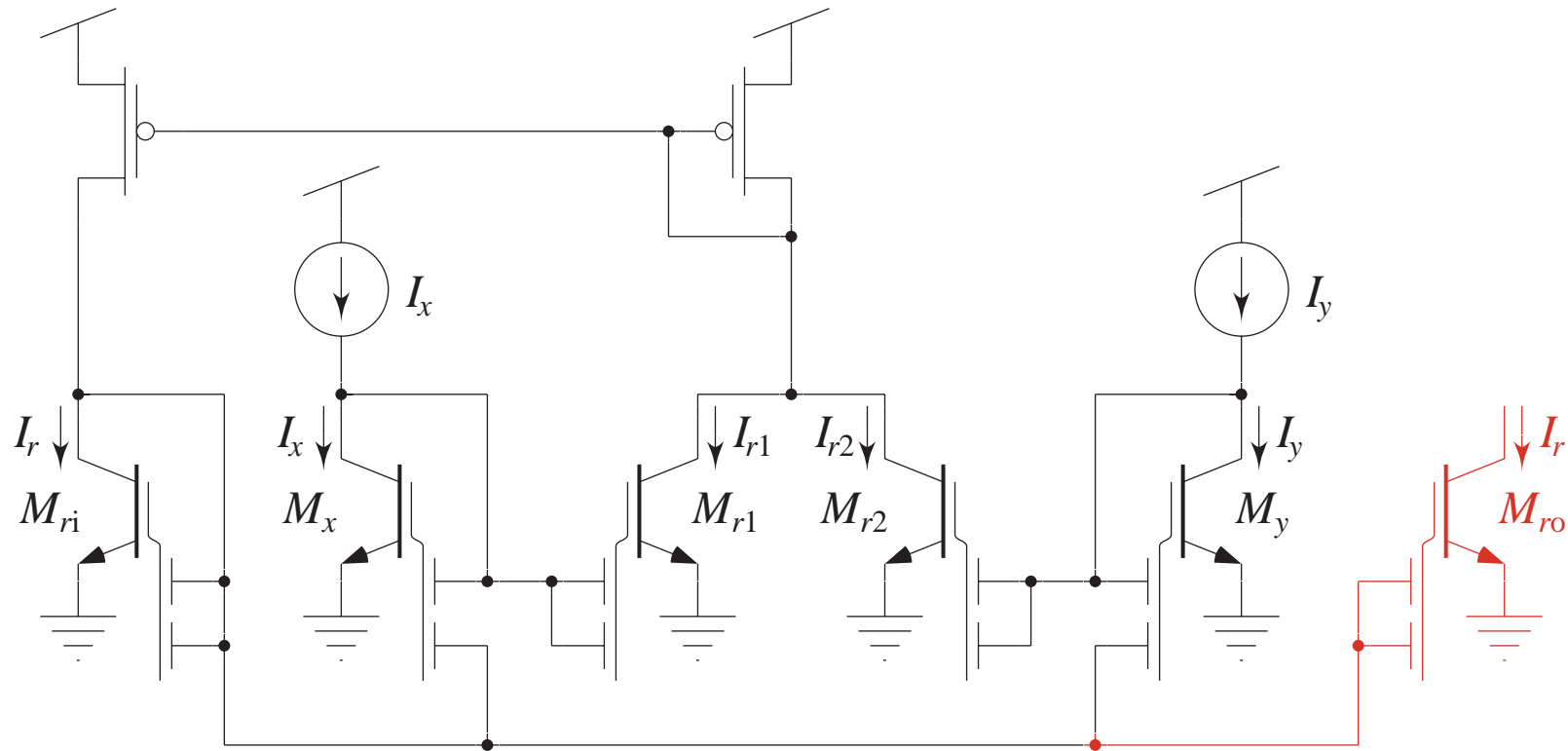
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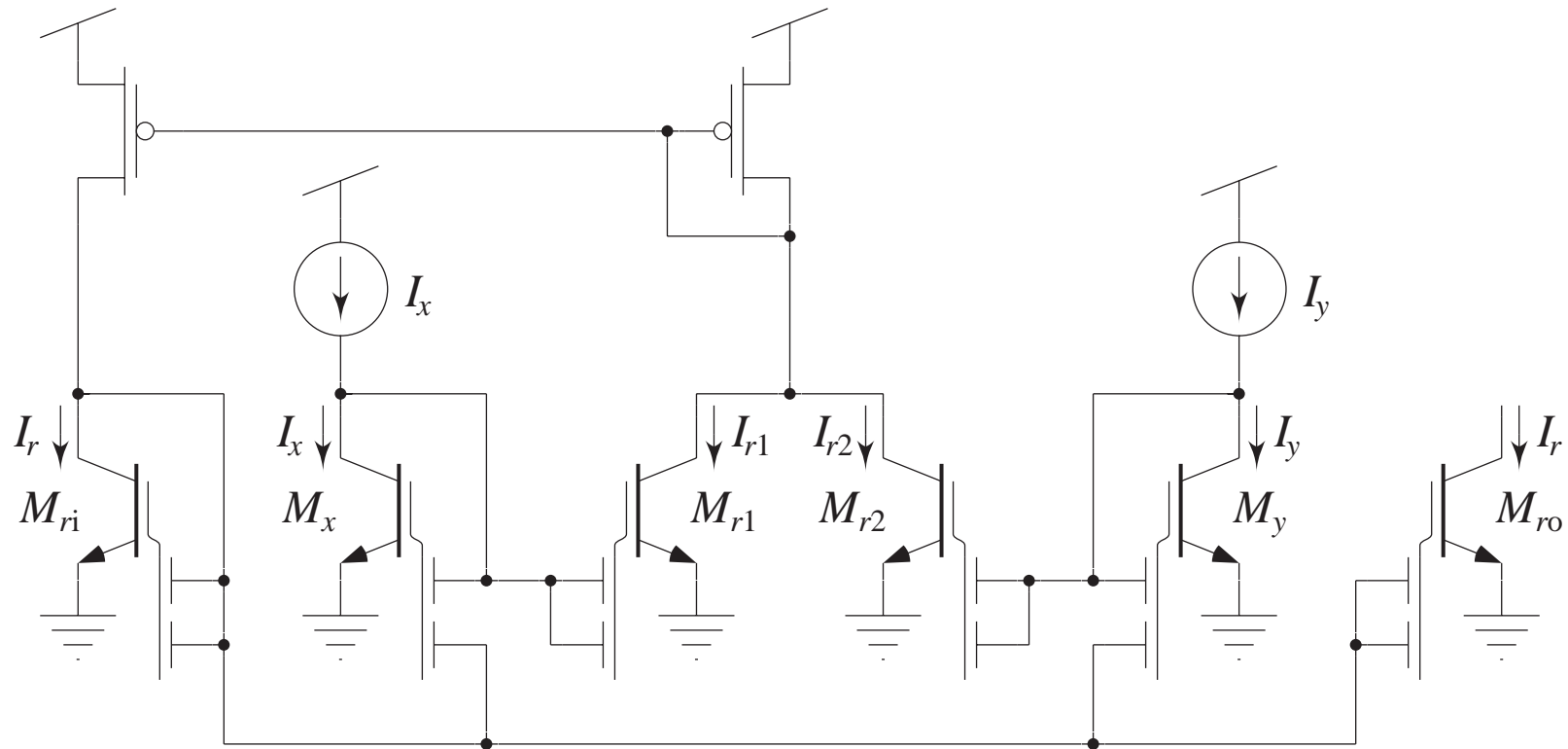




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## Static MITE Network Synthesis: **Vector Normalizer**

Synthesize a two-dimensional vector-normalization circuit implementing

$$u = \frac{x}{\sqrt{x^2 + y^2}} \quad \text{and} \quad v = \frac{y}{\sqrt{x^2 + y^2}}, \quad \text{where } x > 0 \quad \text{and} \quad y > 0.$$

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Each equation shares  $r \equiv \sqrt{x^2 + y^2}$ , which we can use to decompose the system as

$$u = \frac{x}{r}, \quad v = \frac{y}{r}, \quad \text{and} \quad r = \sqrt{x^2 + y^2}, \quad \text{where } x > 0 \quad \text{and} \quad y > 0.$$

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We represent each signal as a ratio of a signal current to the unit current:

$$x \equiv \frac{I_x}{I_1}, \quad y \equiv \frac{I_y}{I_1}, \quad u \equiv \frac{I_u}{I_1}, \quad v \equiv \frac{I_v}{I_1}, \quad \text{and} \quad r \equiv \frac{I_r}{I_1}.$$

---

## Static MITE Network Synthesis: **Vector Normalizer**

We substitute these into the original equations and rearrange to obtain

$$\frac{I_u}{I_1} = \frac{I_x/I_1}{I_r/I_1} \quad \text{and} \quad \frac{I_v}{I_1} = \frac{I_y/I_1}{I_r/I_1}$$

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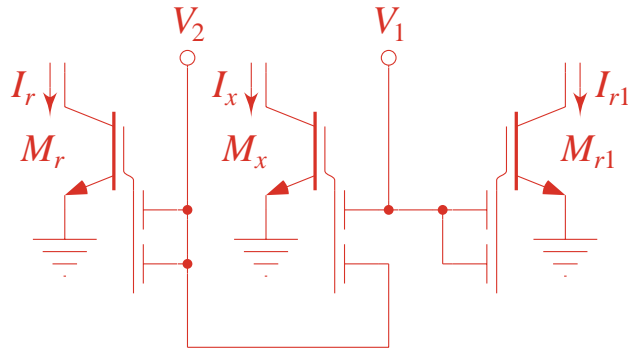
$$\frac{I_r}{I_1} = \sqrt{\left(\frac{I_x}{I_1}\right)^2 + \left(\frac{I_y}{I_1}\right)^2} \quad \Rightarrow \quad I_r^2 = I_x^2 + I_y^2 \quad \Rightarrow \quad I_r = \underbrace{\frac{I_x^2}{I_r}}_{I_{r1}} + \underbrace{\frac{I_y^2}{I_r}}_{I_{r2}}$$

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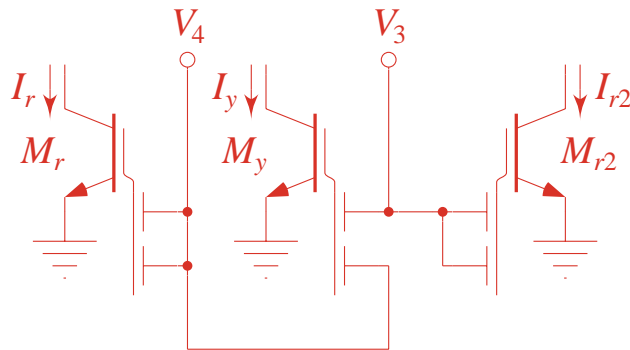
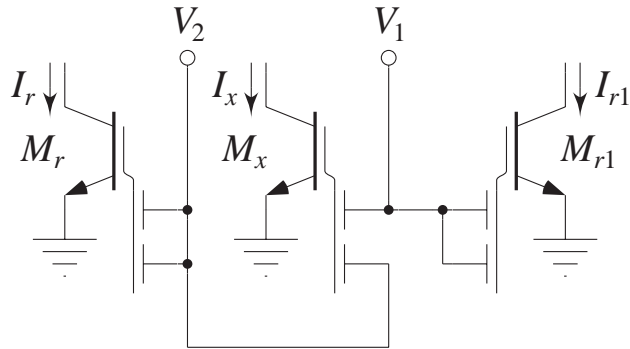
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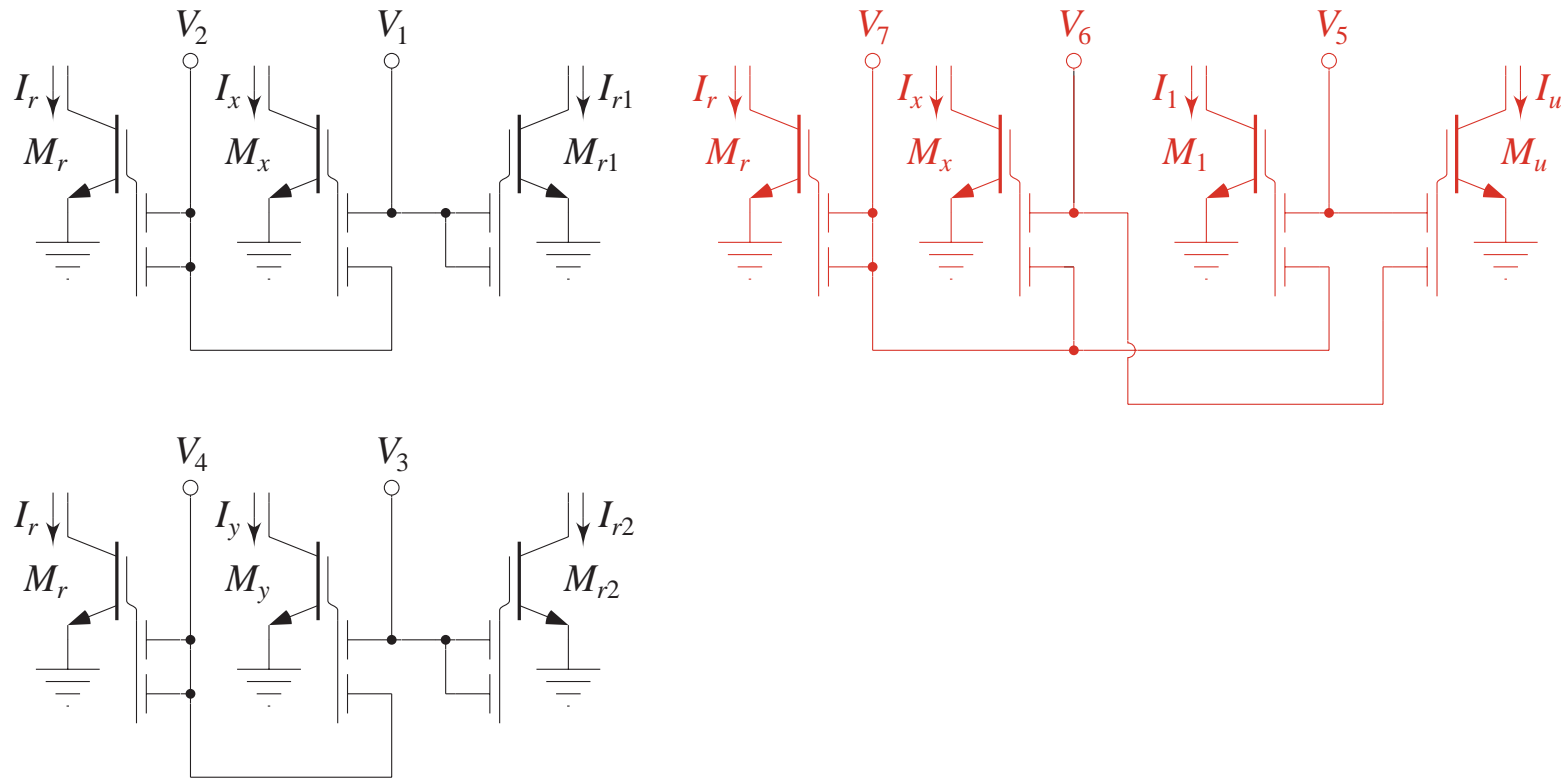
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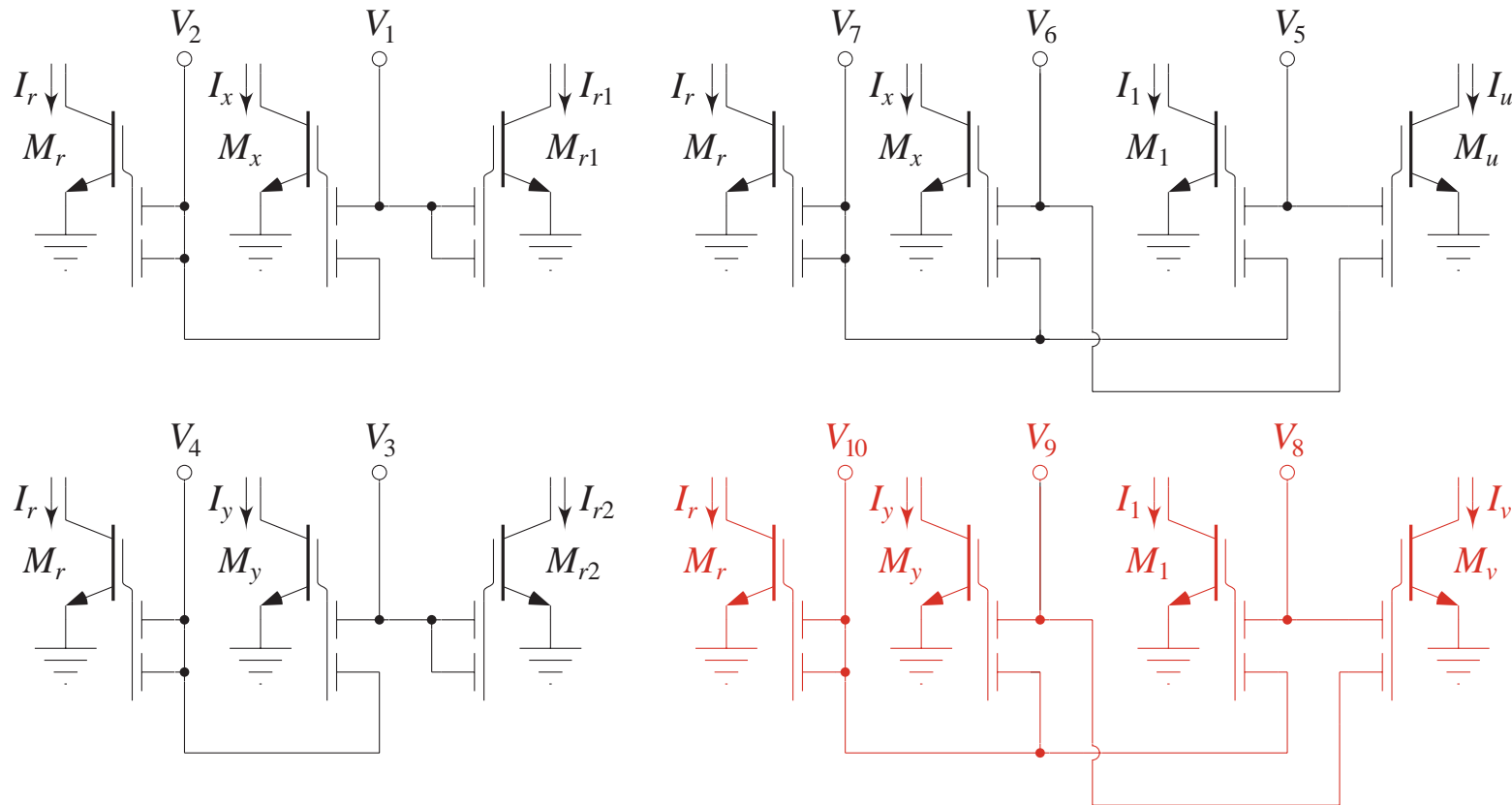
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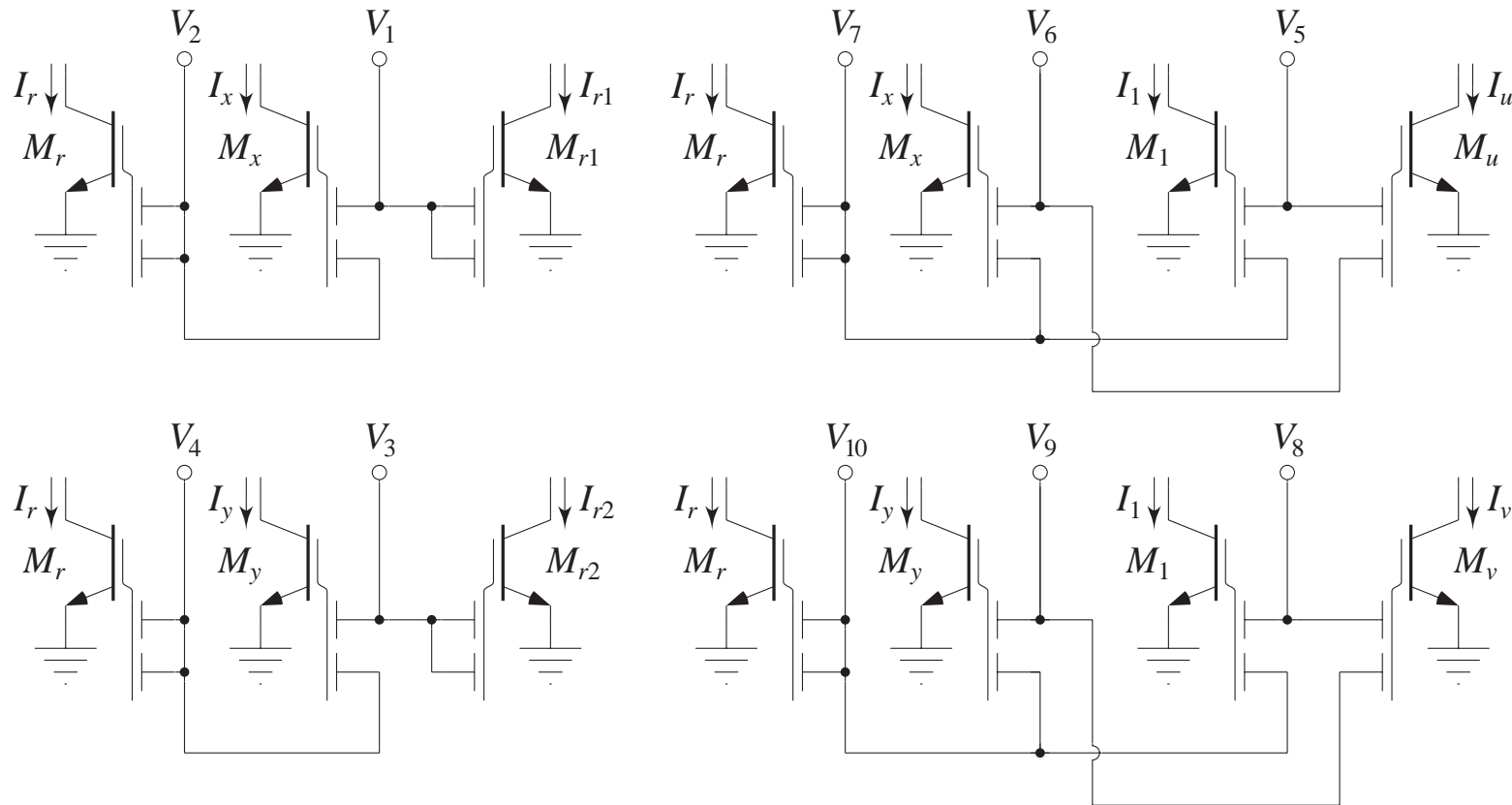
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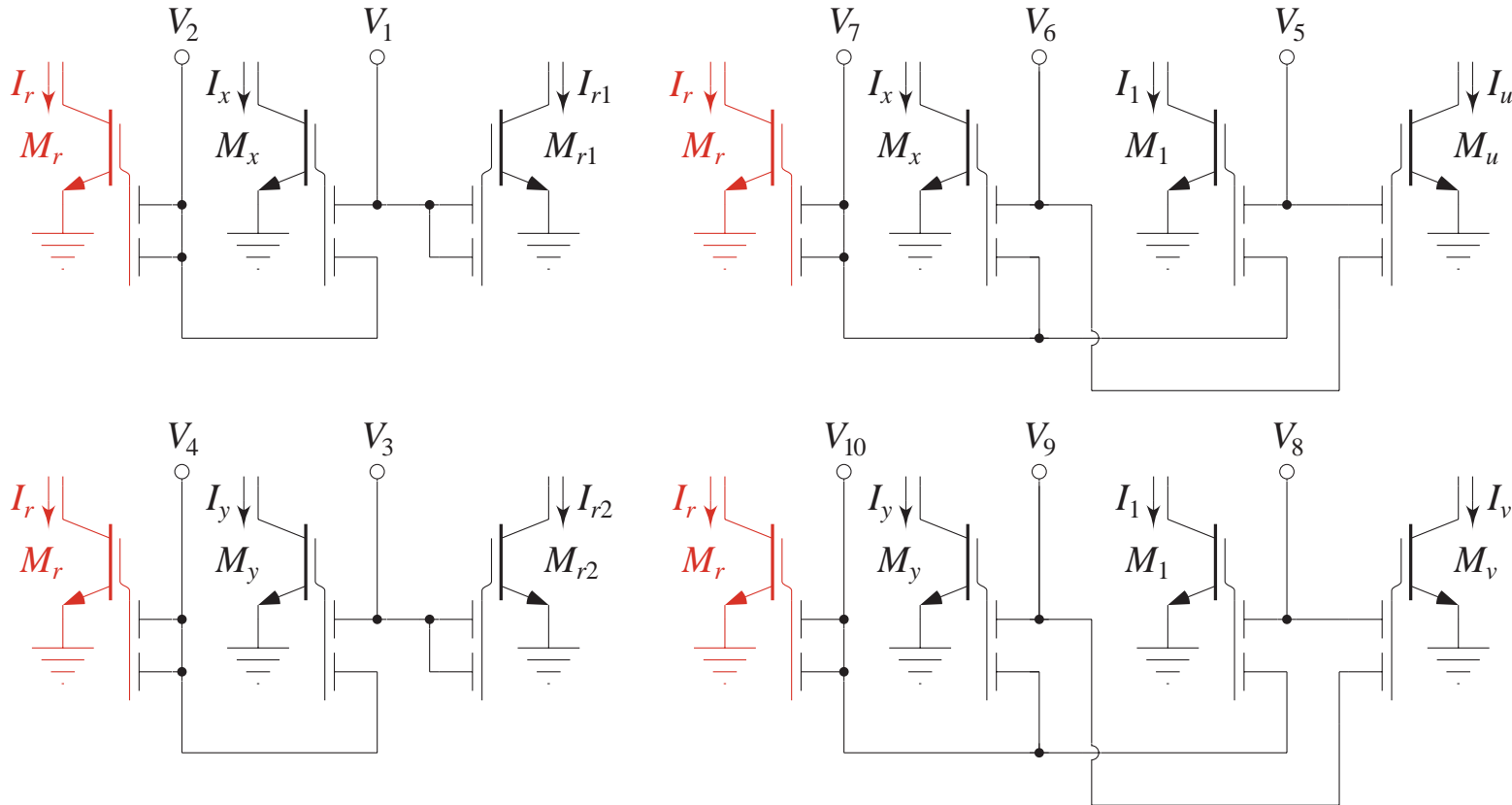
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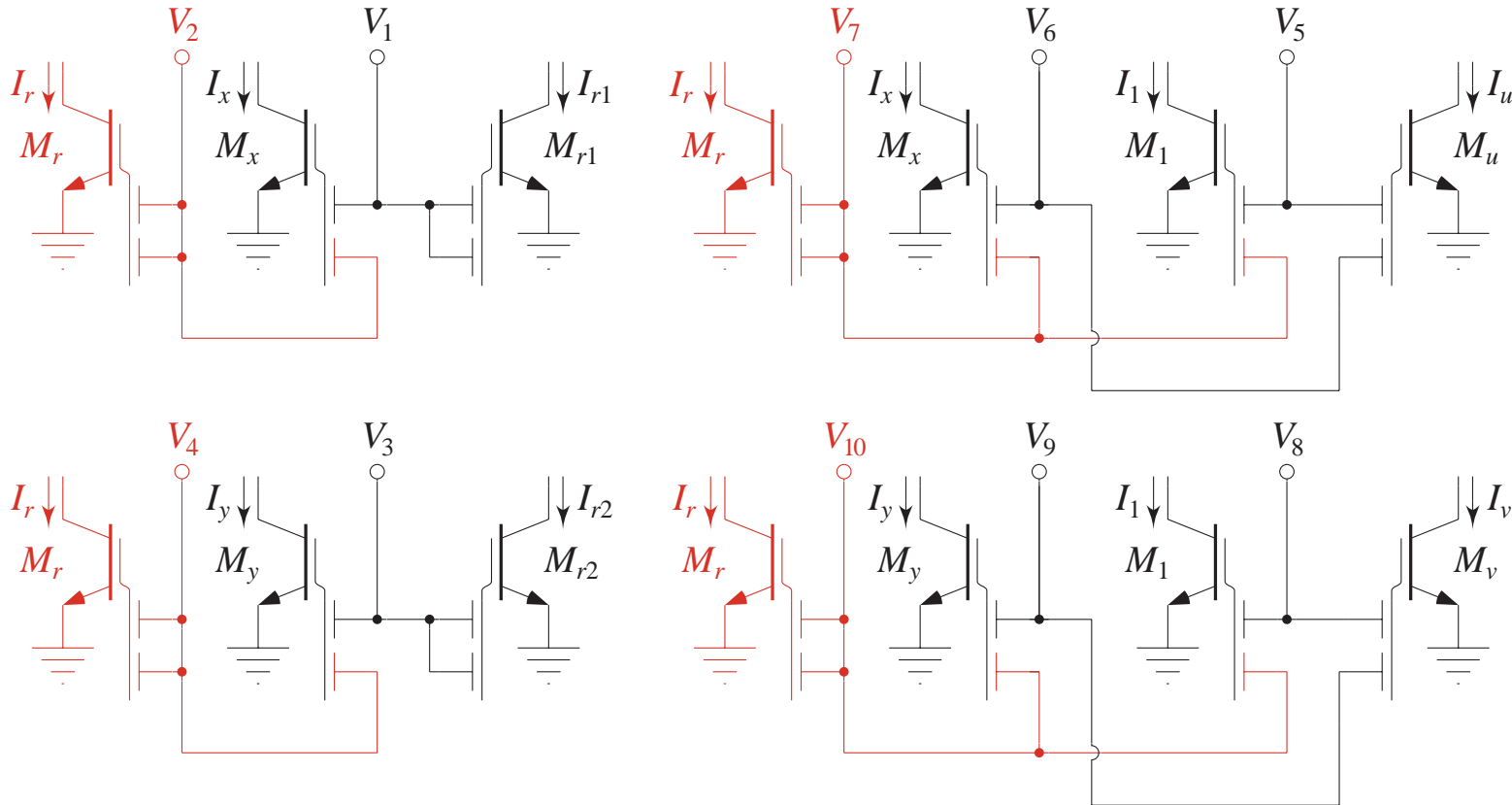
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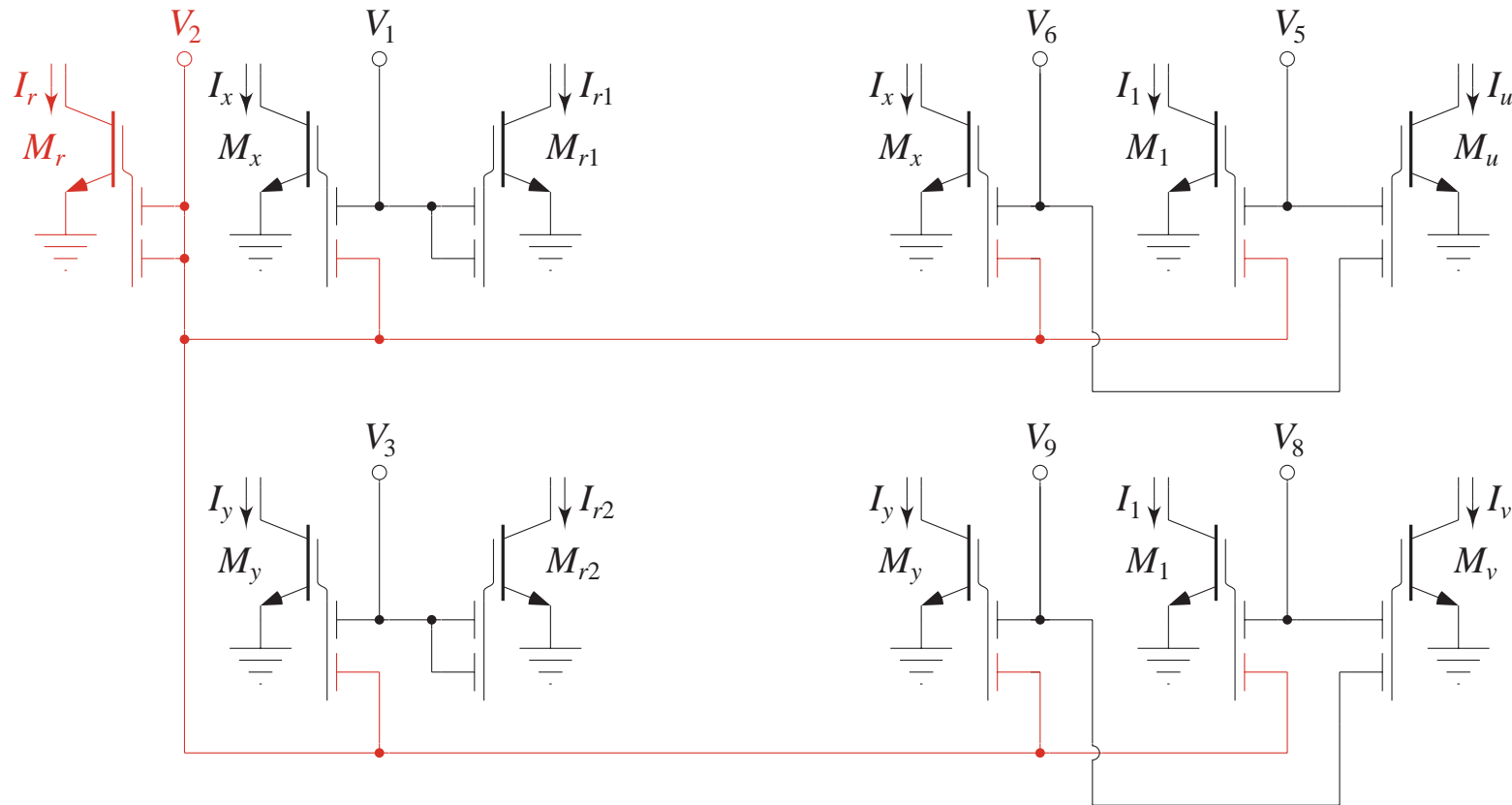
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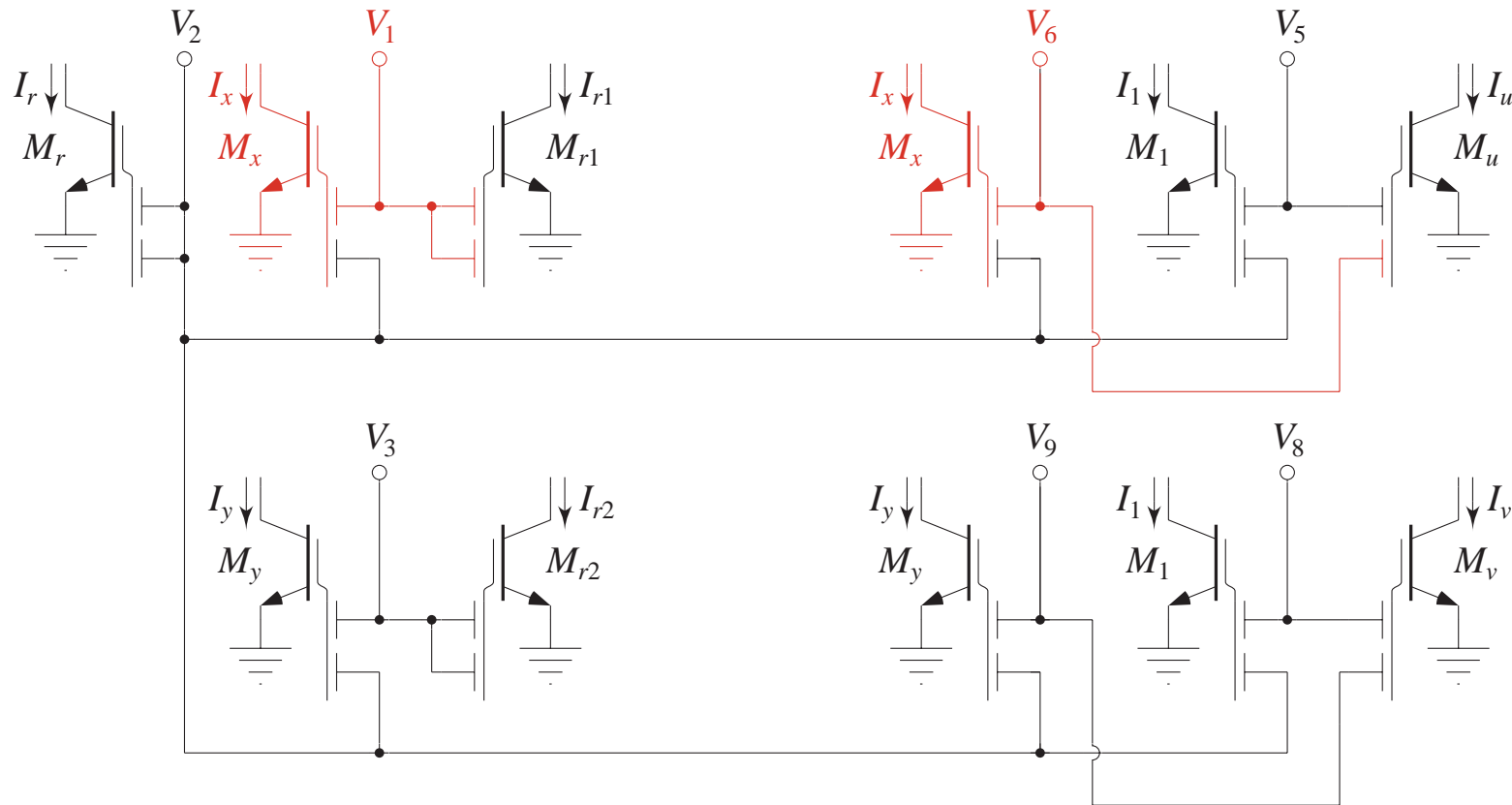
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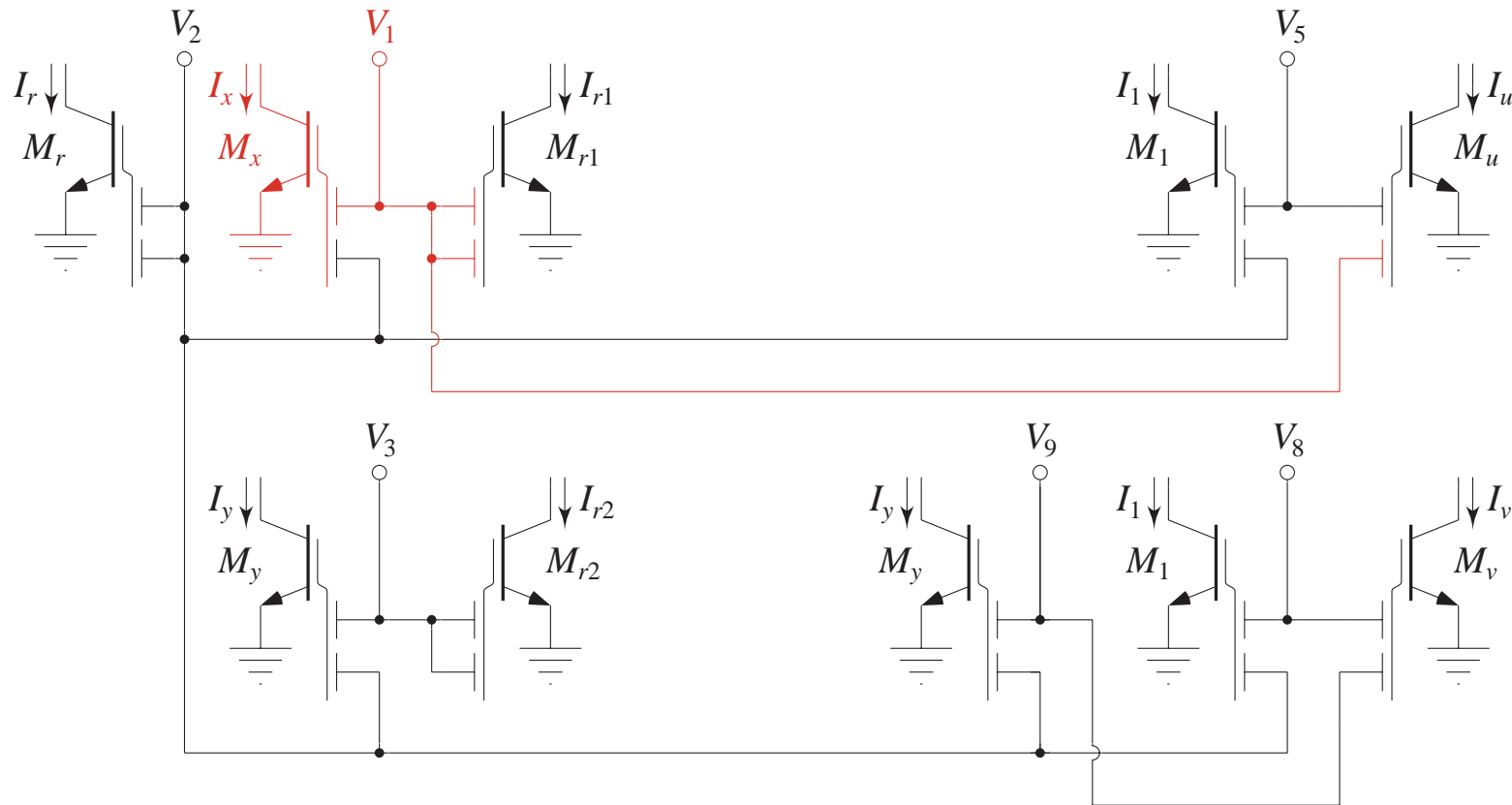
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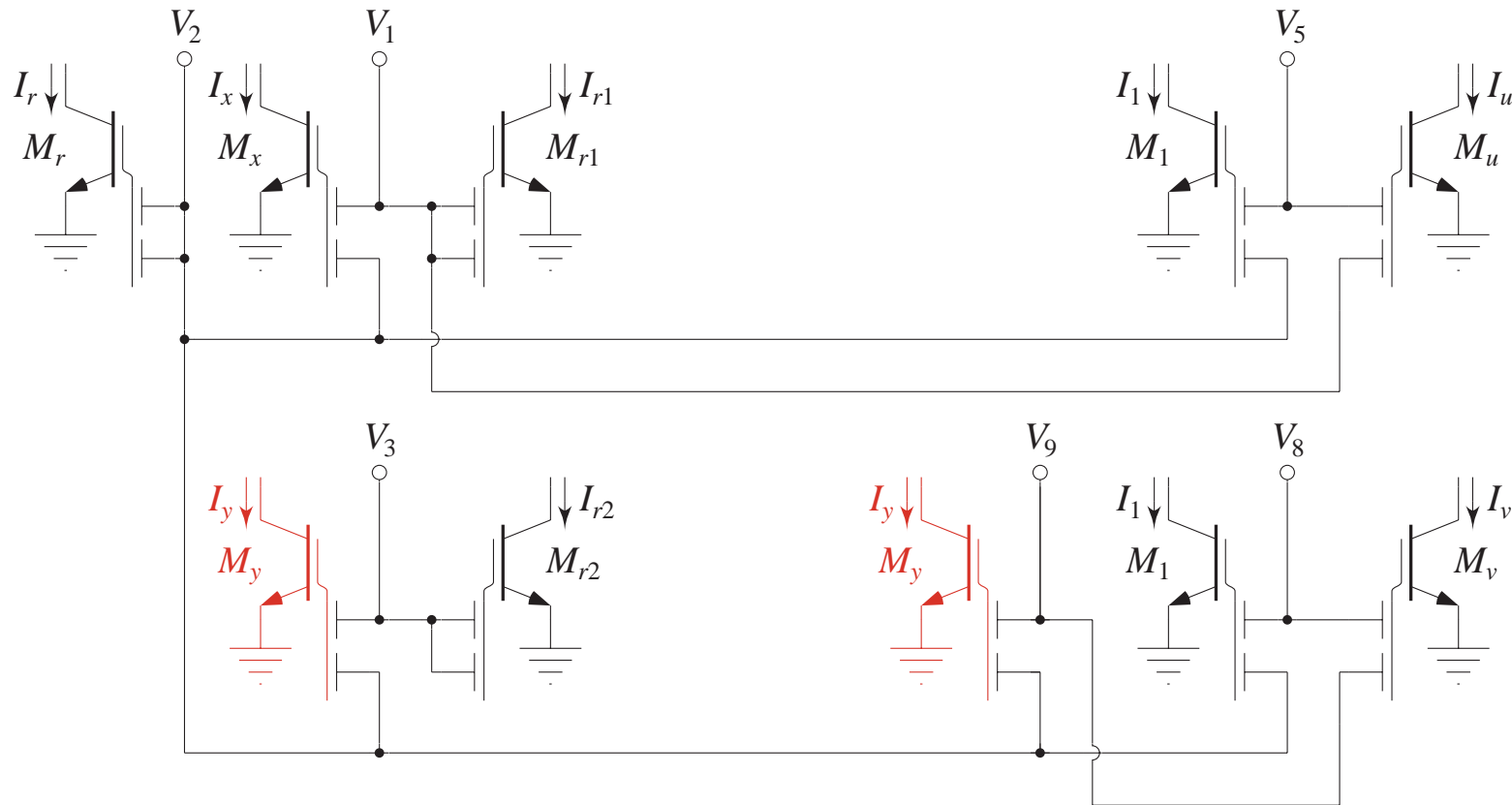
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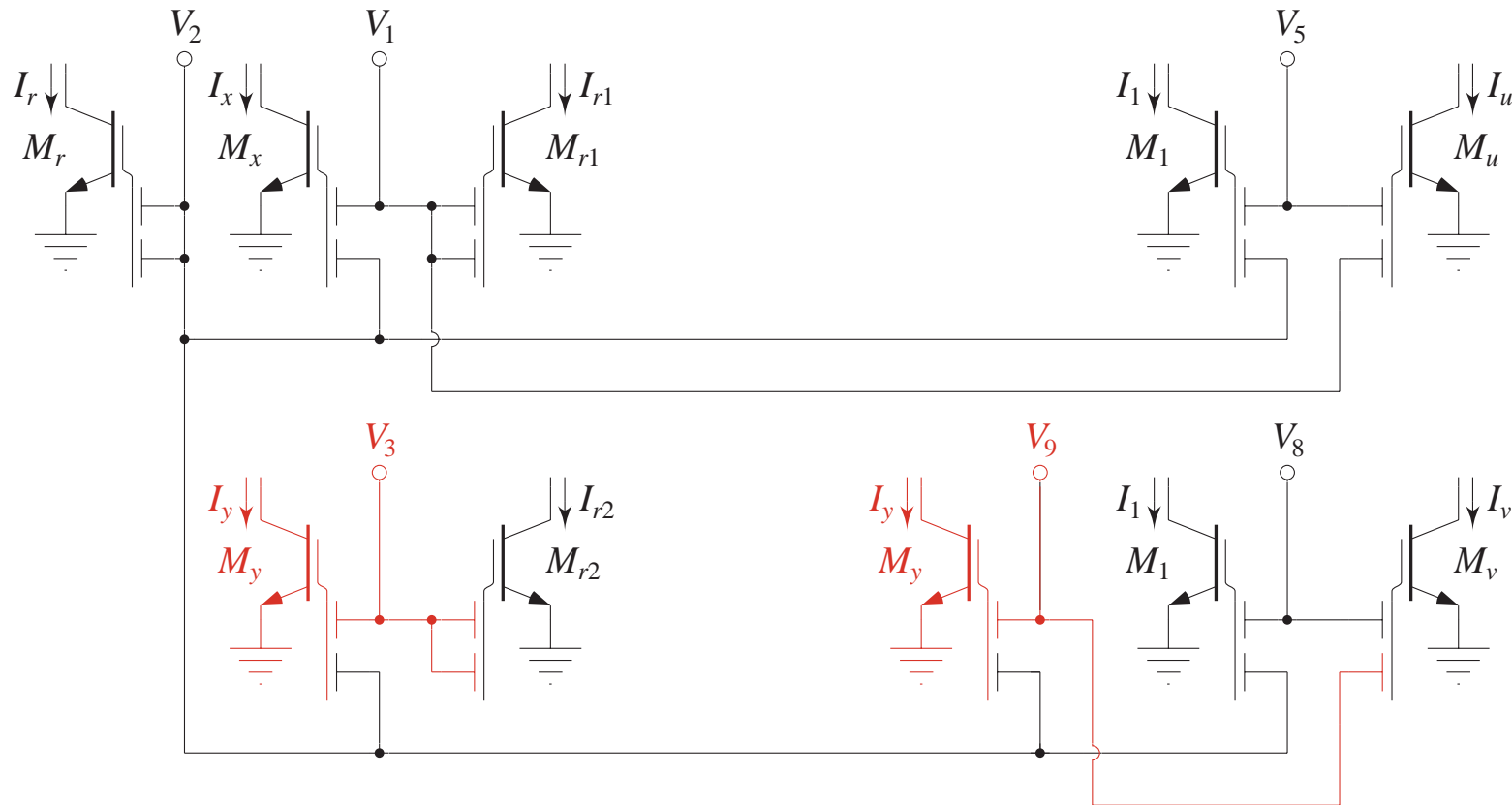
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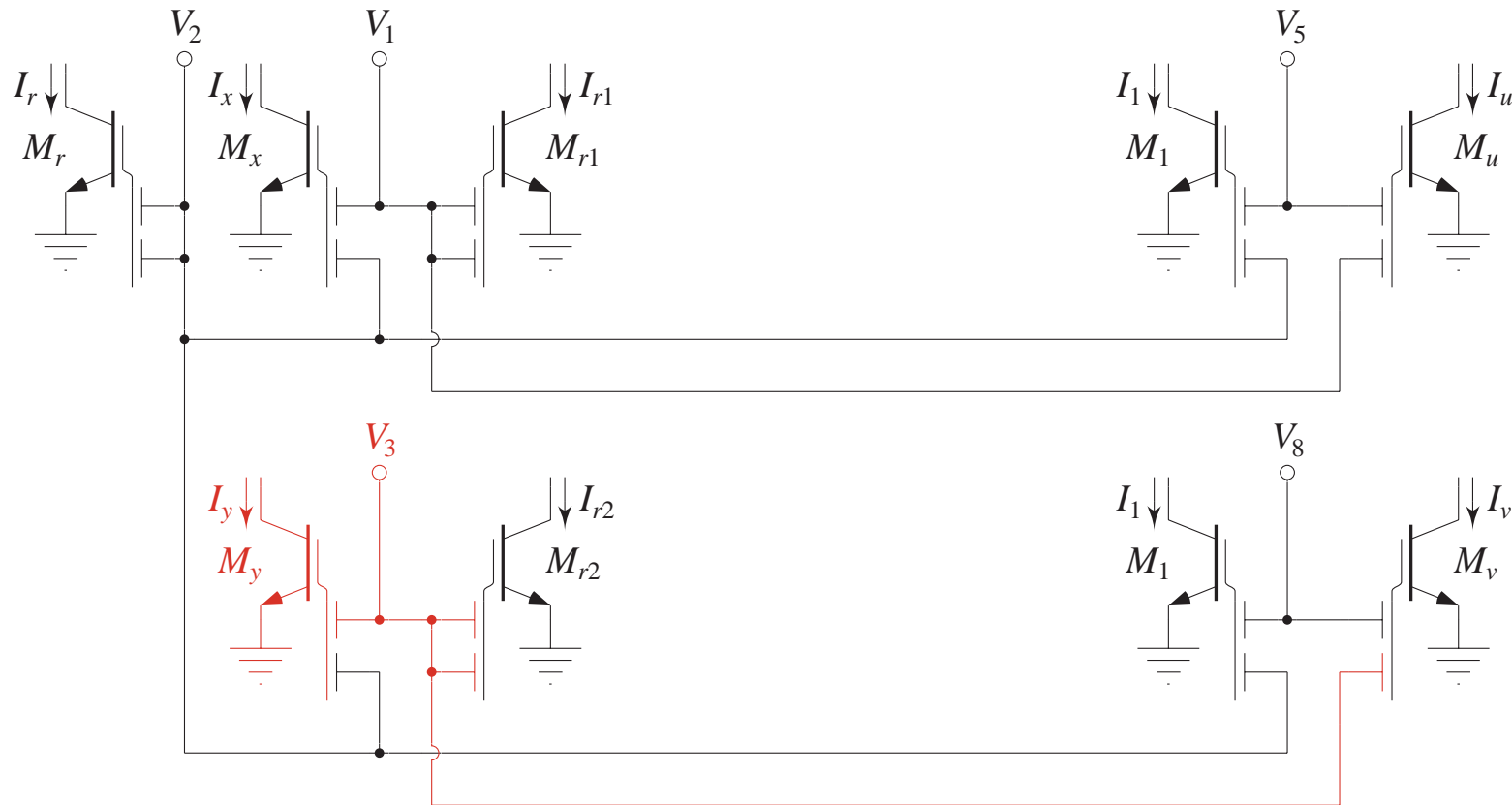
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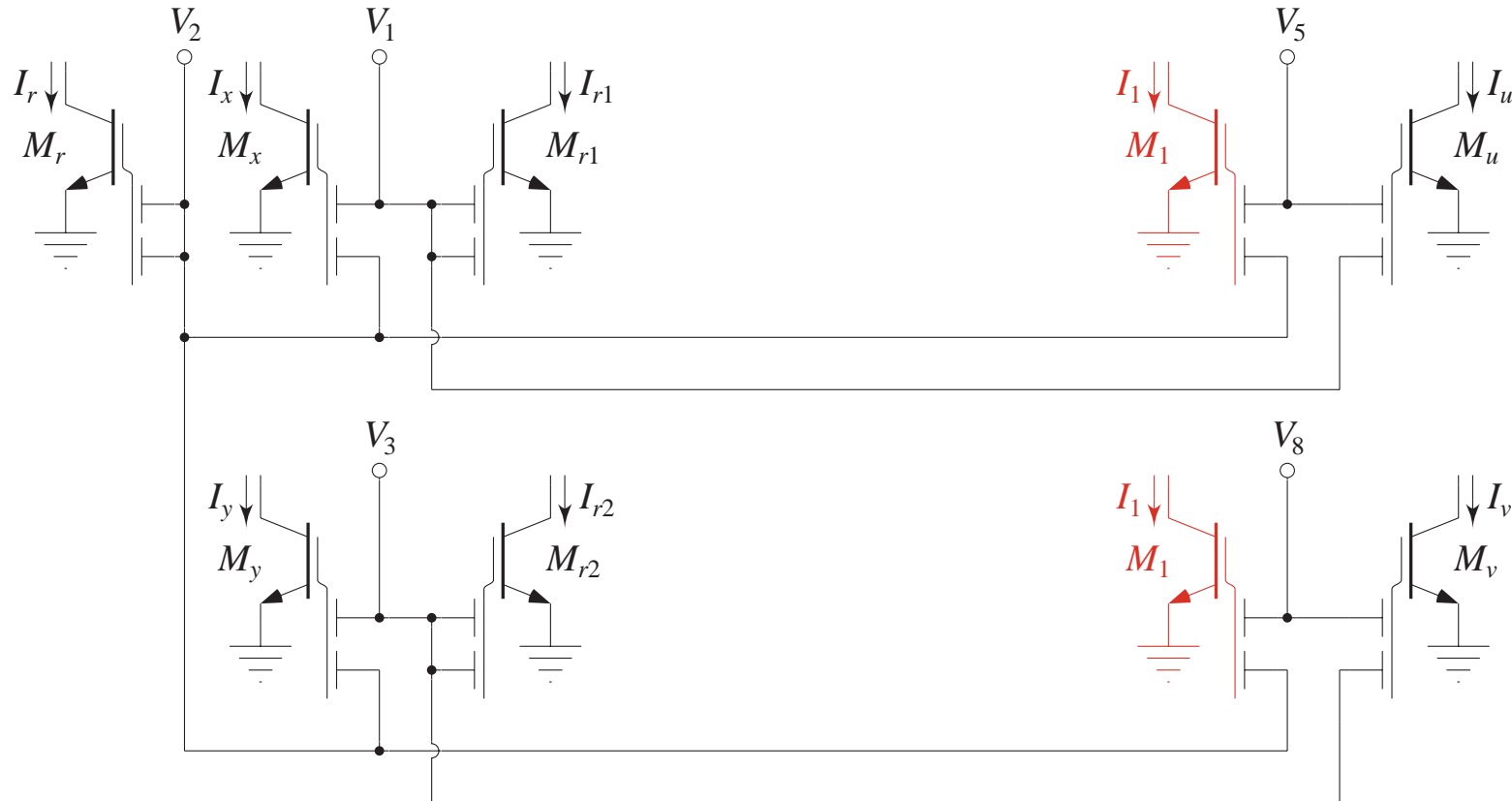
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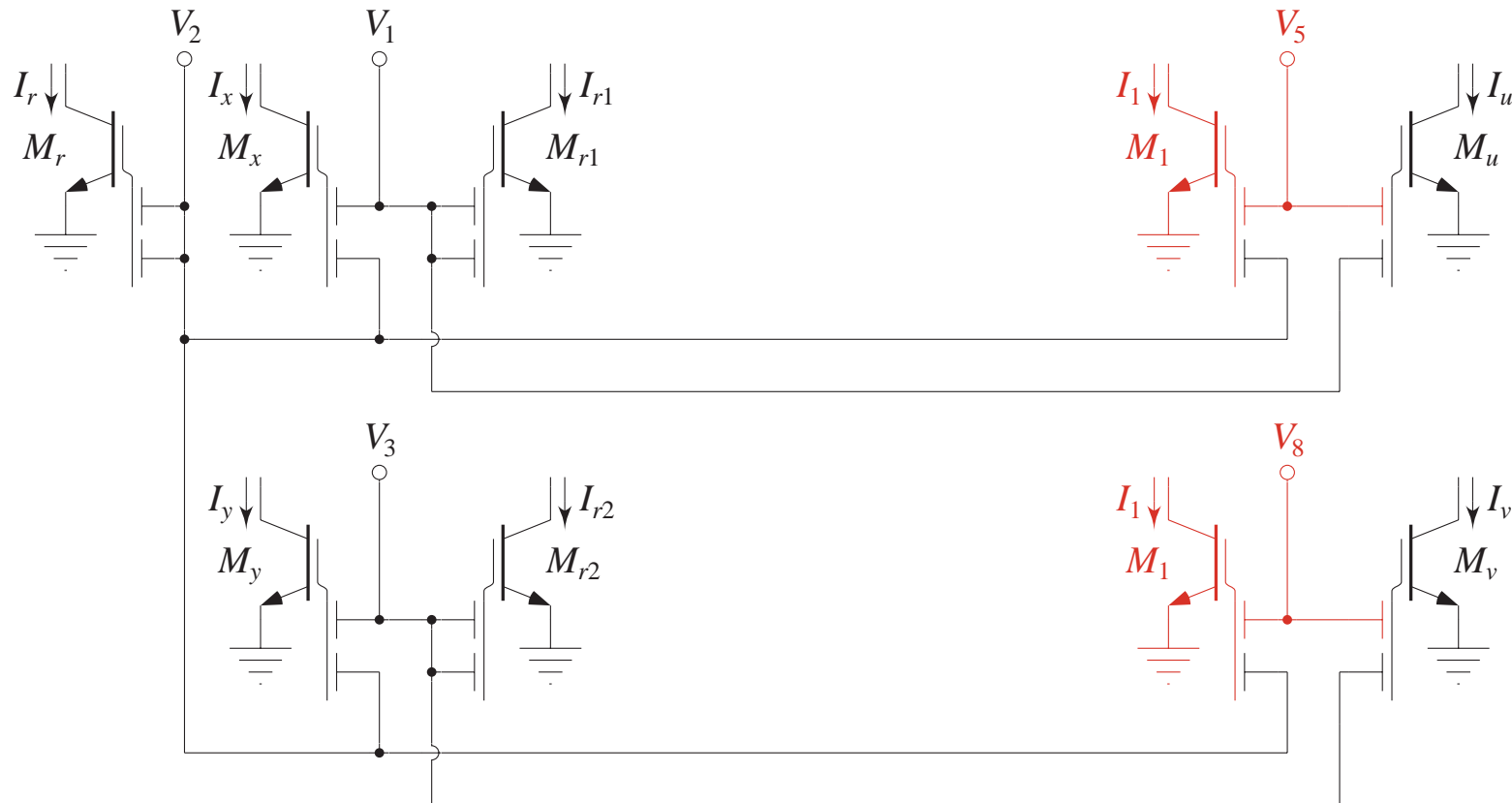
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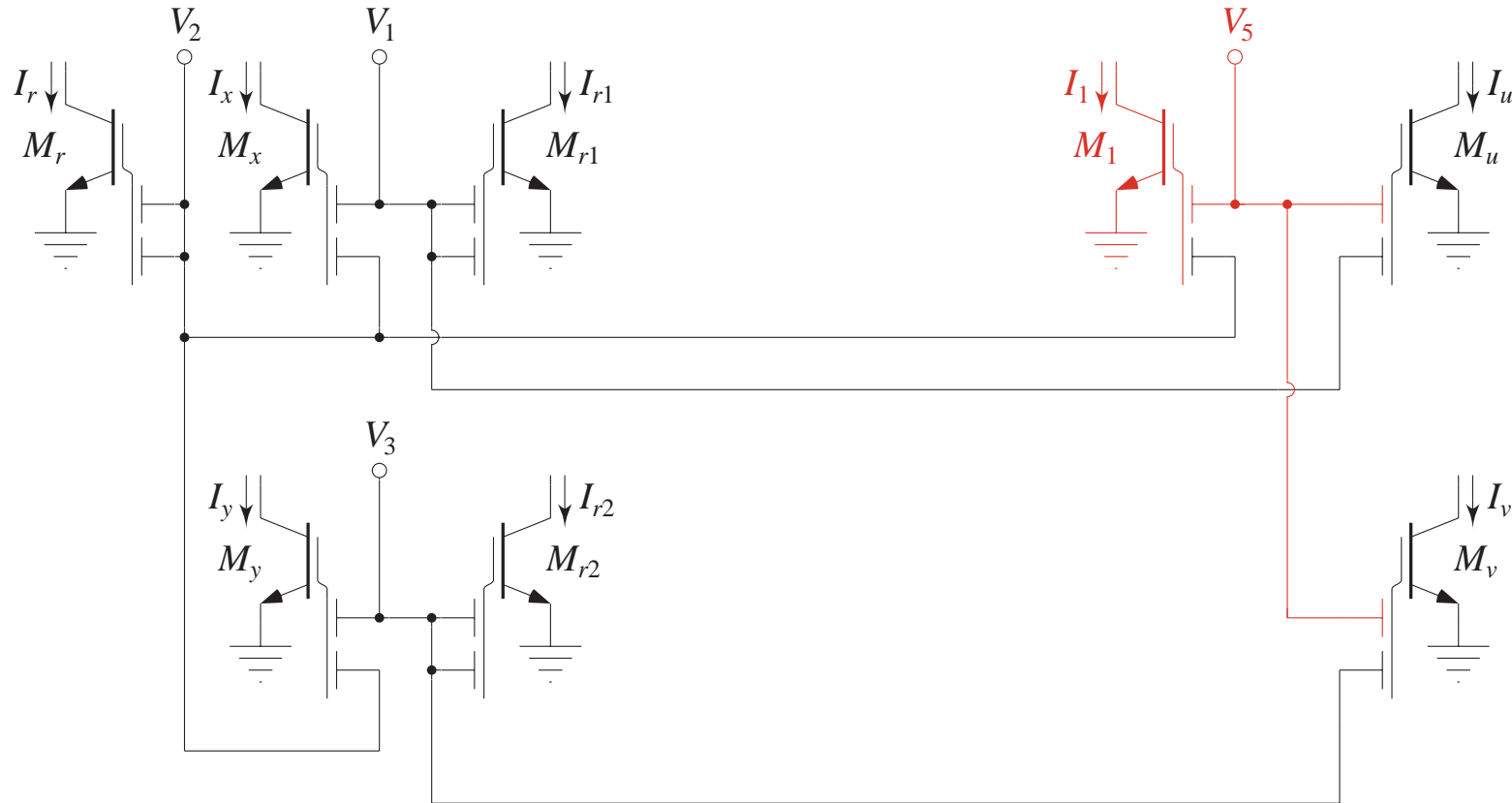
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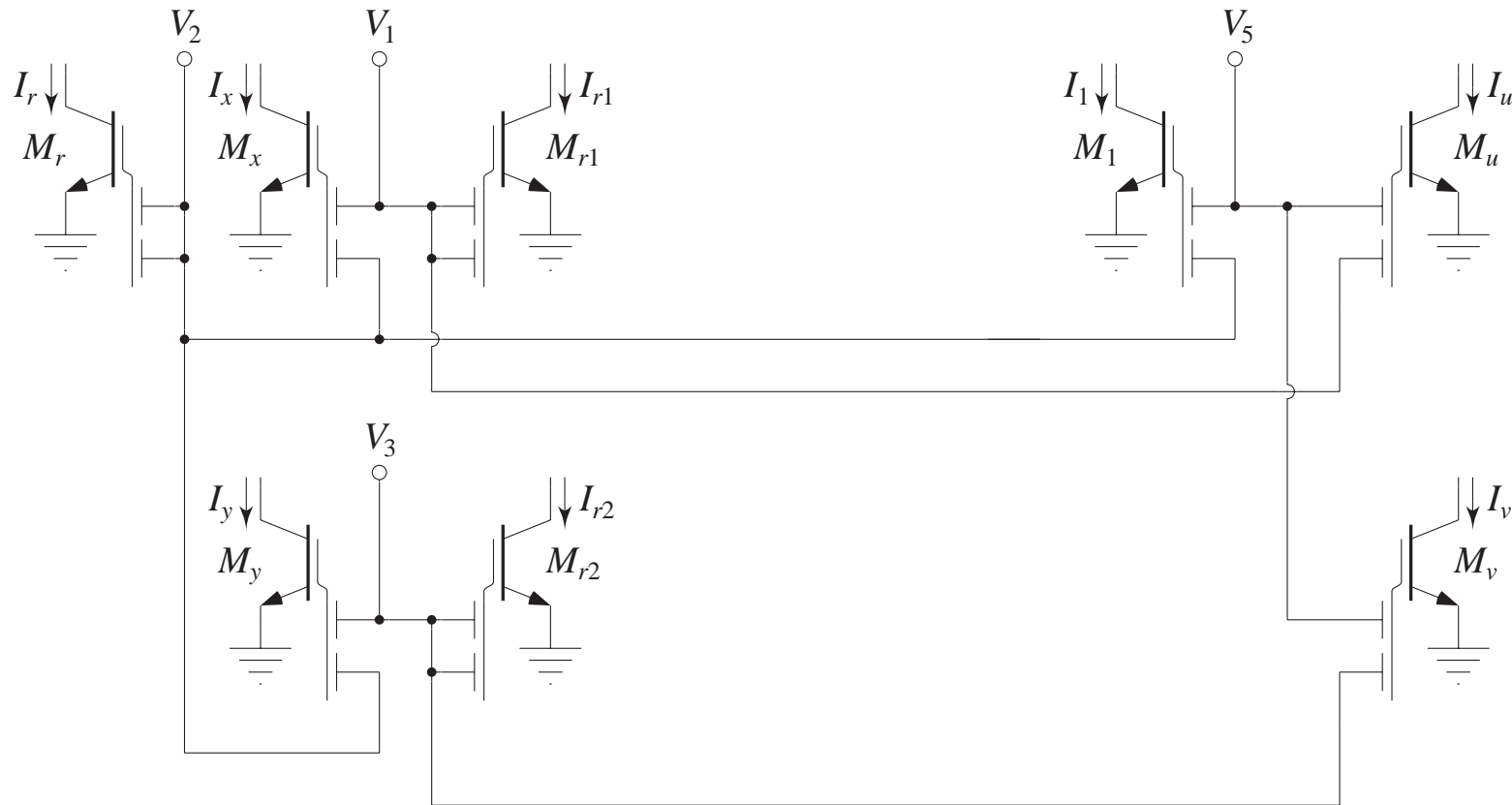
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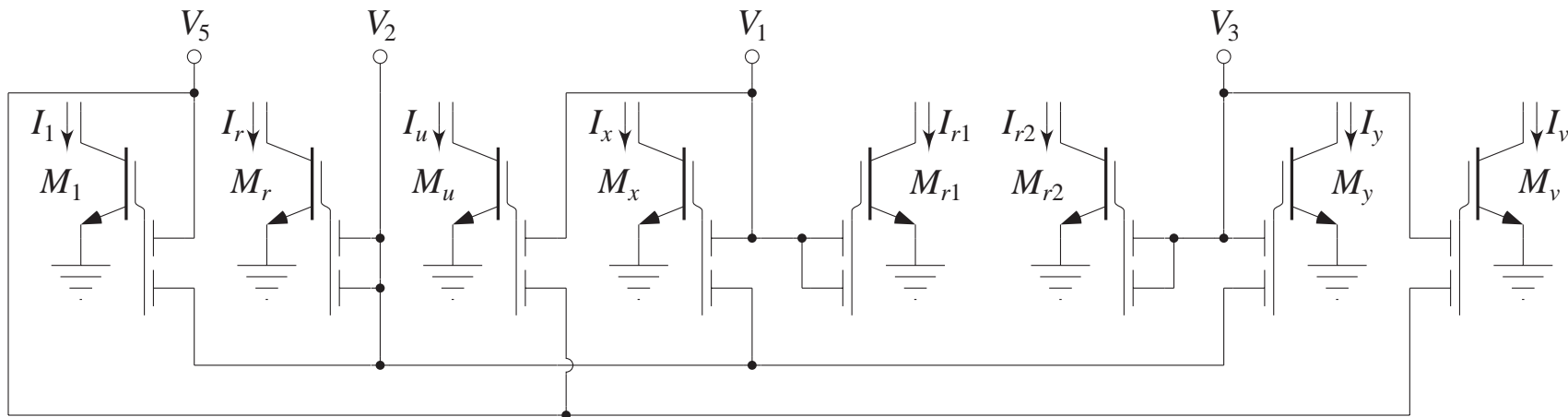
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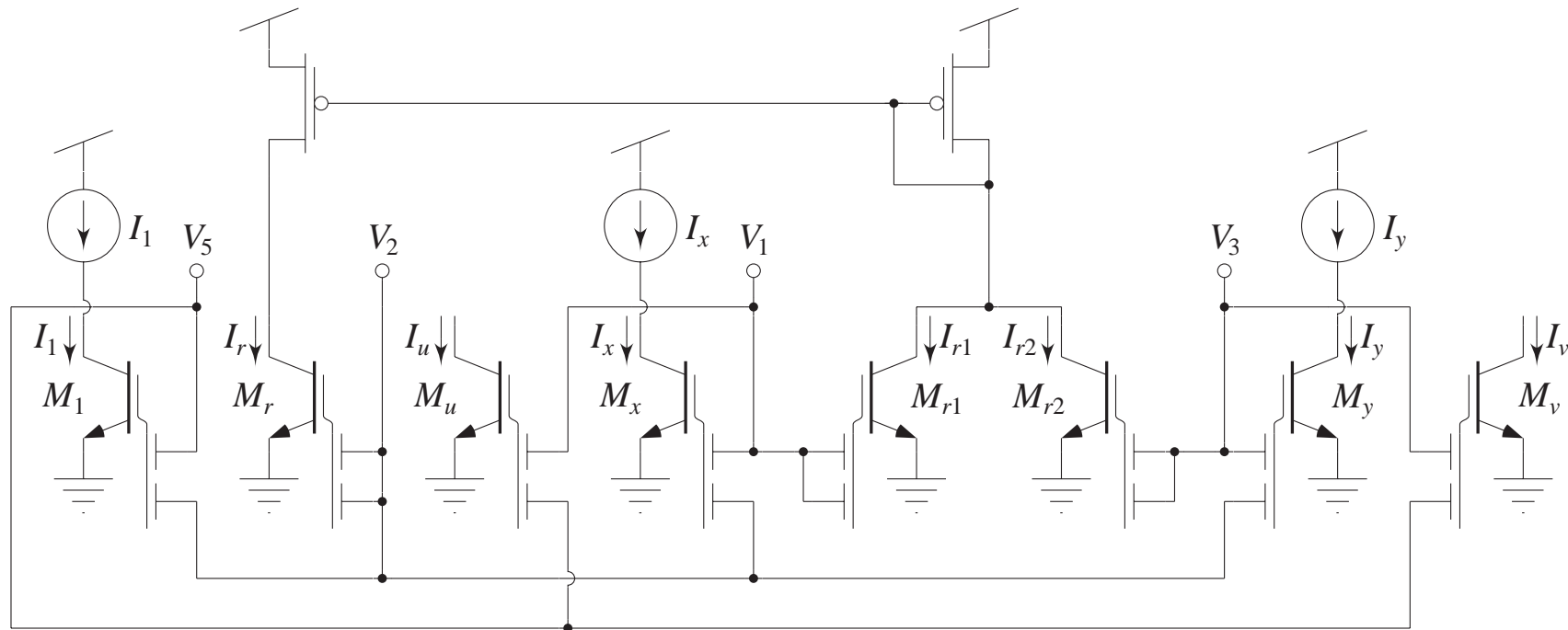






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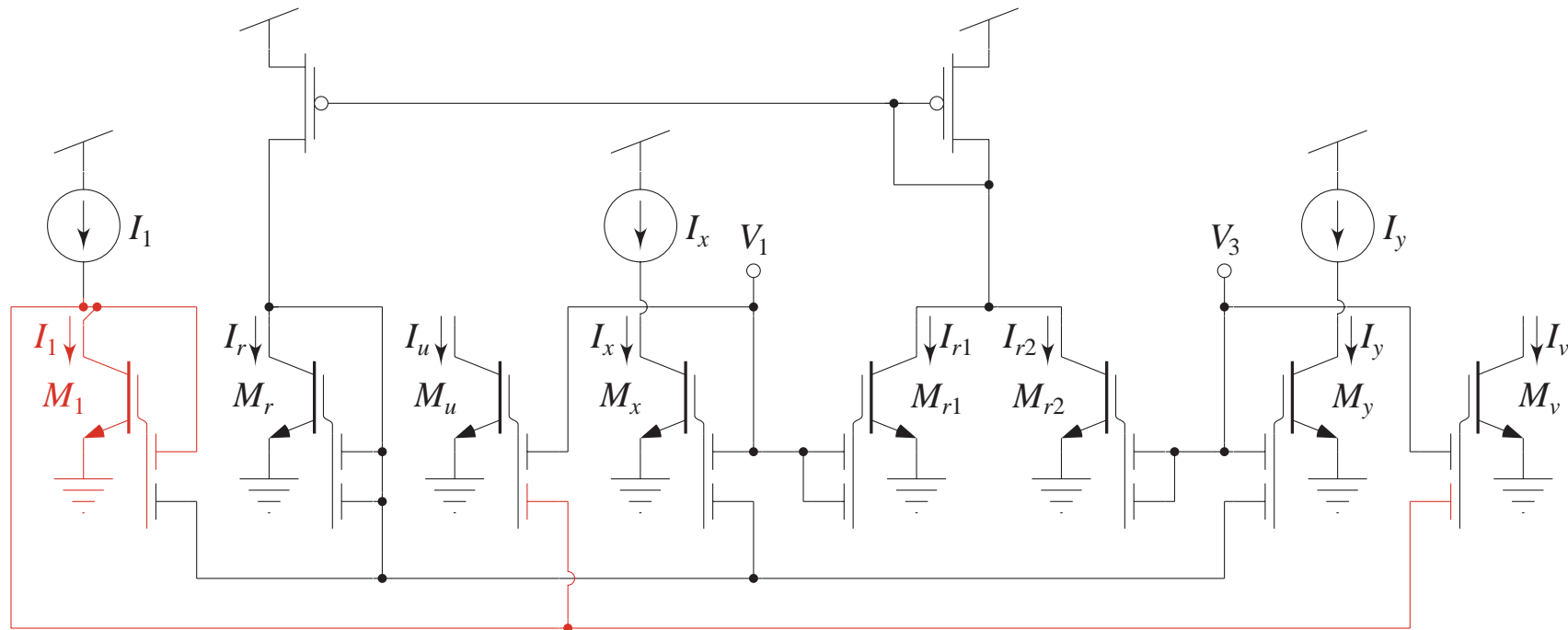
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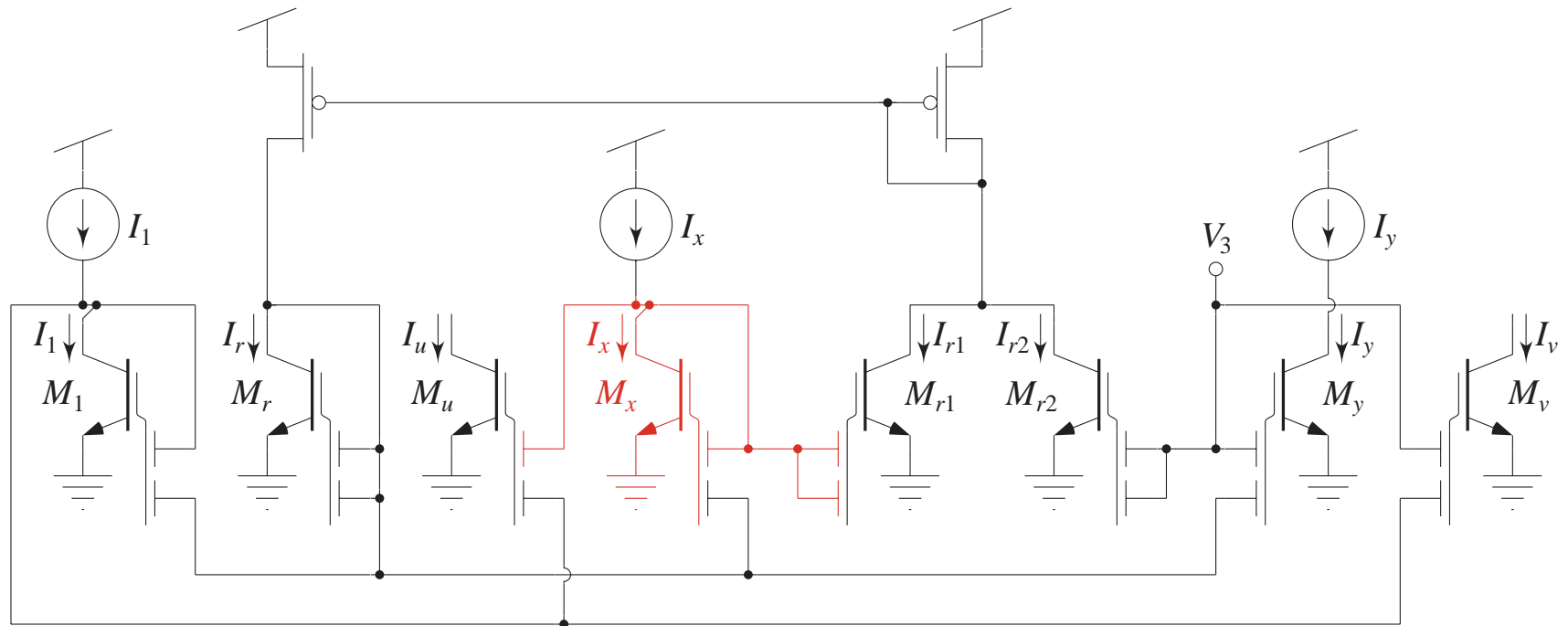
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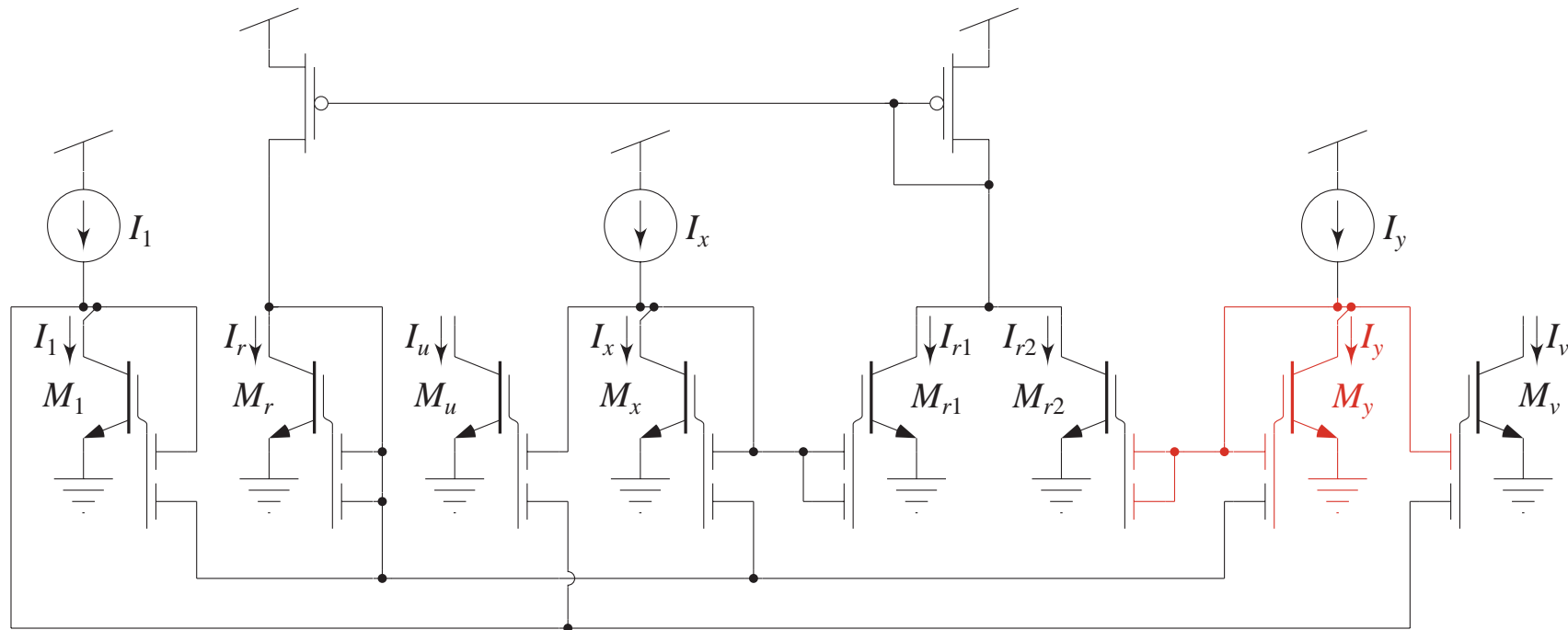
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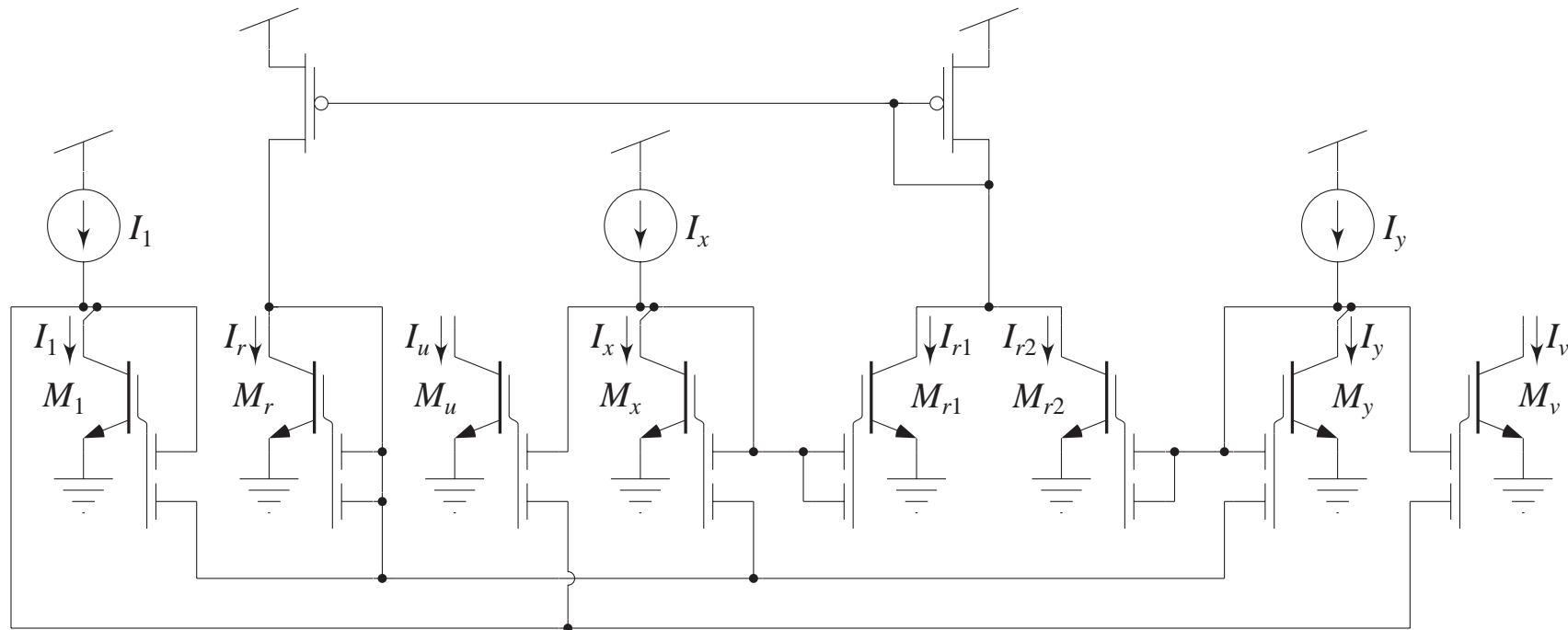
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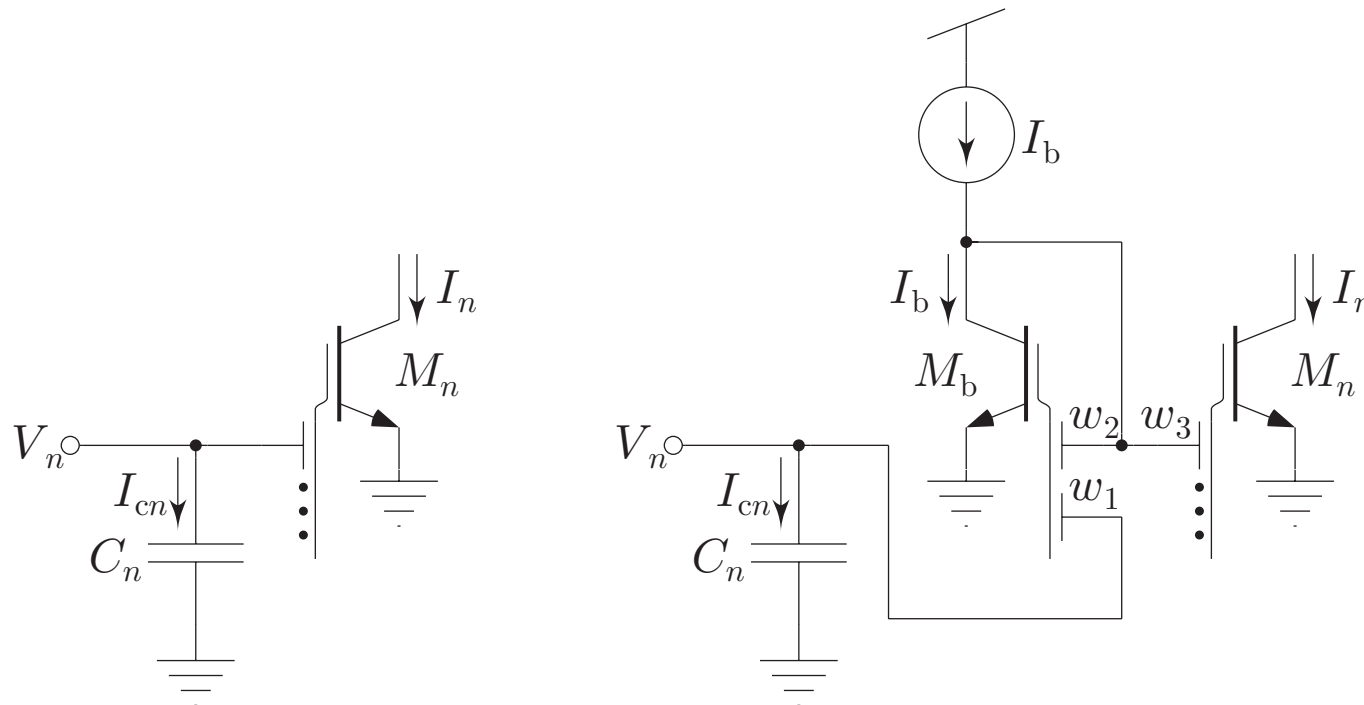


# Static MITE Network Synthesis: **Vector Normalizer**

$$\begin{array}{lll} \text{TLP: } I_{r1}I_r = I_x^2 & I_uI_r = I_xI_1 & \text{KCL: } I_r = I_{r1} + I_{r2} \\ I_{r2}I_r = I_y^2 & I_vI_r = I_yI_1 & \end{array}$$



# Dynamic MITE Network Synthesis: Output Structures



$$I_n \propto e^{wV_n/U_T}$$

$$\frac{\partial I_n}{\partial V_n} = \frac{w}{U_T} I_n$$

$$I_n \propto e^{-wV_n/U_T}$$

$$\frac{\partial I_n}{\partial V_n} = -\frac{w}{U_T} I_n$$

# Dynamic MITE Network Synthesis: **First-Order LPF**

Synthesize a first-order low-pass filter described by

$$\tau \frac{dy}{dt} + y = x, \quad \text{where } x > 0.$$



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We represent each signal as a ratio of a signal current to the unit current:

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Substituting these into the ODE, we obtain

$$\tau \frac{d}{dt} \left( \frac{I_y}{I_1} \right) + \frac{I_y}{I_1} = \frac{I_x}{I_1}$$

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Substituting these into the ODE, we obtain

$$\tau \frac{d}{dt} \left( \frac{I_y}{I_1} \right) + \frac{I_y}{I_1} = \frac{I_x}{I_1} \quad \Rightarrow \quad \tau \frac{dI_y}{dt} + I_y = I_x.$$

## Dynamic MITE Network Synthesis: **First-Order LPF**

To implement the time derivative, we introduce a log-compressed voltage state variable,  $V_y$ . Using the chain rule, we can express the preceding equation as

$$\tau \frac{\partial I_y}{\partial V_y} \cdot \frac{dV_y}{dt} + I_y = I_x$$

## Dynamic MITE Network Synthesis: **First-Order LPF**

To implement the time derivative, we introduce a log-compressed voltage state variable,  $V_y$ . Using the chain rule, we can express the preceding equation as

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$$\Rightarrow -\frac{w\tau}{U_T} \cdot \frac{dV_y}{dt} + 1 = \frac{I_x}{I_y}$$

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$$\Rightarrow -\frac{w\tau}{U_T} \cdot \frac{dV_y}{dt} + 1 = \frac{I_x}{I_y} \quad \Rightarrow \quad -\frac{w\tau}{CU_T} \cdot C \frac{dV_y}{dt} + 1 = \frac{I_x}{I_y}$$

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## Dynamic MITE Network Synthesis: **First-Order LPF**

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$$\Rightarrow -\frac{w\tau}{U_T} \cdot \frac{dV_y}{dt} + 1 = \frac{I_x}{I_y} \quad \Rightarrow \quad -\underbrace{\frac{w\tau}{CU_T}}_{1/I_\tau} \cdot \underbrace{C}_{I_c} \frac{dV_y}{dt} + 1 = \frac{I_x}{I_y}$$

$$\Rightarrow -\frac{I_c}{I_\tau} + 1 = \frac{I_x}{I_y} \quad \Rightarrow \quad I_c - I_\tau = \frac{I_\tau I_x}{I_y}$$

## Dynamic MITE Network Synthesis: **First-Order LPF**

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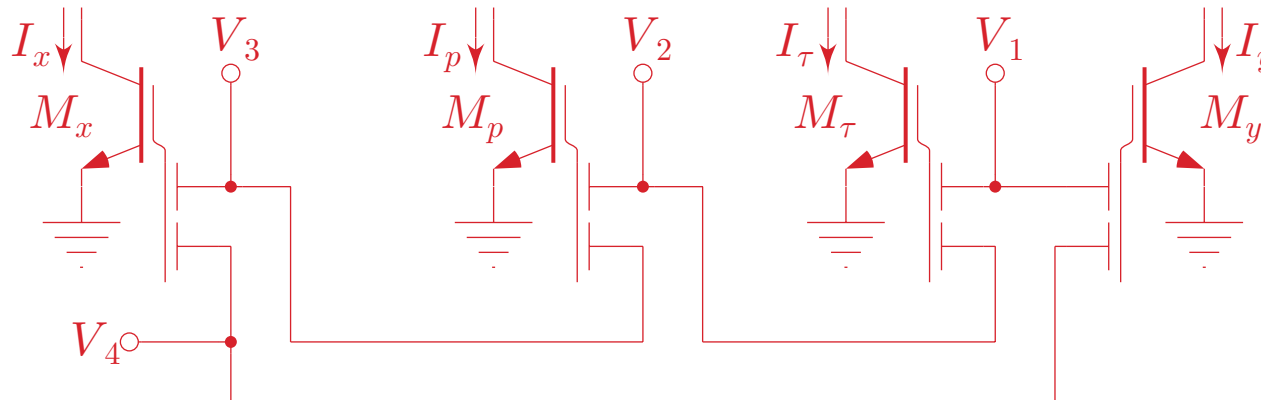
$$\begin{aligned} \tau \frac{\partial I_y}{\partial V_y} \cdot \frac{dV_y}{dt} + I_y = I_x &\implies \tau \left( -\frac{w}{U_T} I_y \right) \frac{dV_y}{dt} + I_y = I_x \\ \implies -\frac{w\tau}{U_T} \cdot \frac{dV_y}{dt} + 1 = \frac{I_x}{I_y} &\implies -\underbrace{\frac{w\tau}{CU_T}}_{1/I_\tau} \cdot \underbrace{C \frac{dV_y}{dt}}_{I_c} + 1 = \frac{I_x}{I_y} \\ \implies -\frac{I_c}{I_\tau} + 1 = \frac{I_x}{I_y} &\implies I_c - I_\tau = \underbrace{\frac{I_\tau I_x}{I_y}}_{I_p} \end{aligned}$$

# Dynamic MITE Network Synthesis: **First-Order LPF**

$$\text{TLP: } I_p I_y = I_x I_\tau \quad \text{KCL: } I_p + I_c = I_\tau$$

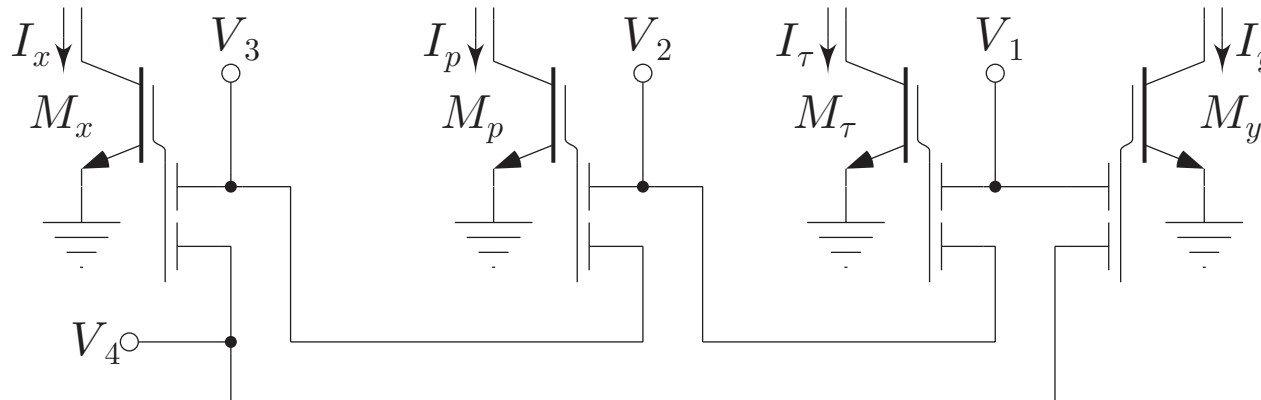
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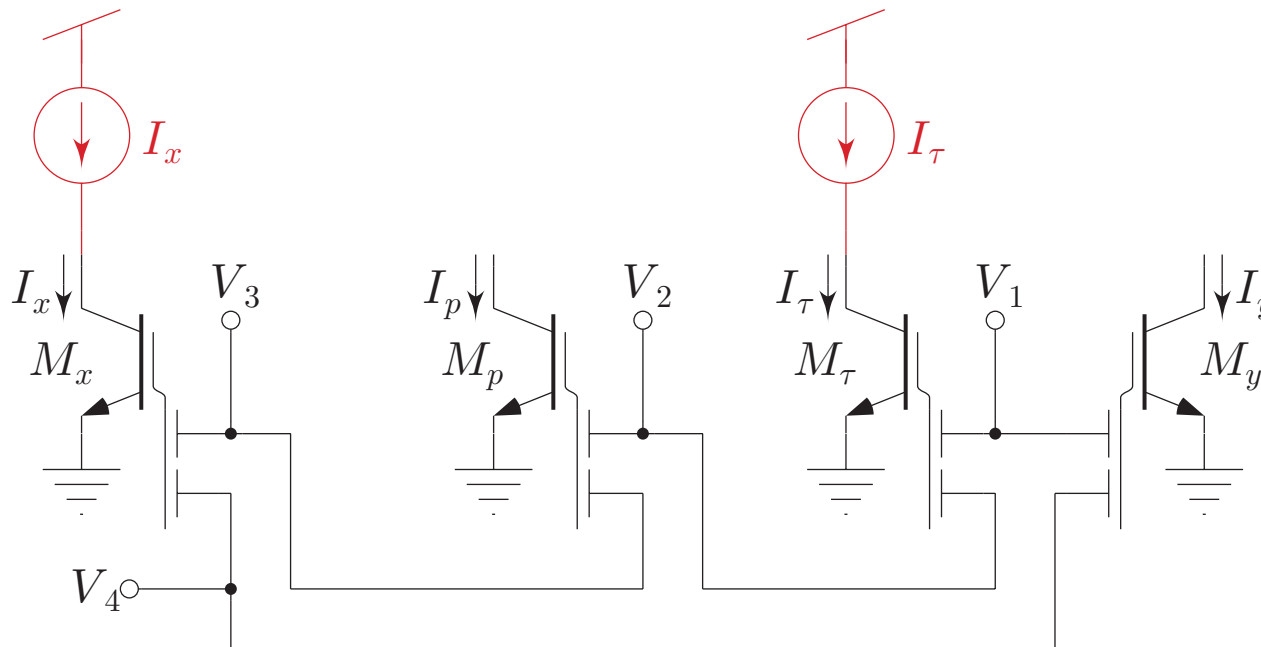
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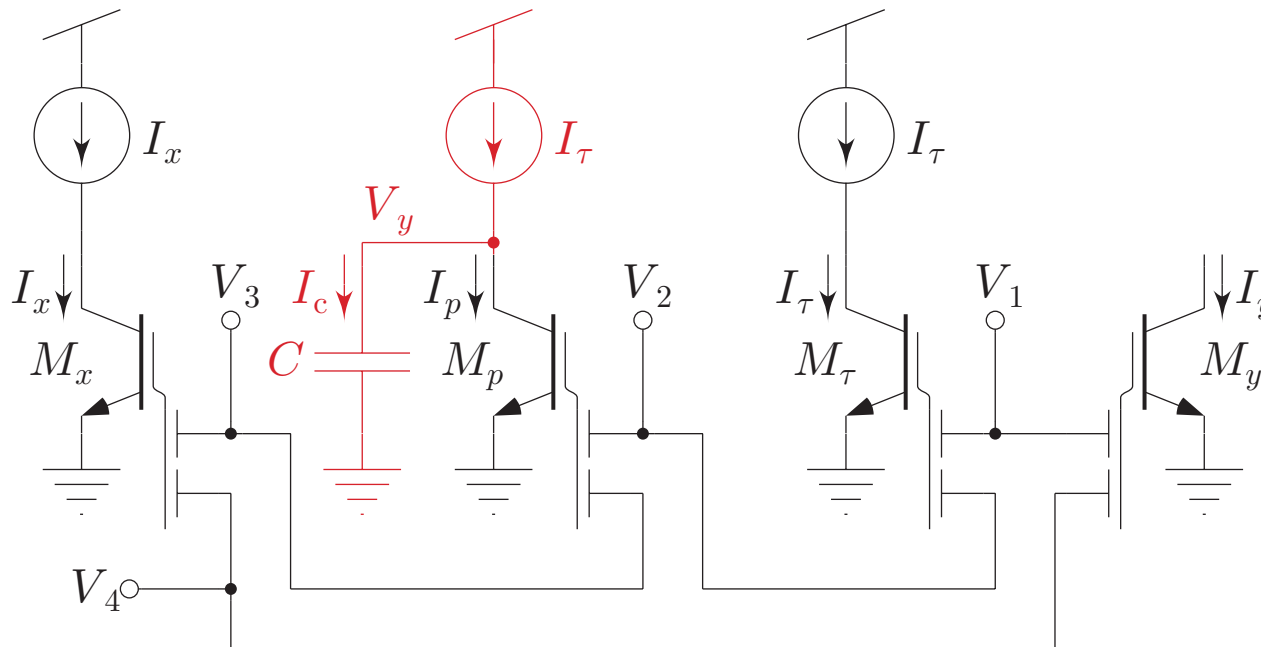
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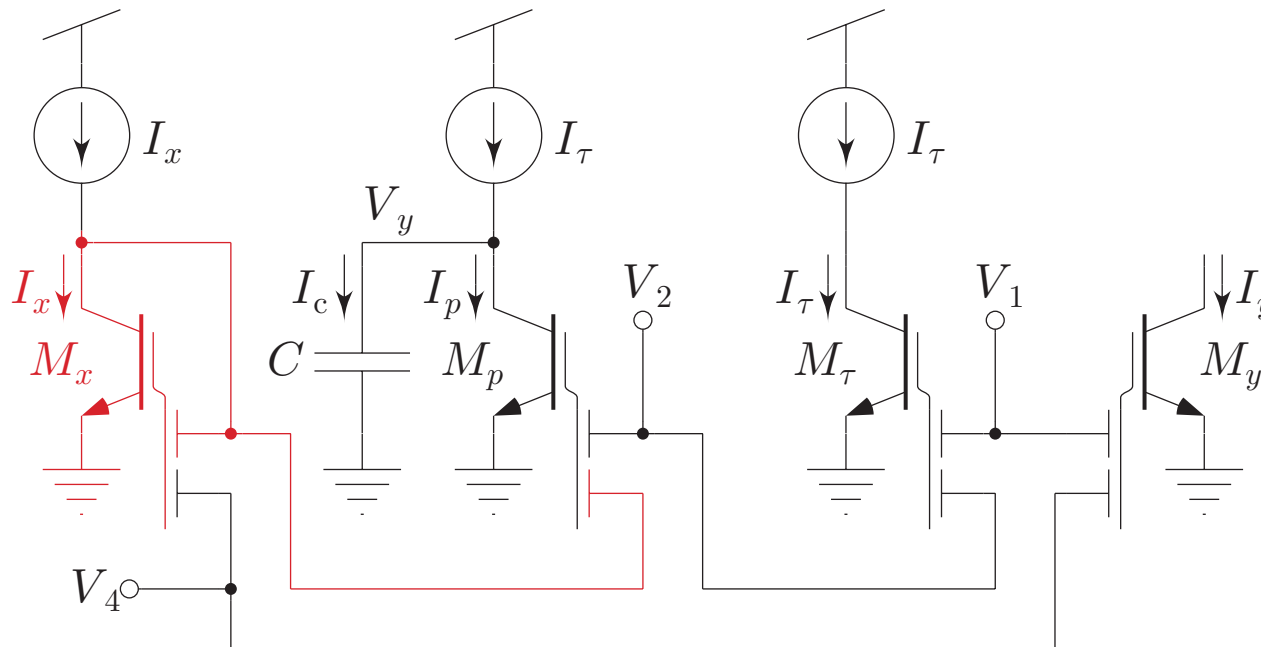
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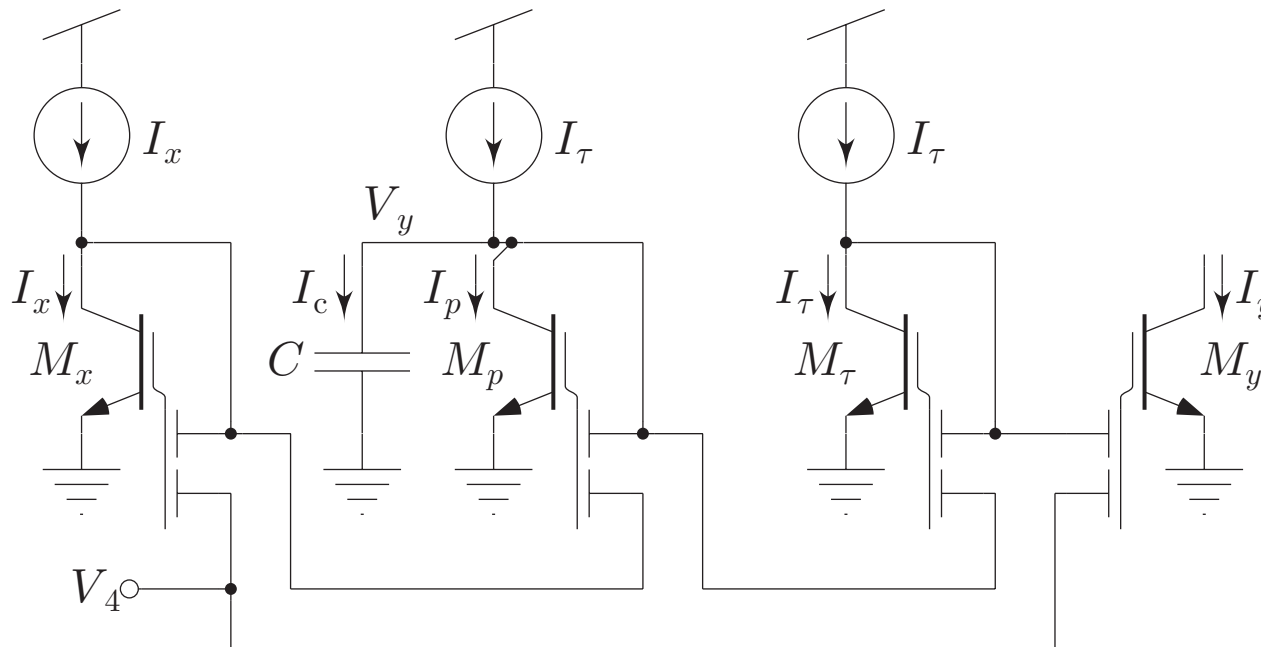






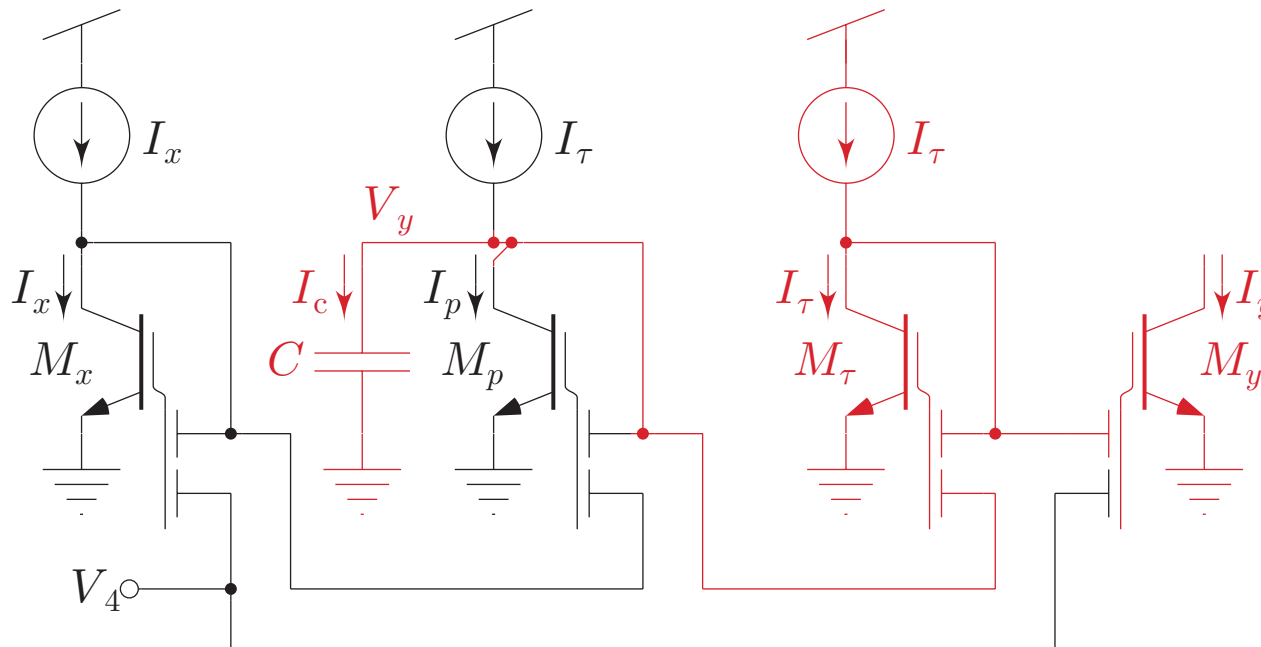
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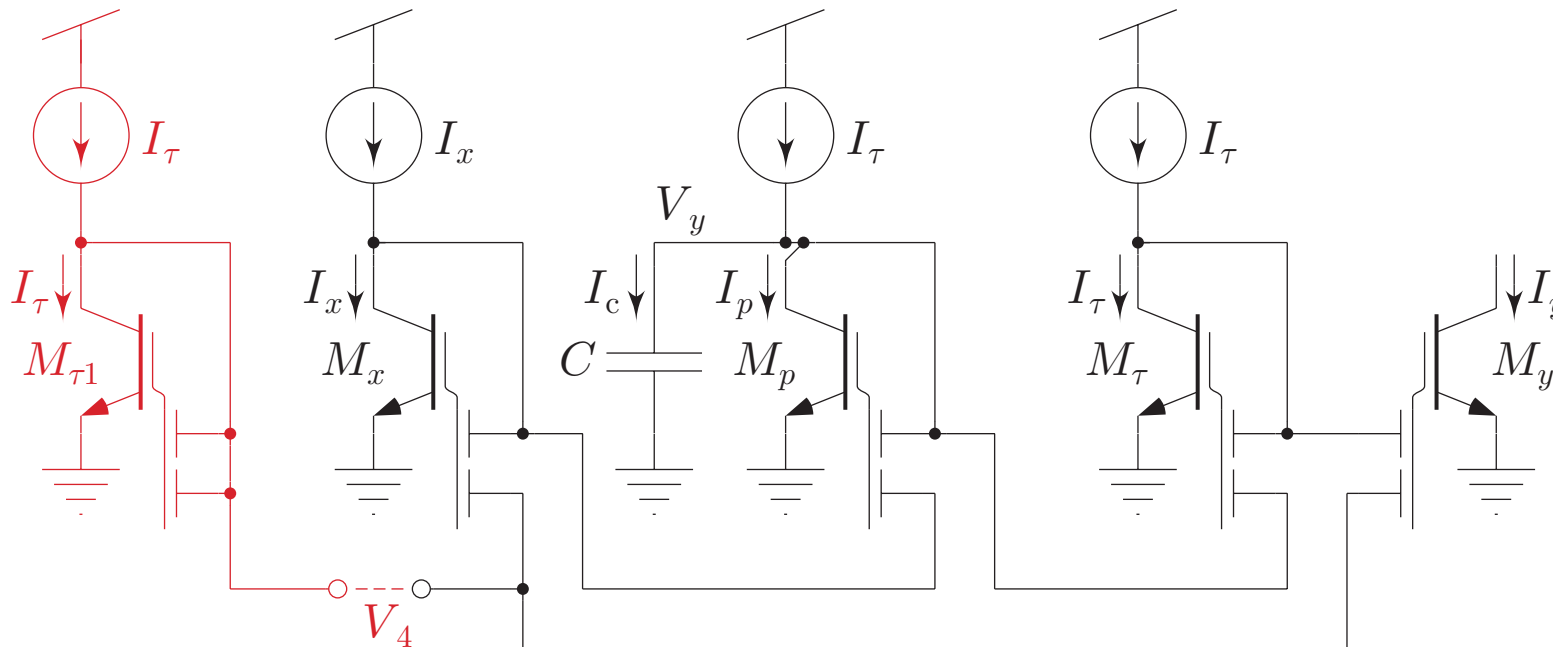
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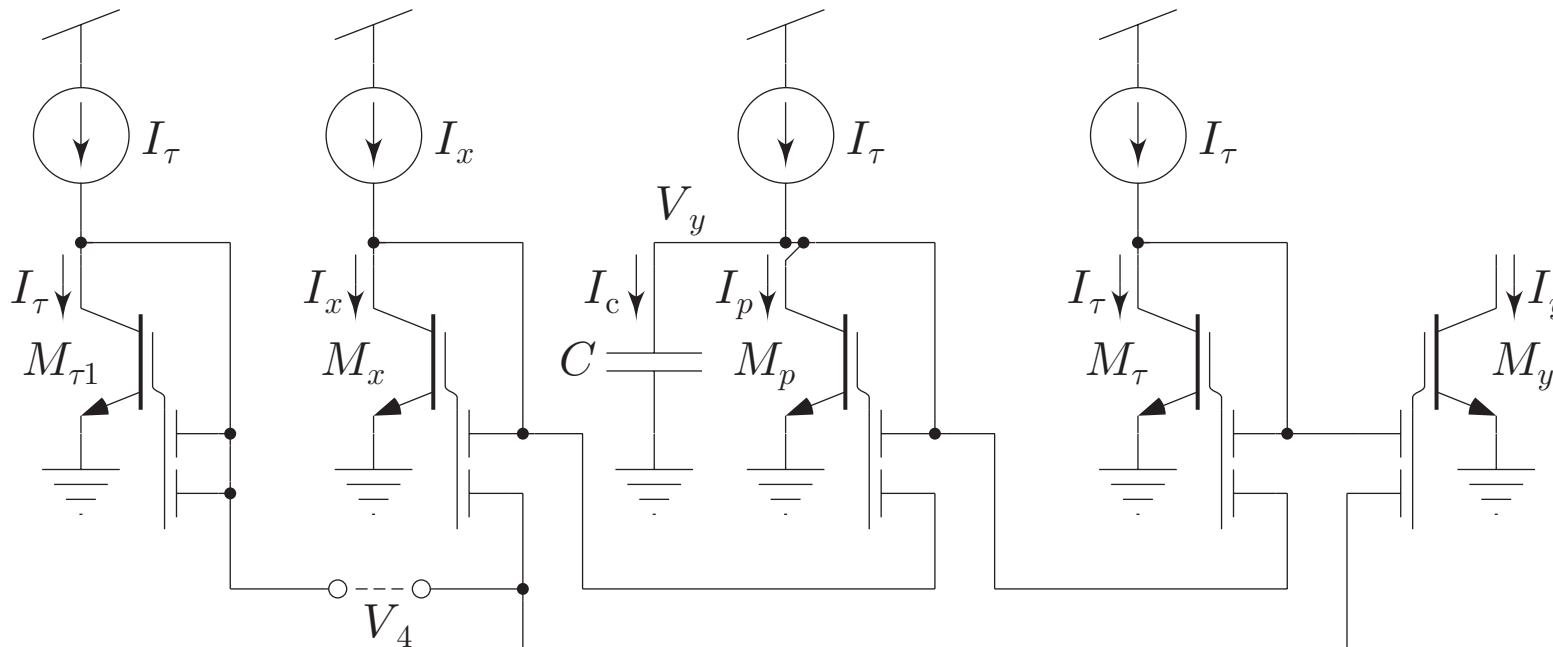
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$$\text{TLP: } I_p I_y = I_x I_\tau \quad \text{KCL: } I_p + I_c = I_\tau$$



# Dynamic MITE Network Synthesis: **Second-Order LPF**

Synthesize a second-order low-pass filter described by

$$\tau^2 \frac{d^2 y}{dt^2} + \frac{\tau}{Q} \cdot \frac{dy}{dt} + y = x, \quad \text{where } x > 0.$$



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$$\tau^2 \frac{d^2 y}{dt^2} + \frac{\tau}{Q} \cdot \frac{dy}{dt} + y = x, \quad \text{where } x > 0.$$

We can decompose this second-order ODE into two coupled first-order ODEs as

$$\tau \frac{d}{dt} \left( \tau \frac{dy}{dt} + \frac{y}{Q} \right) + y = x$$

## Dynamic MITE Network Synthesis: **Second-Order LPF**

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$$\tau^2 \frac{d^2 y}{dt^2} + \frac{\tau}{Q} \cdot \frac{dy}{dt} + y = x, \quad \text{where } x > 0.$$

We can decompose this second-order ODE into two coupled first-order ODEs as

$$\tau \frac{d}{dt} \left( \underbrace{\tau \frac{dy}{dt} + \frac{y}{Q}}_z \right) + y = x \quad \Longrightarrow \quad \begin{cases} \tau \frac{dz}{dt} = x - y \\ \tau \frac{dy}{dt} = z - \frac{y}{Q} \end{cases}$$

## Dynamic MITE Network Synthesis: **Second-Order LPF**

We represent each signal as a ratio of a signal current to the unit current:

$$x \equiv \frac{I_x}{I_1}, \quad y \equiv \frac{I_y}{I_1}, \quad \text{and} \quad z \equiv \frac{I_z}{I_1}.$$

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Substituting these into the pair of ODEs, we obtain

$$\left\{ \begin{array}{l} \tau \frac{d}{dt} \left( \frac{I_z}{I_1} \right) = \frac{I_x}{I_1} - \frac{I_y}{I_1} \\ \tau \frac{d}{dt} \left( \frac{I_y}{I_1} \right) = \frac{I_z}{I_1} - \frac{1}{Q} \cdot \frac{I_y}{I_1} \end{array} \right.$$

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## Dynamic MITE Network Synthesis: **Second-Order LPF**

To implement the time derivatives, we introduce log-compressed voltage state variables,  $V_z$  and  $V_y$ . Using the chain rule, we can express the preceding pair of equations as

$$\left\{ \begin{array}{l} \tau \frac{\partial I_z}{\partial V_z} \cdot \frac{dV_z}{dt} = I_x - I_y \\ \tau \frac{\partial I_y}{\partial V_y} \cdot \frac{dV_y}{dt} = I_z - \frac{I_y}{Q} \end{array} \right.$$

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$$\implies \left\{ \begin{array}{l} \frac{w\tau}{U_T} \cdot \frac{dV_z}{dt} = \frac{I_y}{I_z} - \frac{I_x}{I_z} \\ \frac{w\tau}{U_T} \cdot \frac{dV_y}{dt} = \frac{1}{Q} - \frac{I_z}{I_y} \end{array} \right.$$

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## Dynamic MITE Network Synthesis: **Second-Order LPF**

By introducing

$$I_\tau \equiv \frac{CU_T}{w\tau}, \quad I_{cz} \equiv C \frac{dV_z}{dt}, \quad \text{and} \quad I_{cy} \equiv C \frac{dV_y}{dt},$$

we can express this pair of equations as

$$\left\{ \begin{array}{l} \frac{I_{cz}}{I_\tau} = \frac{I_y}{I_z} - \frac{I_x}{I_z} \\ \frac{I_{cy}}{I_\tau} = \frac{1}{Q} - \frac{I_z}{I_y} \end{array} \right.$$

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where we have further introduced

$$I_w \equiv \frac{I_y I_\tau}{I_z}, \quad I_{pz} \equiv \frac{I_x I_\tau}{I_z}, \quad \text{and} \quad I_{py} \equiv \frac{I_z I_\tau}{I_y}.$$

## Dynamic MITE Network Synthesis: **Second-Order LPF**

$$\text{TLP: } \begin{aligned} I_z I_{pz} &= I_x I_\tau \\ I_z I_w &= I_y I_\tau \end{aligned}$$

$$I_y I_{py} = I_z I_\tau$$

$$\text{KCL: } \begin{aligned} I_{pz} + I_{cz} &= I_w \\ I_{py} + I_{cy} &= I_\tau / Q \end{aligned}$$

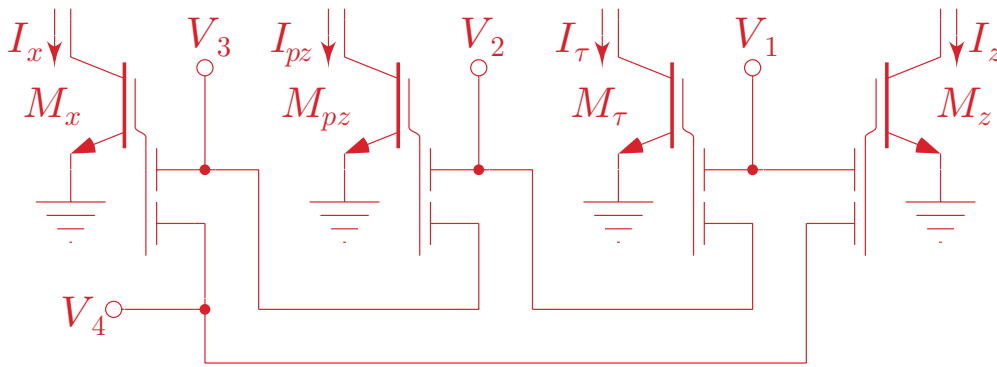
# Dynamic MITE Network Synthesis: **Second-Order LPF**

$$\text{TLP: } I_z I_{pz} = I_x I_\tau \quad I_y I_{py} = I_z I_\tau$$

$$I_z I_w = I_y I_\tau$$

$$\text{KCL: } I_{pz} + I_{cz} = I_w$$

$$I_{py} + I_{cy} = I_\tau / Q$$



# Dynamic MITE Network Synthesis: **Second-Order LPF**

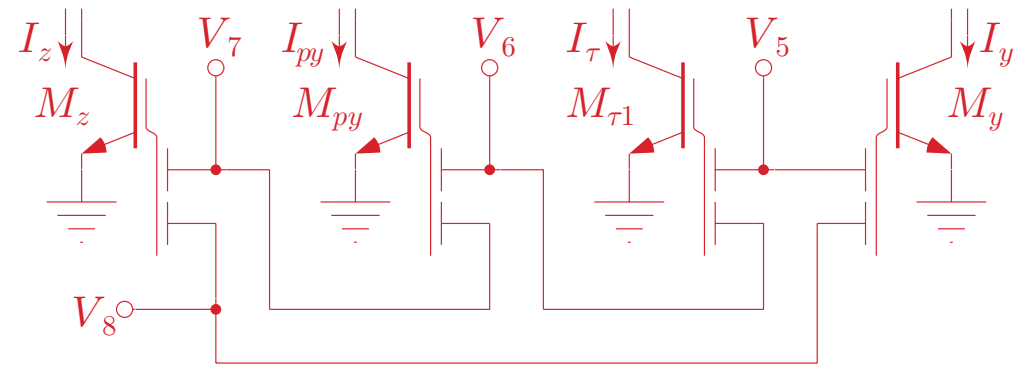
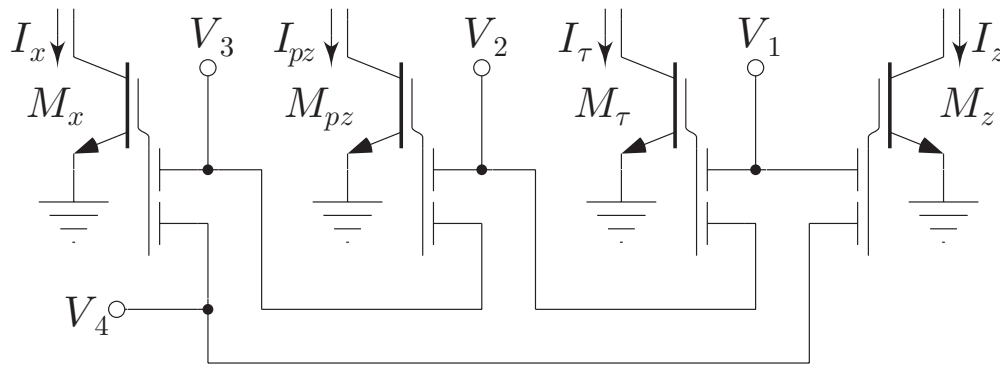
$$\text{TLP: } I_z I_{pz} = I_x I_\tau$$

$$I_z I_w = I_y I_\tau$$

$$I_y I_{py} = I_z I_\tau$$

$$\text{KCL: } I_{pz} + I_{cz} = I_w$$

$$I_{py} + I_{cy} = I_\tau / Q$$





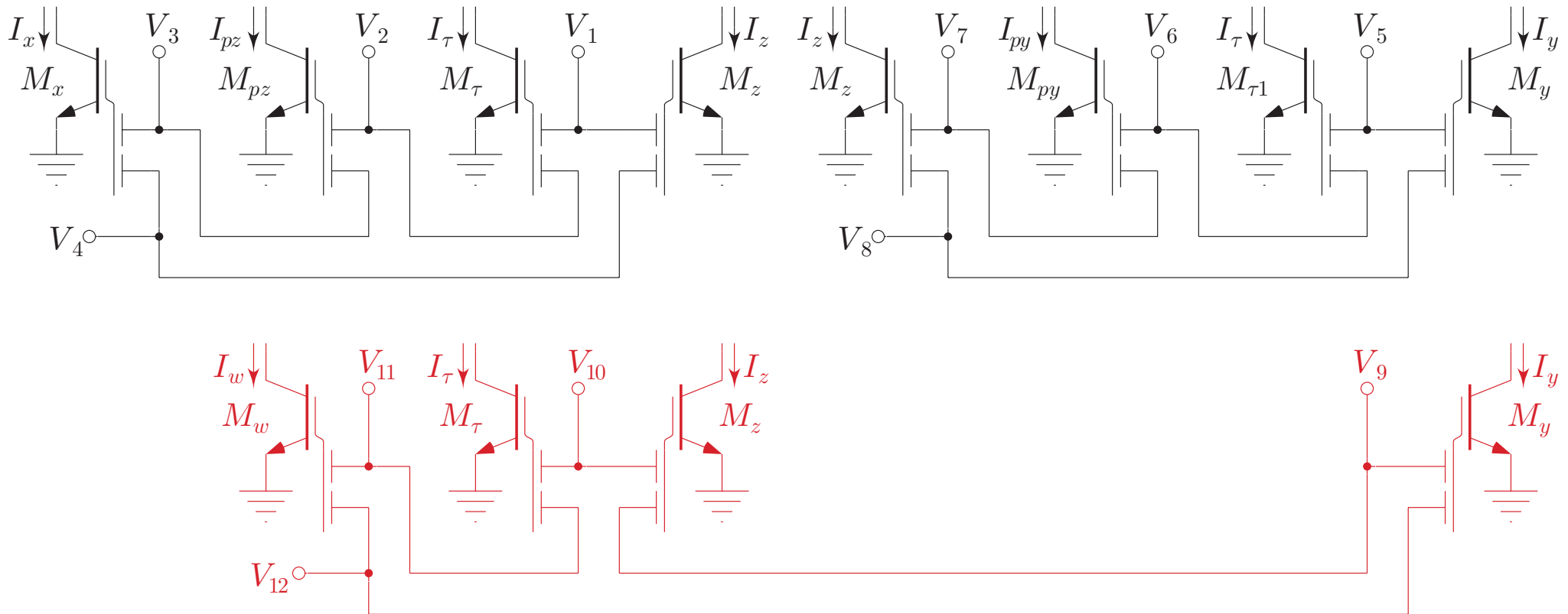
# Dynamic MITE Network Synthesis: **Second-Order LPF**

$$\text{TLP: } I_z I_{pz} = I_x I_\tau \quad I_y I_{py} = I_z I_\tau$$

$$I_z I_w = I_y I_\tau$$

$$\text{KCL: } I_{pz} + I_{cz} = I_w$$

$$I_{py} + I_{cy} = I_\tau / Q$$



# Dynamic MITE Network Synthesis: **Second-Order LPF**

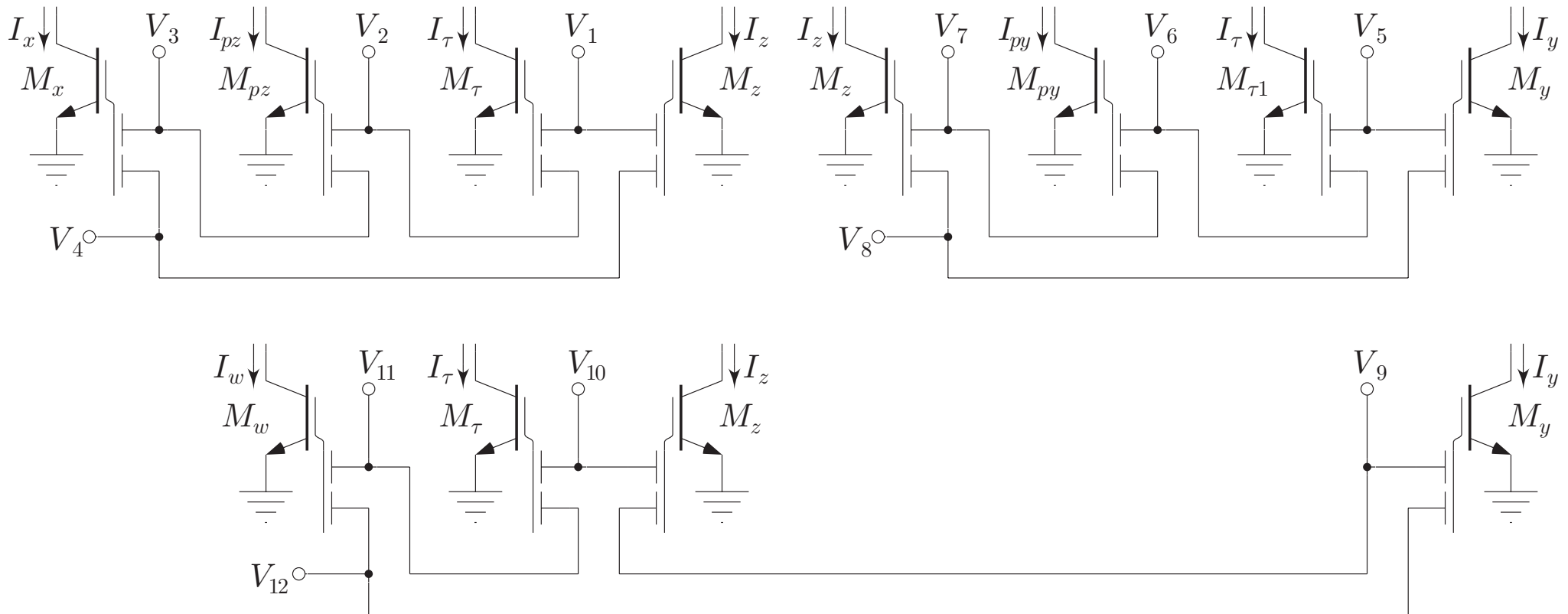
$$\text{TLP: } I_z I_{pz} = I_x I_\tau$$

$$I_z I_w = I_y I_\tau$$

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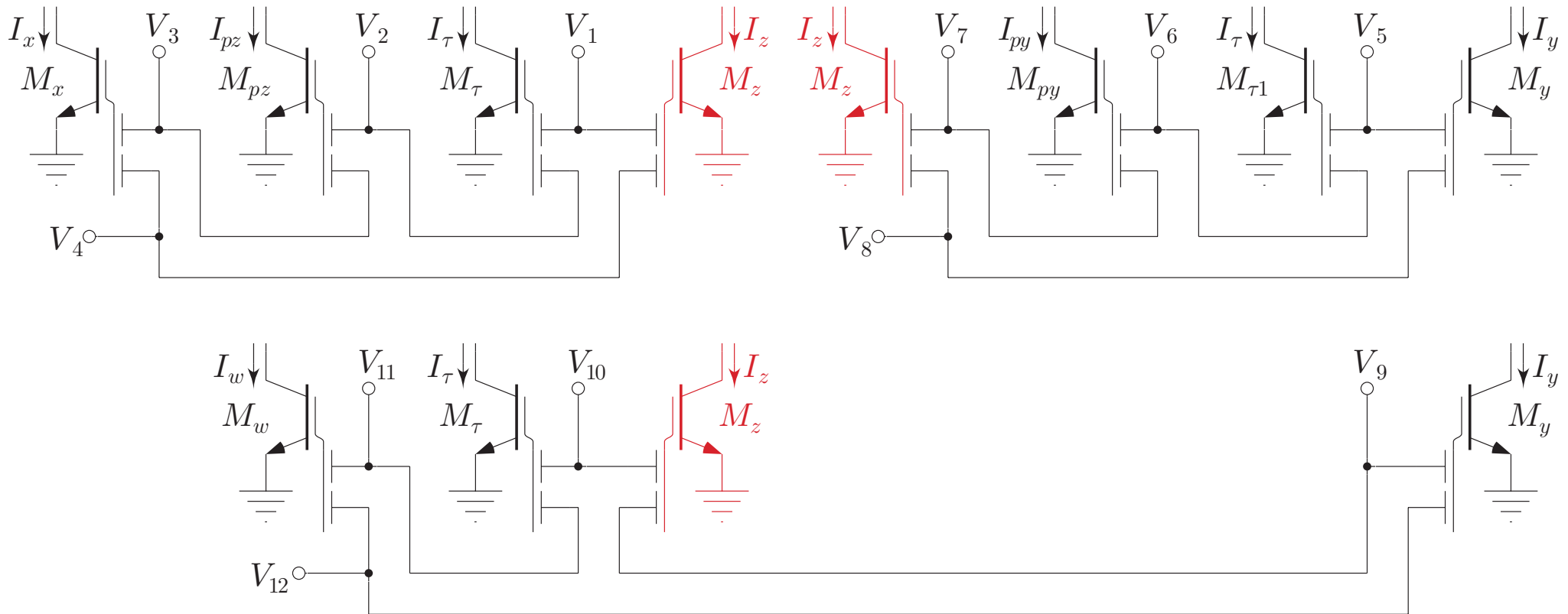
$$\text{TLP: } I_z I_{pz} = I_x I_\tau$$

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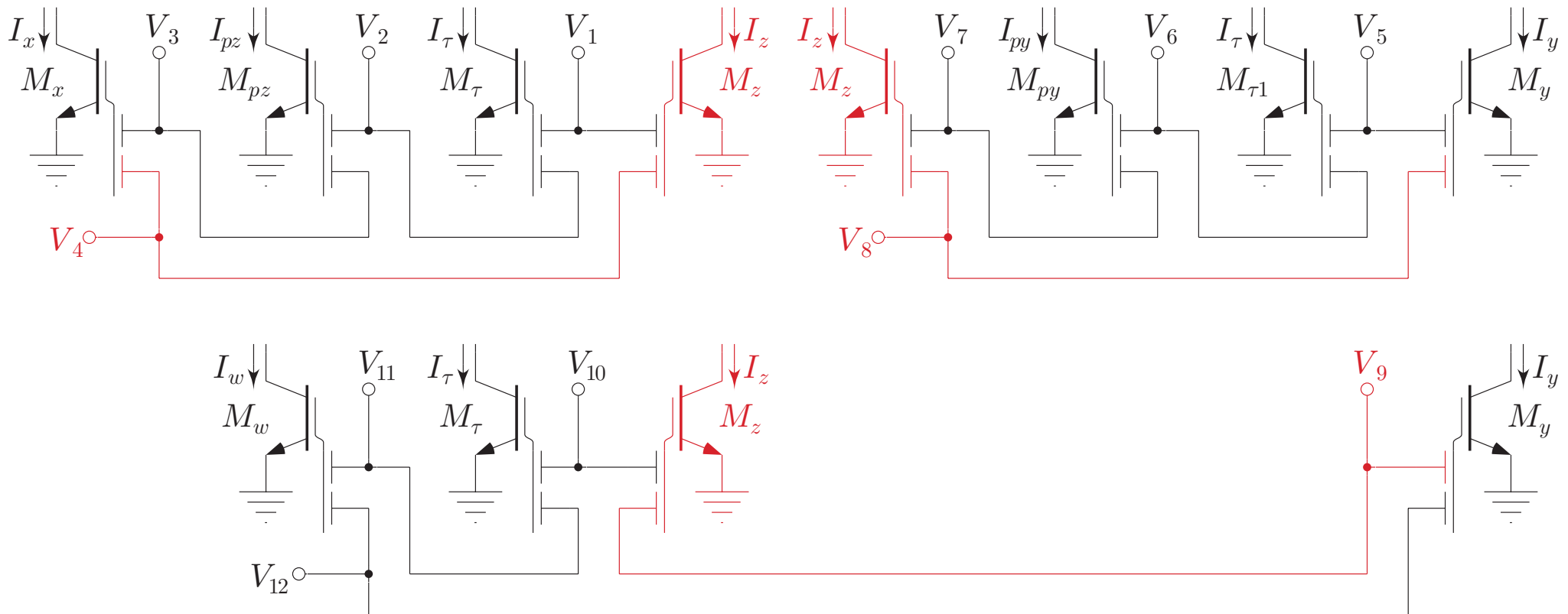
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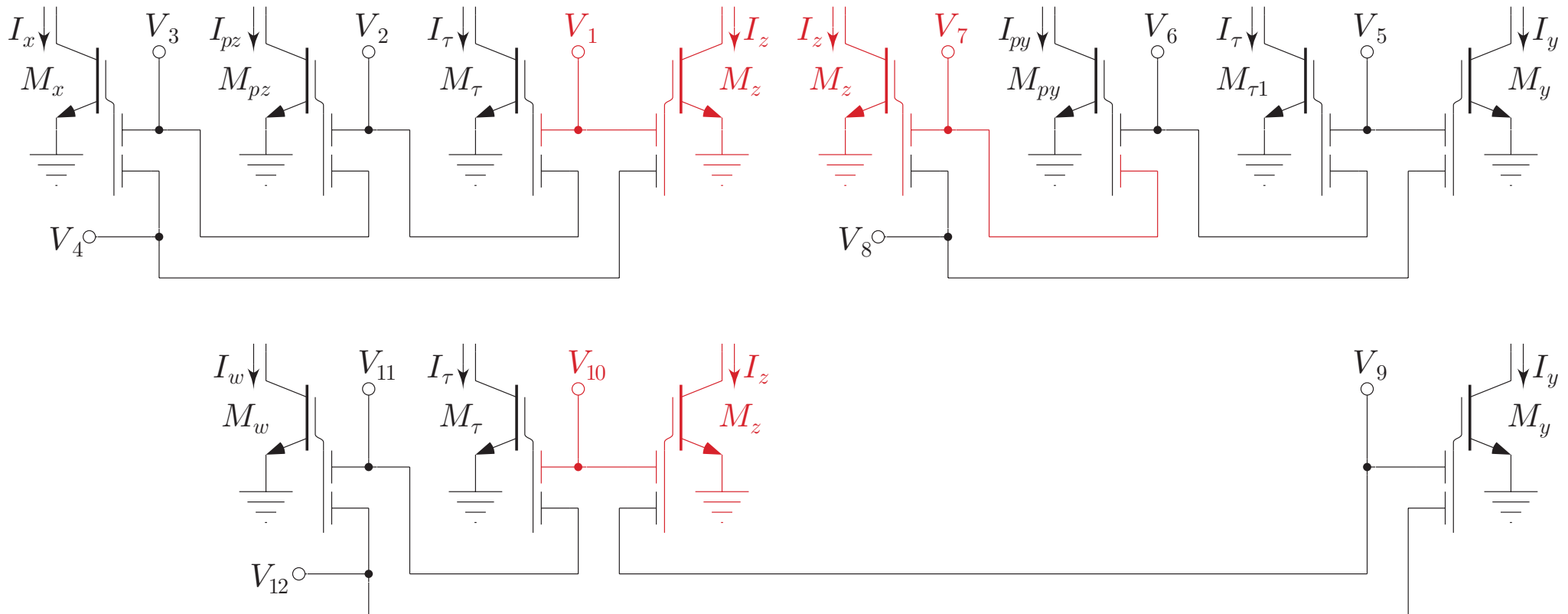
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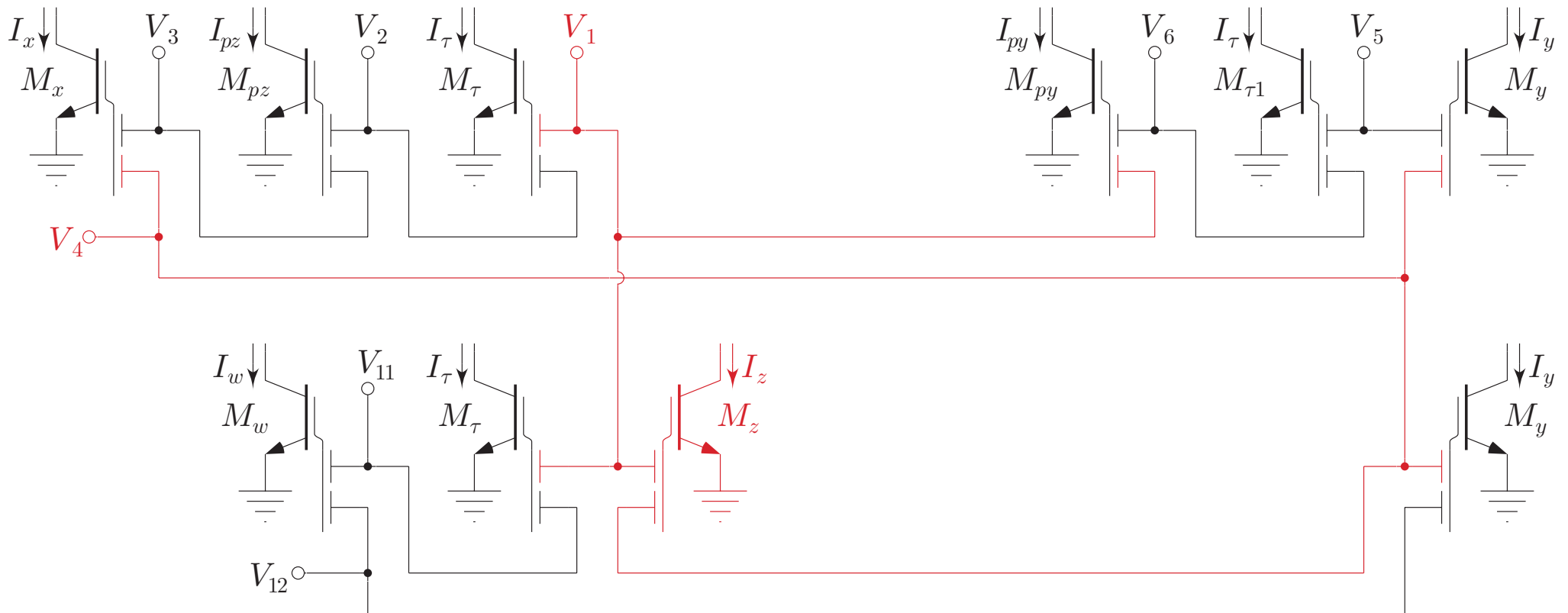
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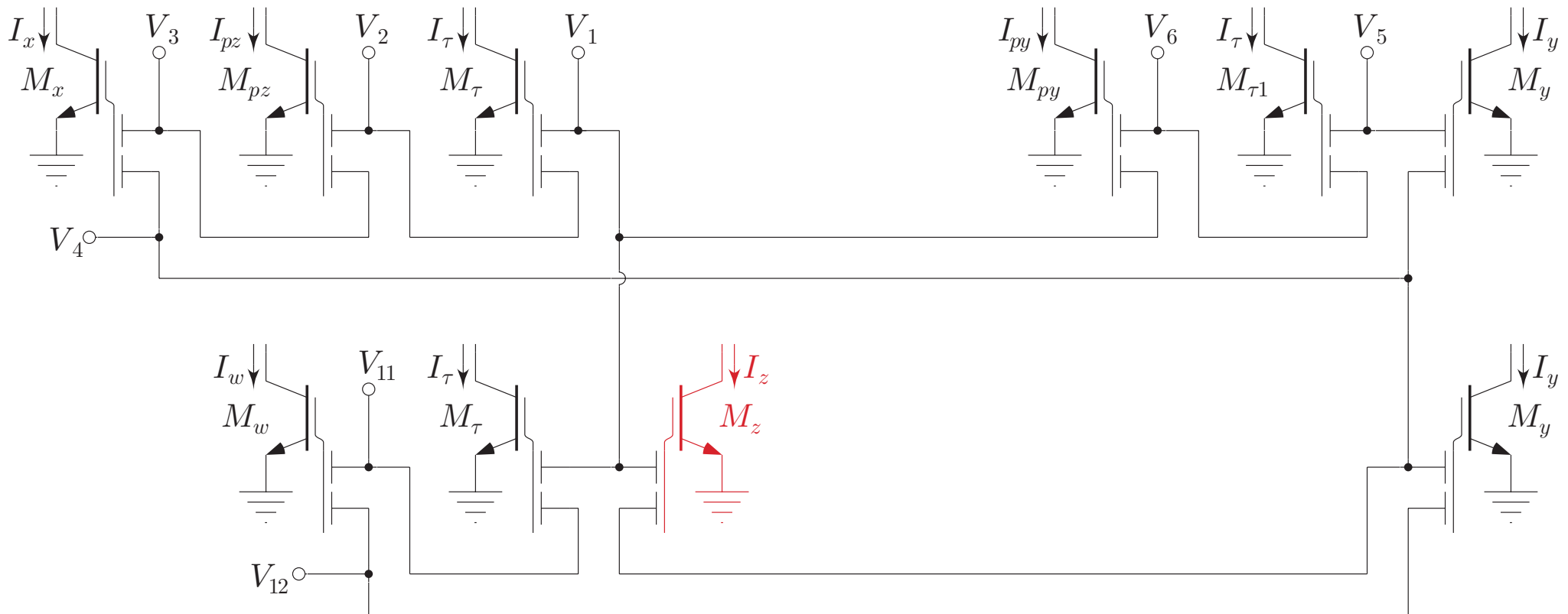
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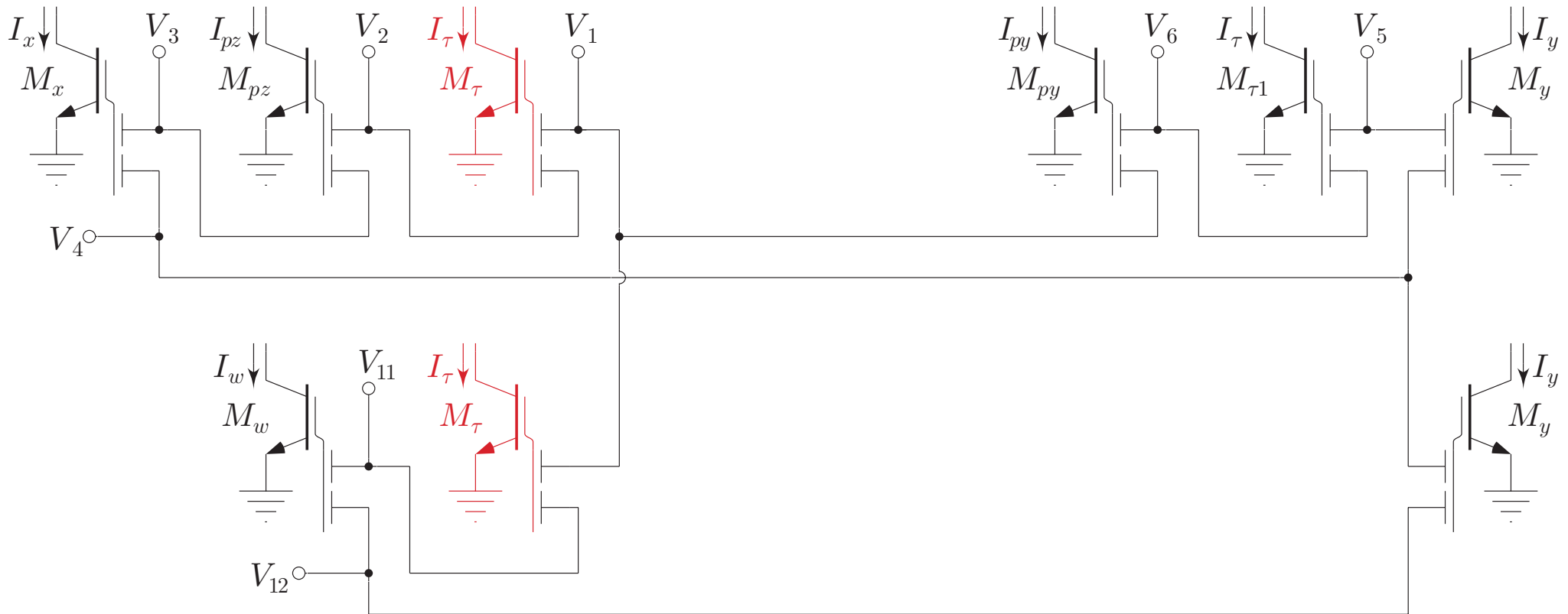
$$I_{py} + I_{cy} = I_\tau / Q$$



# Dynamic MITE Network Synthesis: **Second-Order LPF**

$$\begin{aligned} \text{TLP: } I_z I_{pz} &= I_x I_\tau & I_y I_{py} &= I_z I_\tau \\ I_z I_w &= I_y I_\tau \end{aligned}$$

$$\begin{aligned} \text{KCL: } I_{pz} + I_{cz} &= I_w \\ I_{py} + I_{cy} &= I_\tau / Q \end{aligned}$$





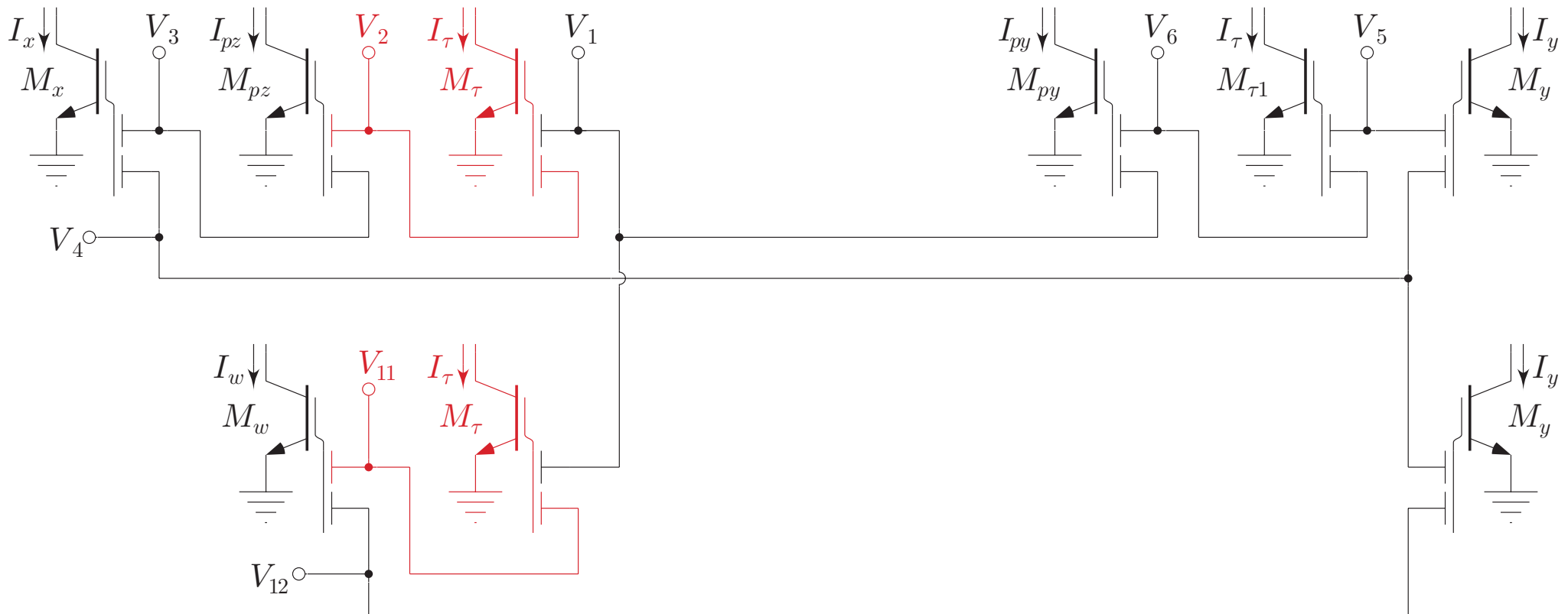
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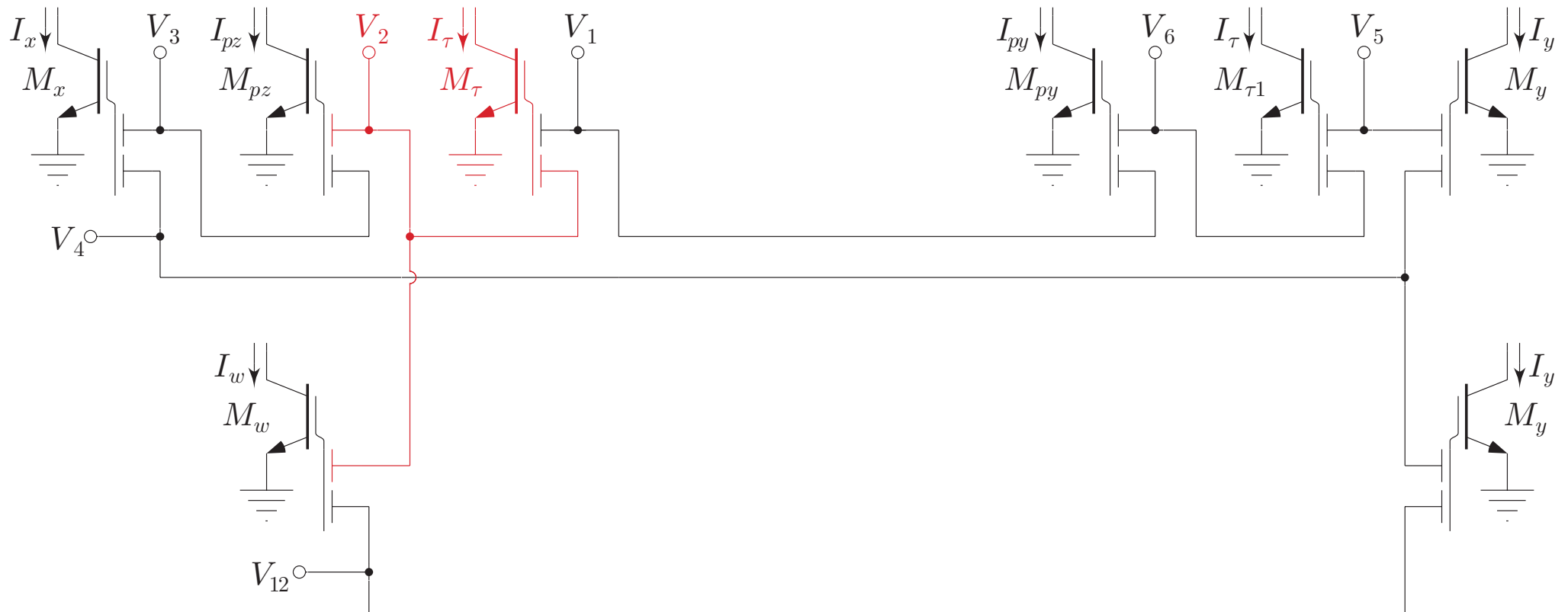
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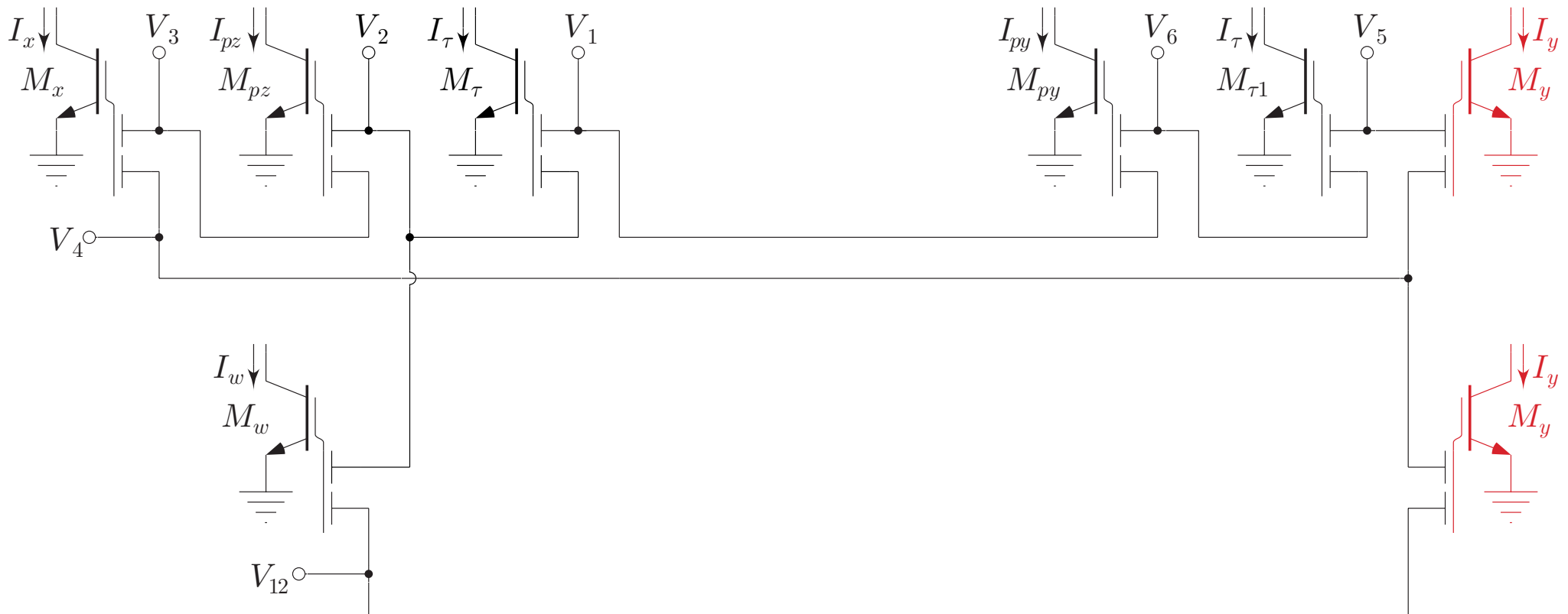
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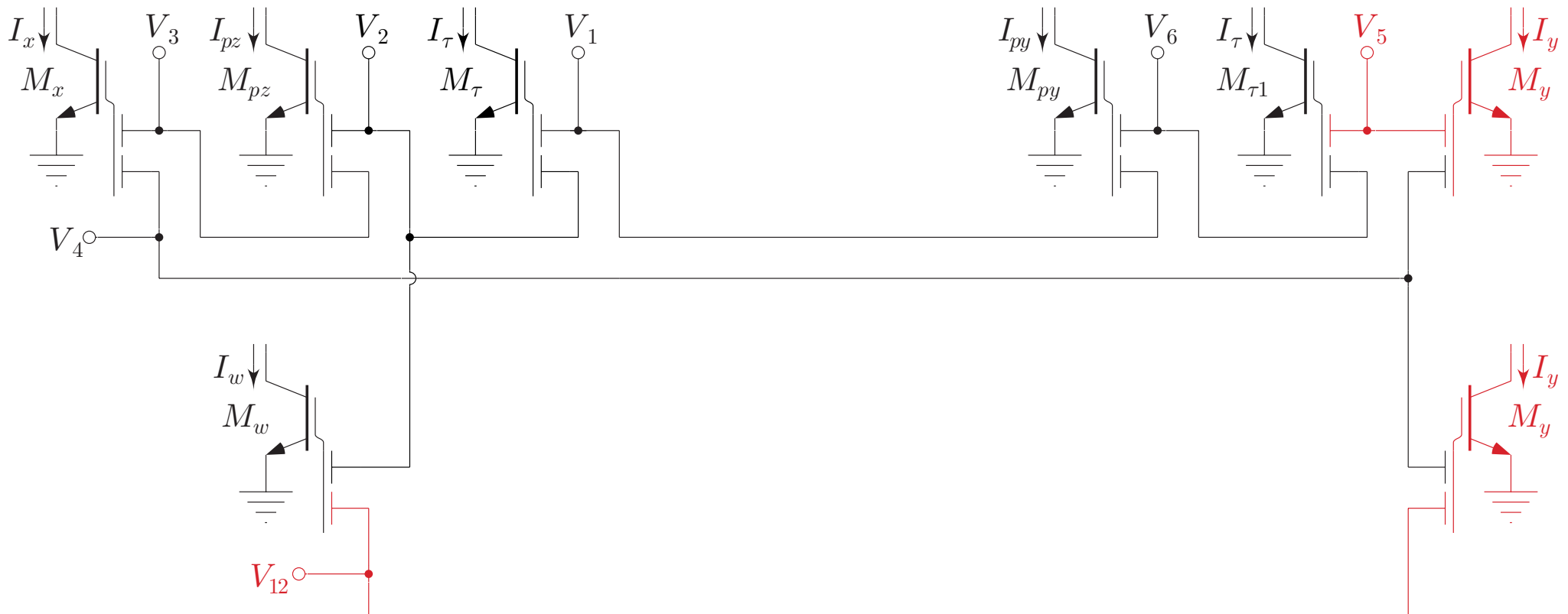
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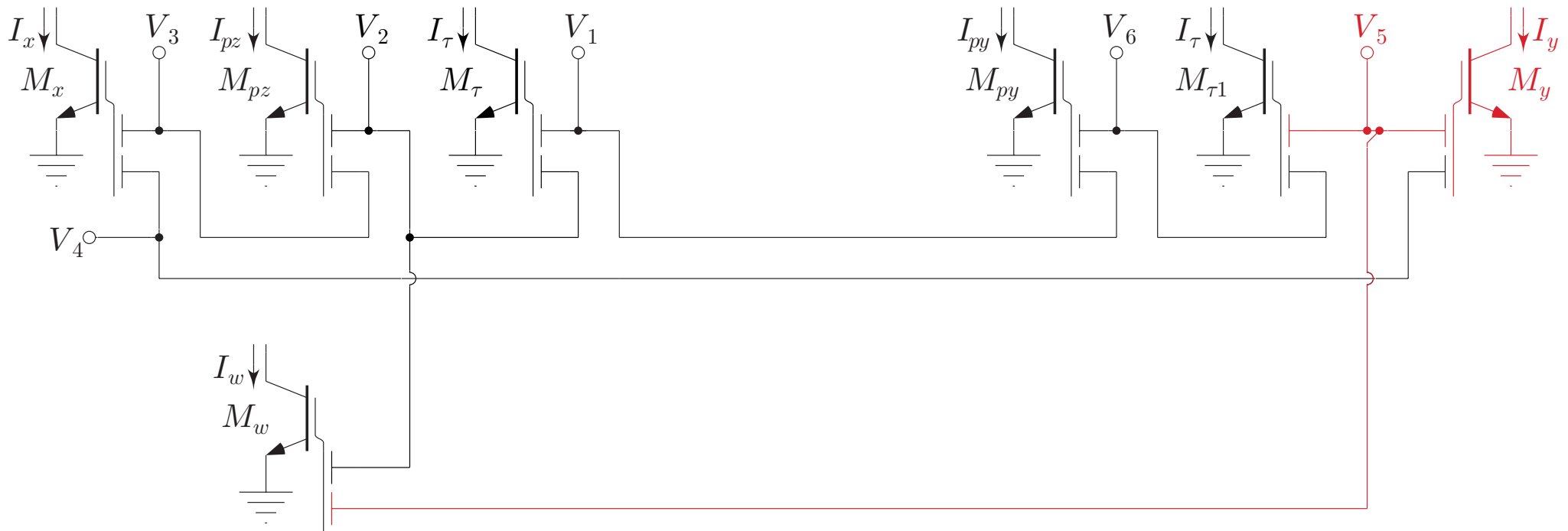
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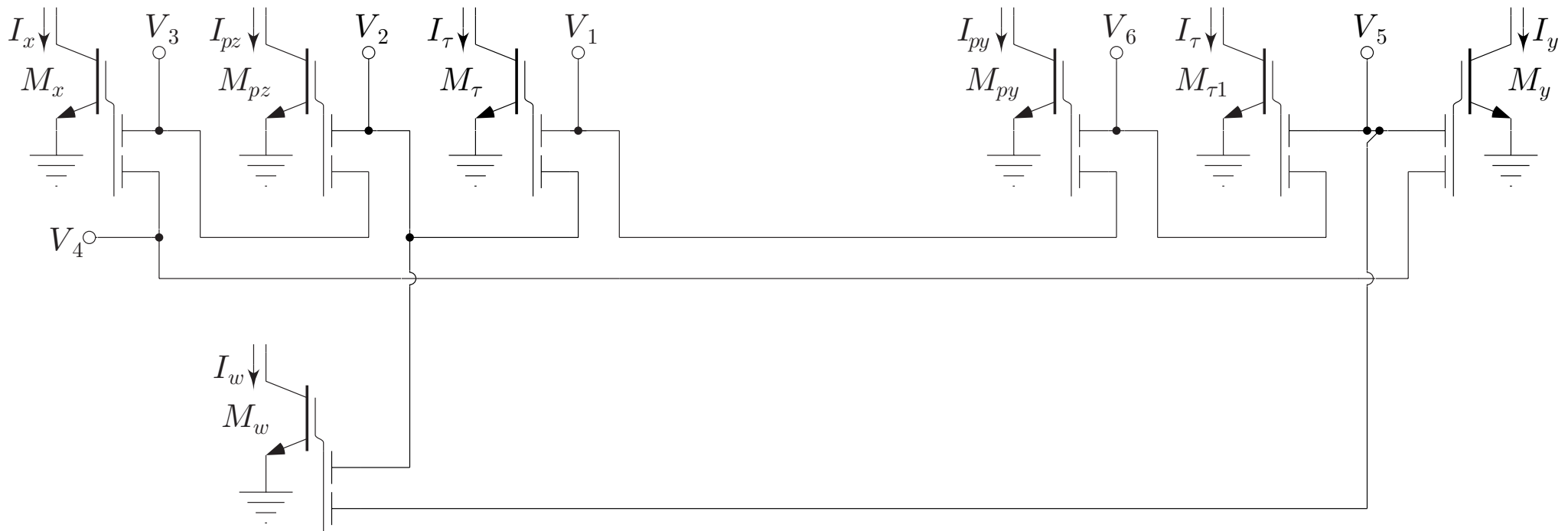
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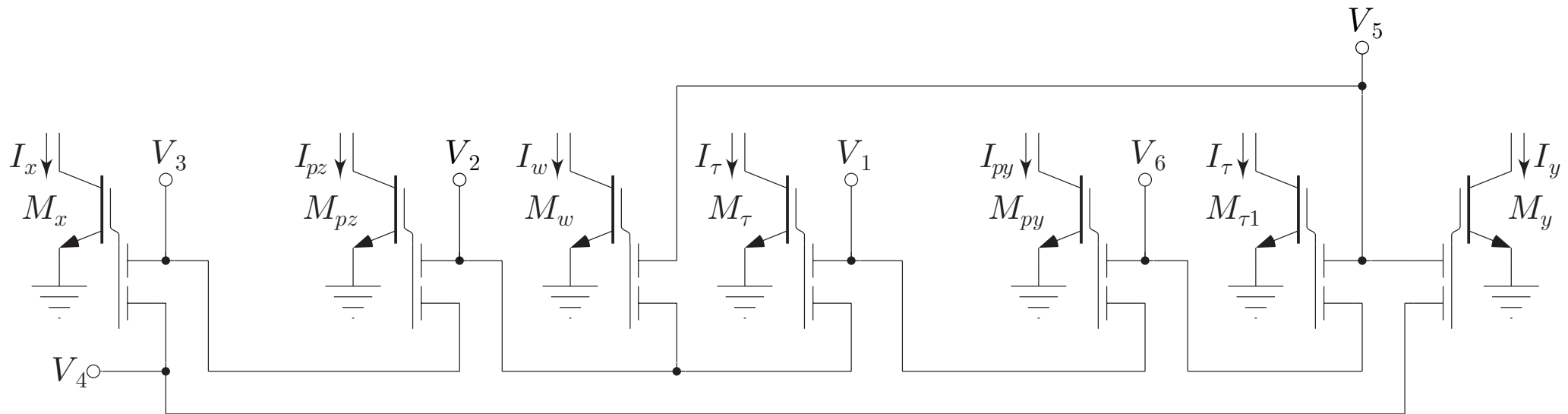
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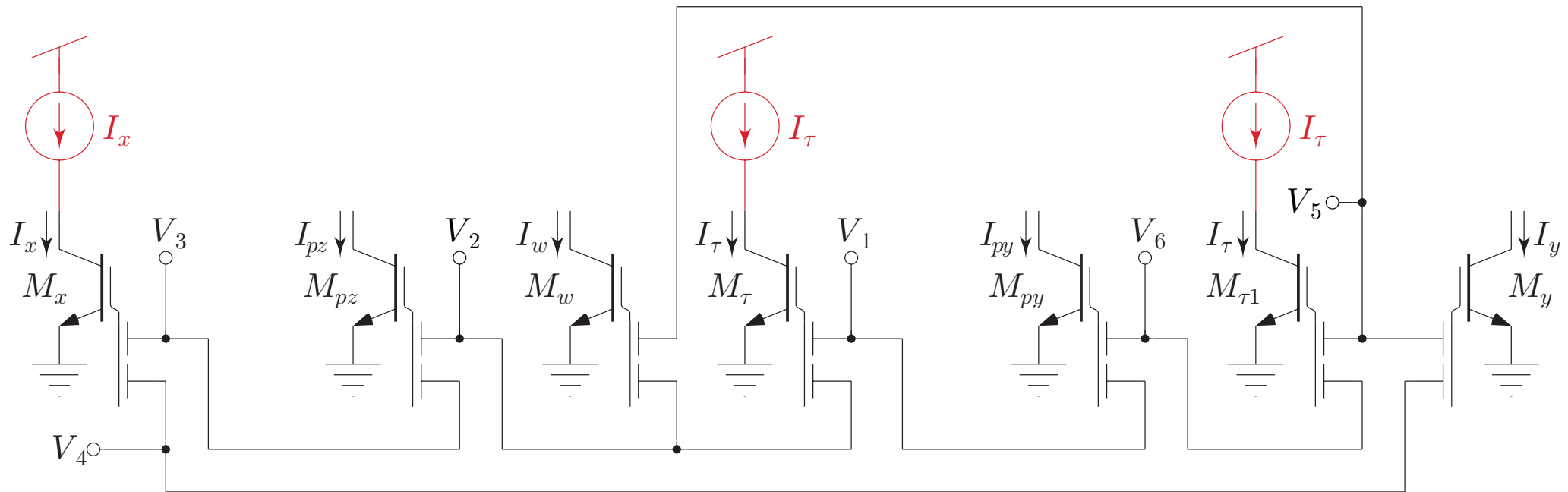
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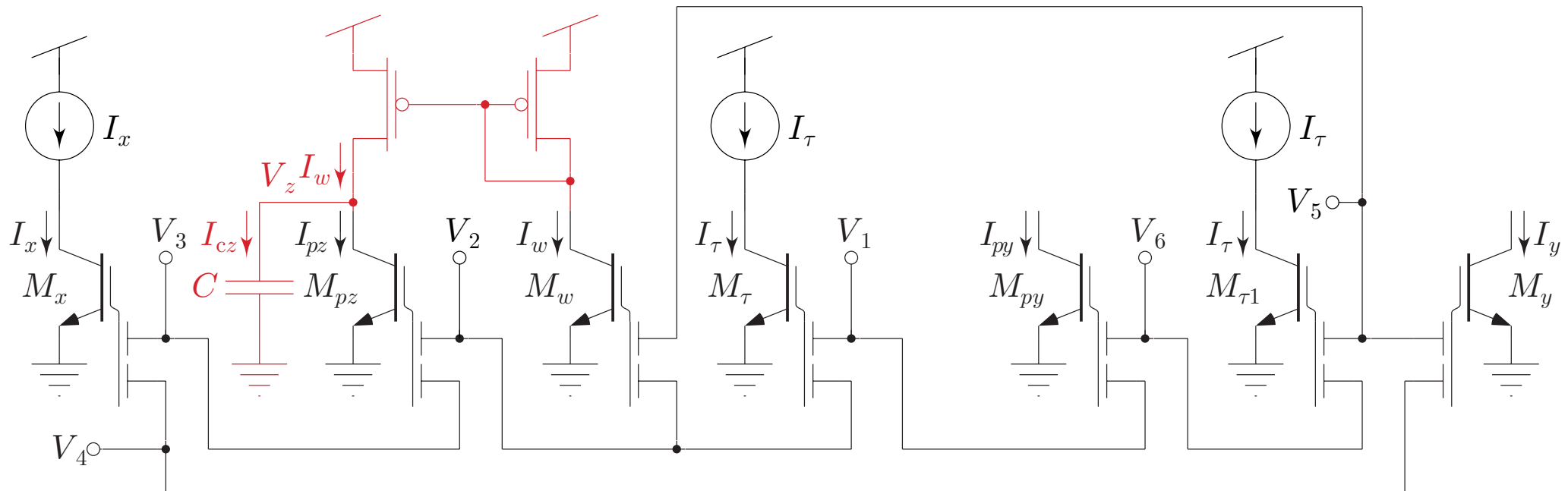
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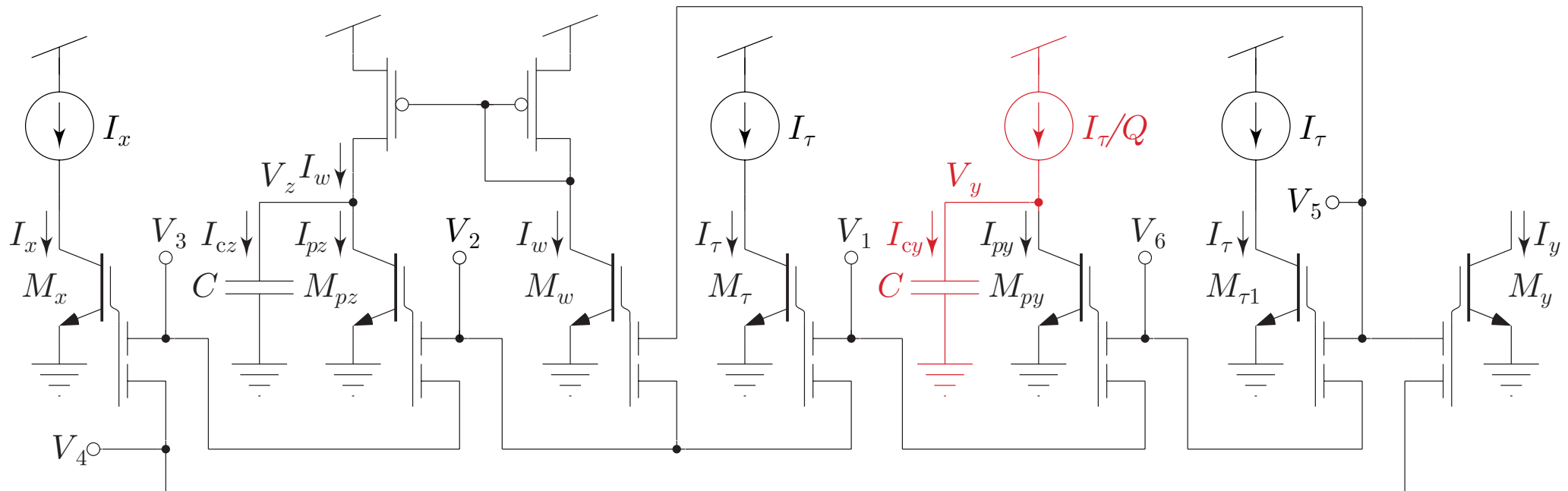
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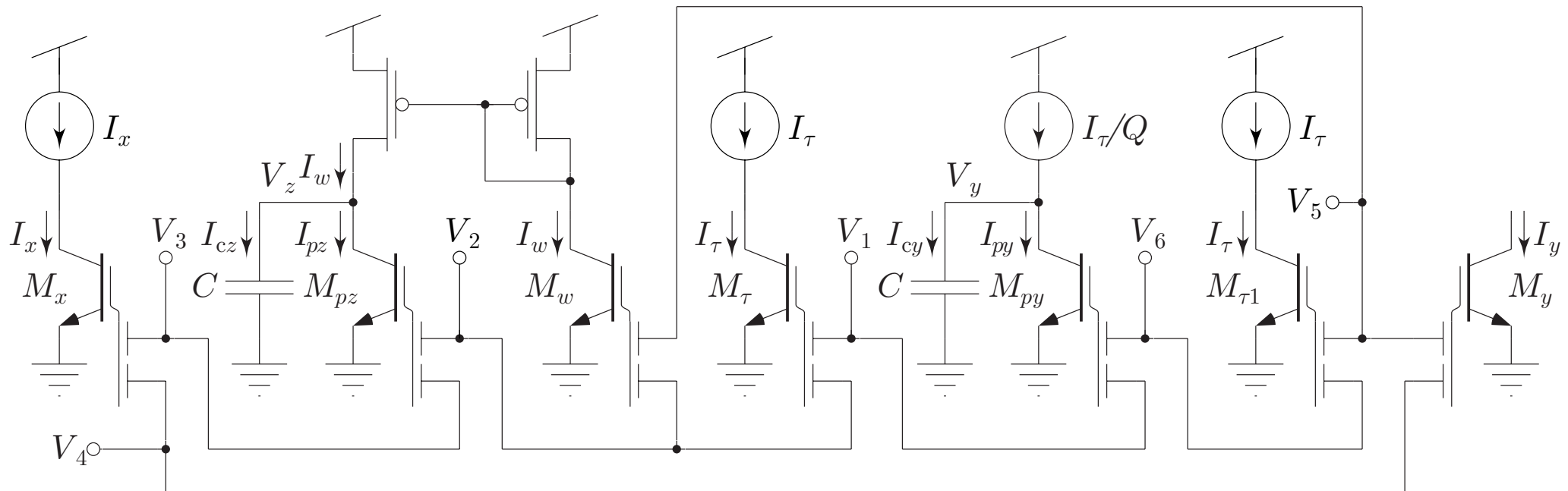
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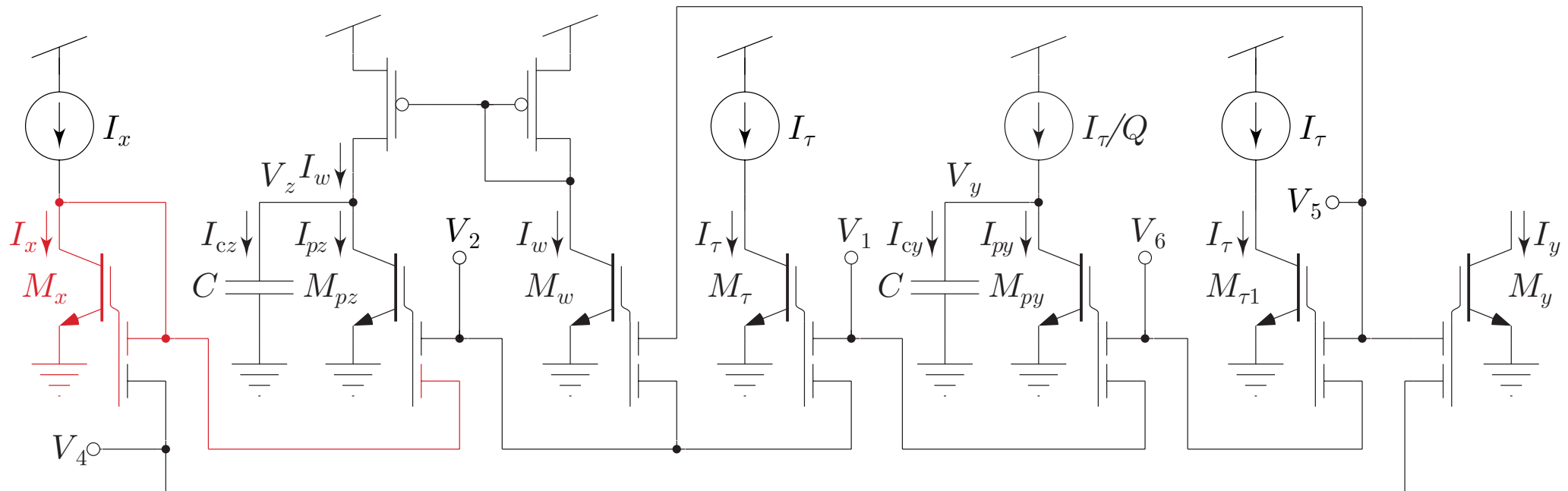
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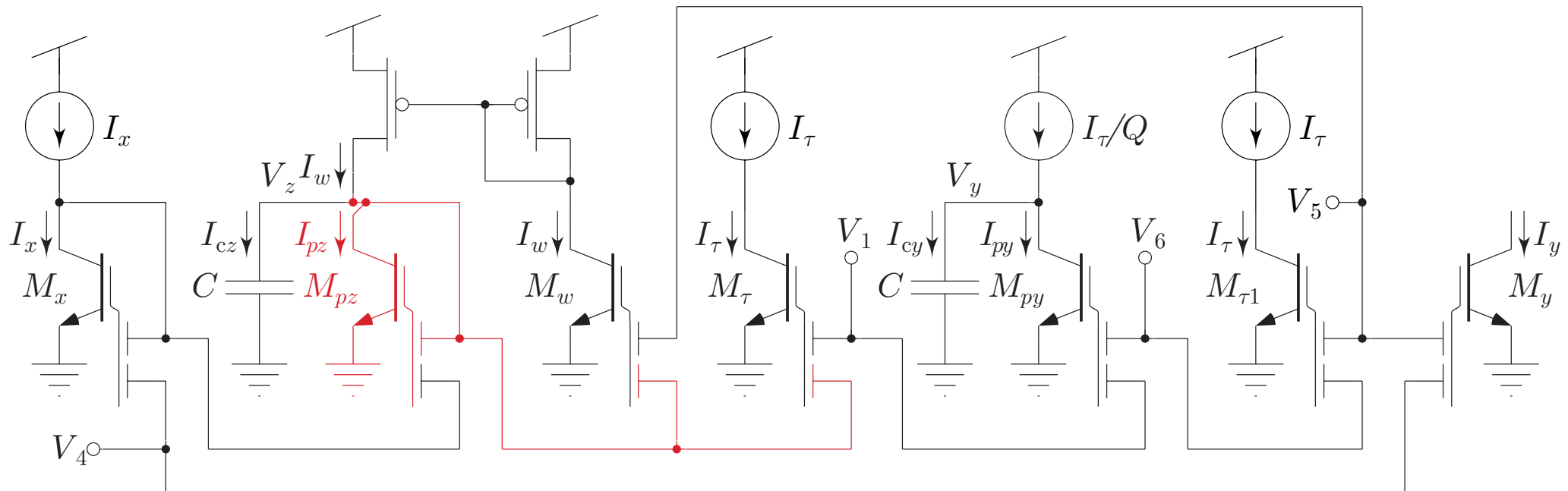
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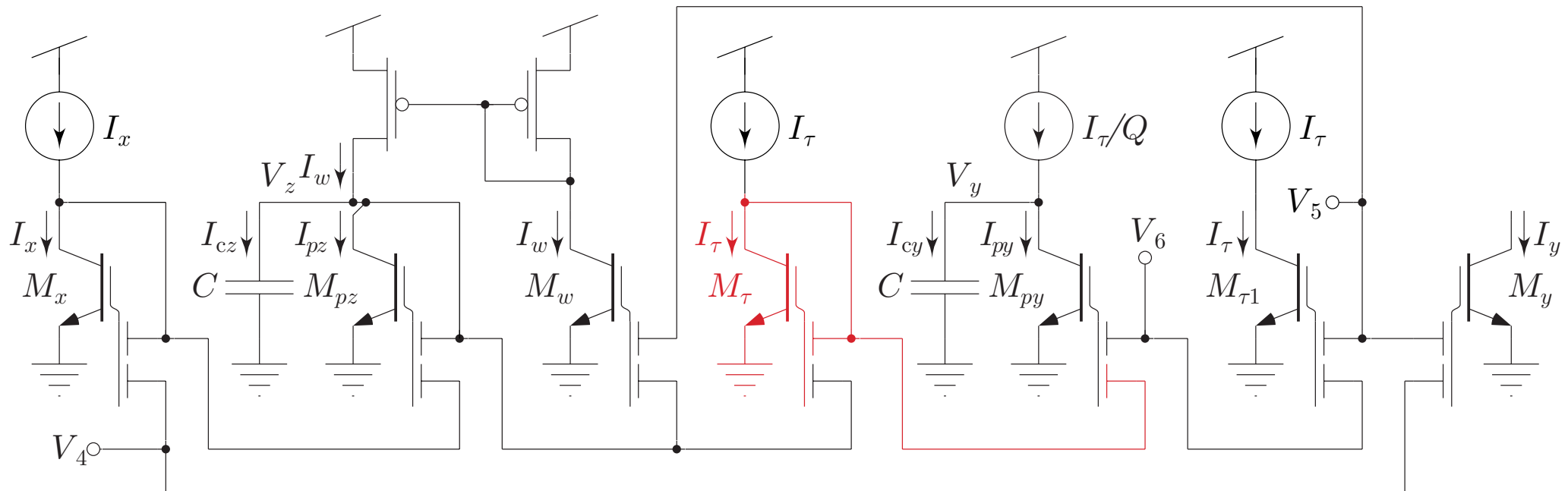
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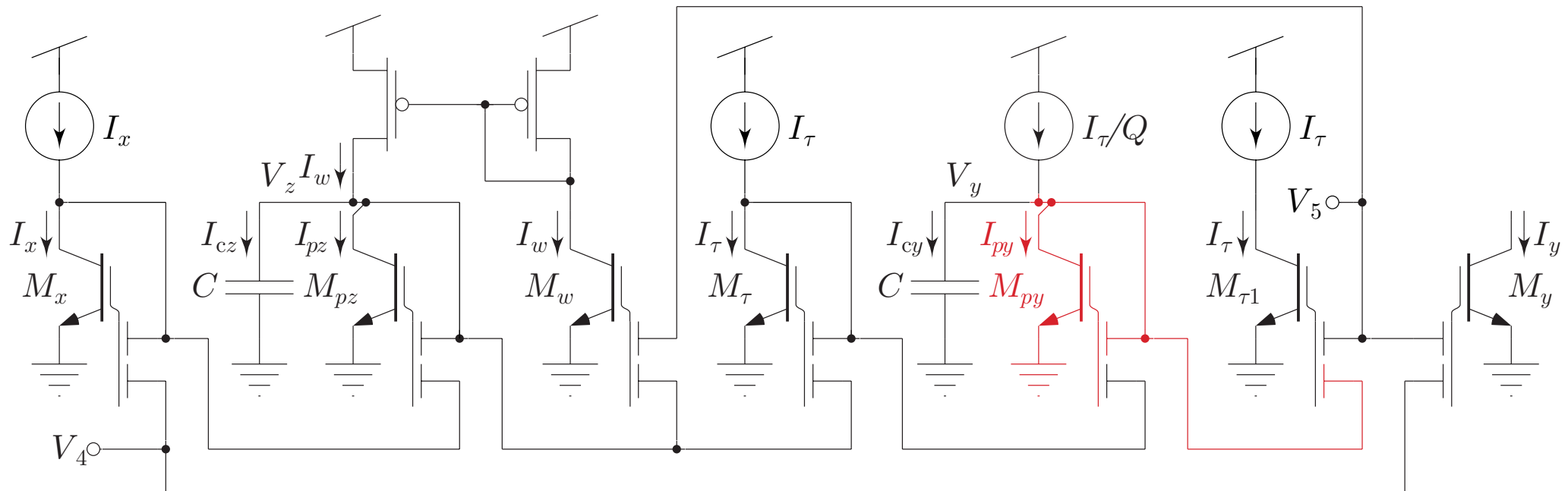
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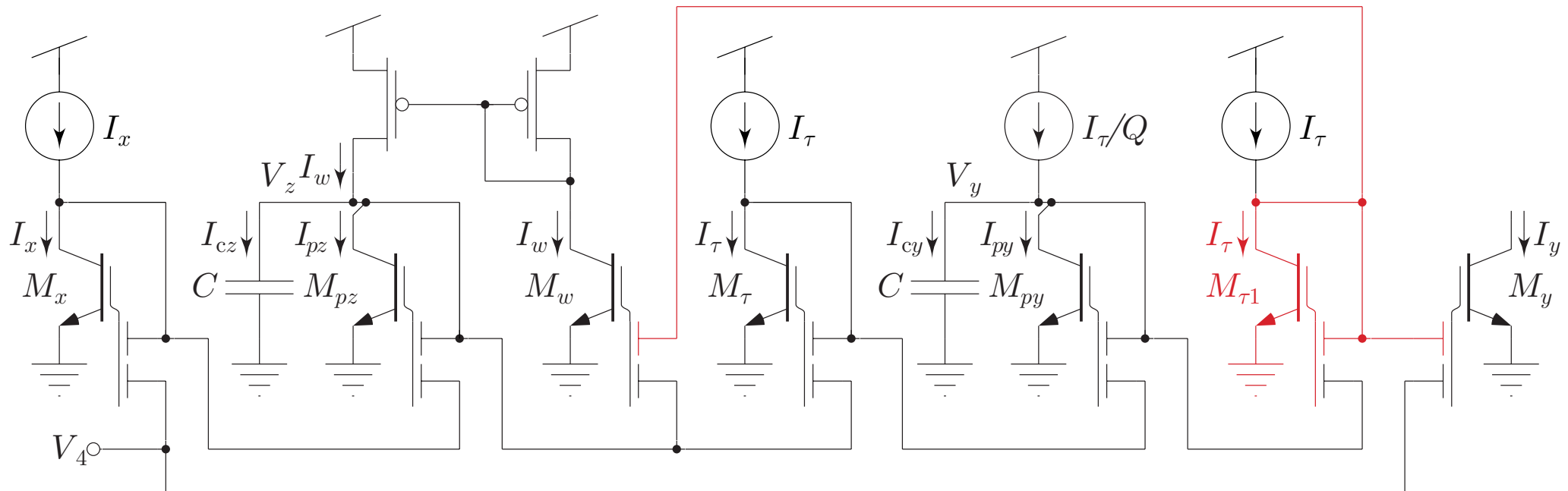
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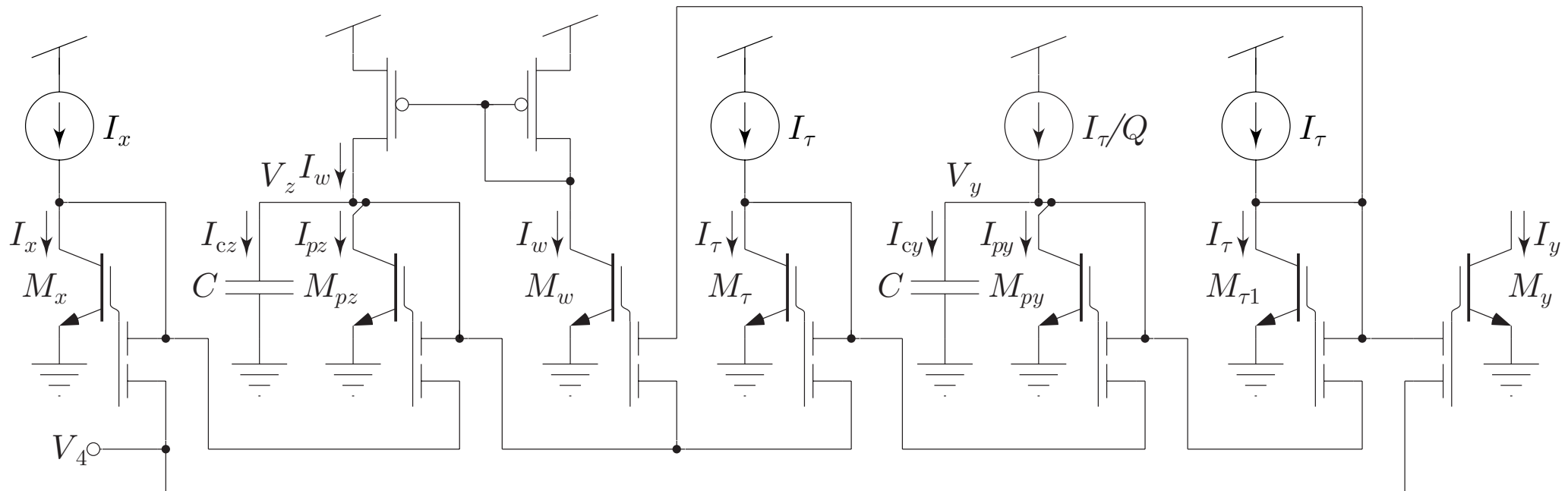
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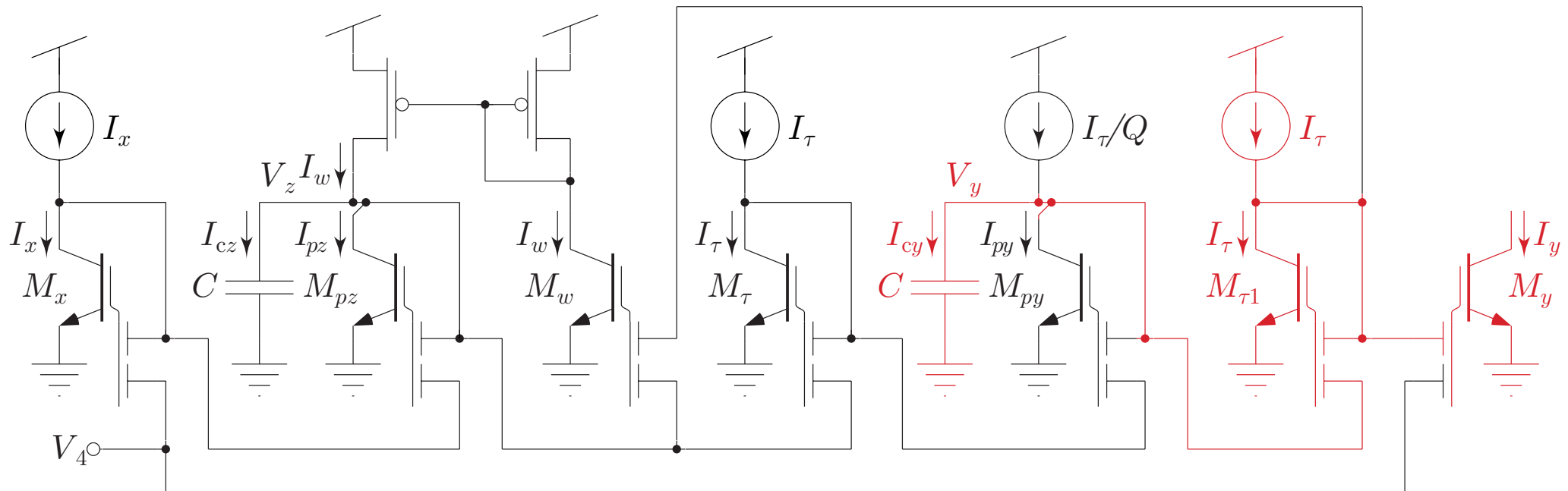
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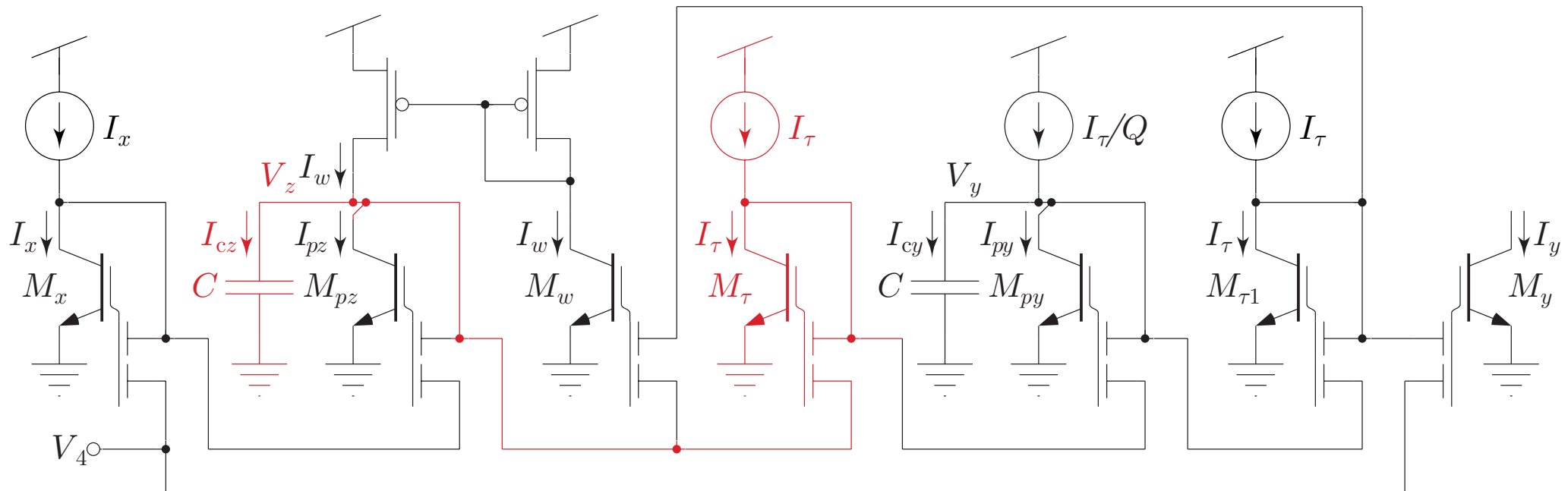
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# Dynamic MITE Network Synthesis: **RMS-to-DC Converter**

Synthesize an RMS-to-DC converter described by

$$x = w^2, \quad \tau \frac{dy}{dt} + y = x, \quad \text{and} \quad z = \sqrt{y}.$$

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Synthesize an RMS-to-DC converter described by

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We can eliminate  $x$  and  $y$  from the system description by substituting

$$x = w^2, \quad y = z^2, \quad \text{and} \quad \frac{dy}{dt} = 2z \frac{dz}{dt}$$

into the linear ODE describing the low-pass filter, obtaining a first-order nonlinear ODE describing the system given by

$$2\tau z \frac{dz}{dt} + z^2 = w^2.$$

## Dynamic MITE Network Synthesis: **RMS-to-DC Converter**

The input signal,  $w$ , can be positive or negative. To remedy this situation, we can offset  $w$  by defining  $u \equiv w + v$ , such that  $u > 0$  and  $v > 0$ . Substituting  $w = u - v$  into the nonlinear ODE, we obtain

$$2\tau z \frac{dz}{dt} + z^2 = u^2 - 2uv + v^2.$$

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We represent each signal as a ratio of a signal current to the unit current:

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$$2\tau \frac{I_z}{I_1} \cdot \frac{d}{dt} \left( \frac{I_z}{I_1} \right) + \left( \frac{I_z}{I_1} \right)^2 = \left( \frac{I_u}{I_1} \right)^2 - 2 \cdot \frac{I_u}{I_1} \cdot \frac{I_v}{I_1} + \left( \frac{I_v}{I_1} \right)^2$$



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## Dynamic MITE Network Synthesis: **RMS-to-DC Converter**

To implement the time derivative, we introduce a log-compressed voltage state variable,  $V_z$ . Using the chain rule, we can express the preceding equation as

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To implement the time derivative, we introduce a log-compressed voltage state variable,  $V_z$ . Using the chain rule, we can express the preceding equation as

$$2\tau I_z \frac{\partial I_z}{\partial V_z} \cdot \frac{dV_z}{dt} + I_z^2 = I_u^2 - 2I_u I_v + I_v^2 \quad \Rightarrow \quad 2\tau I_z \left( -\frac{w}{U_T} I_z \right) \frac{dV_z}{dt} + I_z^2 = I_u^2 - 2I_u I_v + I_v^2$$

$$\Rightarrow -\frac{2w\tau}{U_T} \cdot \frac{dV_z}{dt} + 1 = \frac{I_u^2}{I_z^2} - \frac{2I_u I_v}{I_z^2} + \frac{I_v^2}{I_z^2} \quad \Rightarrow \quad -\underbrace{\frac{2w\tau}{CU_T}}_{1/I_\tau} \cdot \underbrace{C}_{I_c} \frac{dV_z}{dt} + 1 = \frac{I_u^2}{I_z^2} - \frac{2I_u I_v}{I_z^2} + \frac{I_v^2}{I_z^2}$$

$$\Rightarrow -\frac{I_c}{I_\tau} + 1 = \frac{I_u^2}{I_z^2} - \frac{2I_u I_v}{I_z^2} + \frac{I_v^2}{I_z^2}$$

## Dynamic MITE Network Synthesis: **RMS-to-DC Converter**

To implement the time derivative, we introduce a log-compressed voltage state variable,  $V_z$ . Using the chain rule, we can express the preceding equation as

$$2\tau I_z \frac{\partial I_z}{\partial V_z} \cdot \frac{dV_z}{dt} + I_z^2 = I_u^2 - 2I_u I_v + I_v^2 \quad \Rightarrow \quad 2\tau I_z \left( -\frac{w}{U_T} I_z \right) \frac{dV_z}{dt} + I_z^2 = I_u^2 - 2I_u I_v + I_v^2$$

$$\Rightarrow -\frac{2w\tau}{U_T} \cdot \frac{dV_z}{dt} + 1 = \frac{I_u^2}{I_z^2} - \frac{2I_u I_v}{I_z^2} + \frac{I_v^2}{I_z^2} \quad \Rightarrow \quad -\underbrace{\frac{2w\tau}{CU_T}}_{1/I_\tau} \cdot \underbrace{C}_{I_c} \frac{dV_z}{dt} + 1 = \frac{I_u^2}{I_z^2} - \frac{2I_u I_v}{I_z^2} + \frac{I_v^2}{I_z^2}$$

$$\Rightarrow -\frac{I_c}{I_\tau} + 1 = \frac{I_u^2}{I_z^2} - \frac{2I_u I_v}{I_z^2} + \frac{I_v^2}{I_z^2} \quad \Rightarrow \quad I_c - I_\tau = \frac{I_\tau I_u^2}{I_z^2} - \frac{I_\tau I_u (2I_v)}{I_z^2} + \frac{I_\tau I_v^2}{I_z^2}.$$



## Dynamic MITE Network Synthesis: RMS-to-DC Converter

To implement the time derivative, we introduce a log-compressed voltage state variable,  $V_z$ . Using the chain rule, we can express the preceding equation as

$$2\tau I_z \frac{\partial I_z}{\partial V_z} \cdot \frac{dV_z}{dt} + I_z^2 = I_u^2 - 2I_u I_v + I_v^2 \quad \Rightarrow \quad 2\tau I_z \left( -\frac{w}{U_T} I_z \right) \frac{dV_z}{dt} + I_z^2 = I_u^2 - 2I_u I_v + I_v^2$$

$$\Rightarrow -\frac{2w\tau}{U_T} \cdot \frac{dV_z}{dt} + 1 = \frac{I_u^2}{I_z^2} - \frac{2I_u I_v}{I_z^2} + \frac{I_v^2}{I_z^2} \quad \Rightarrow \quad -\underbrace{\frac{2w\tau}{CU_T}}_{1/I_\tau} \cdot \underbrace{C}_{I_c} \frac{dV_z}{dt} + 1 = \frac{I_u^2}{I_z^2} - \frac{2I_u I_v}{I_z^2} + \frac{I_v^2}{I_z^2}$$

$$\Rightarrow -\frac{I_c}{I_\tau} + 1 = \frac{I_u^2}{I_z^2} - \frac{2I_u I_v}{I_z^2} + \frac{I_v^2}{I_z^2} \quad \Rightarrow \quad I_c - I_\tau = \frac{I_\tau I_u^2}{I_z^2} - \frac{I_\tau I_u \overbrace{(2I_v)}^{I_{2v}}}{I_z^2} + \frac{I_\tau I_v^2}{I_z^2}.$$

## Dynamic MITE Network Synthesis: RMS-to-DC Converter

To implement the time derivative, we introduce a log-compressed voltage state variable,  $V_z$ . Using the chain rule, we can express the preceding equation as

$$2\tau I_z \frac{\partial I_z}{\partial V_z} \cdot \frac{dV_z}{dt} + I_z^2 = I_u^2 - 2I_u I_v + I_v^2 \quad \Rightarrow \quad 2\tau I_z \left( -\frac{w}{U_T} I_z \right) \frac{dV_z}{dt} + I_z^2 = I_u^2 - 2I_u I_v + I_v^2$$

$$\Rightarrow -\frac{2w\tau}{U_T} \cdot \frac{dV_z}{dt} + 1 = \frac{I_u^2}{I_z^2} - \frac{2I_u I_v}{I_z^2} + \frac{I_v^2}{I_z^2} \quad \Rightarrow \quad -\underbrace{\frac{2w\tau}{CU_T}}_{1/I_\tau} \cdot \underbrace{C \frac{dV_z}{dt}}_{I_c} + 1 = \frac{I_u^2}{I_z^2} - \frac{2I_u I_v}{I_z^2} + \frac{I_v^2}{I_z^2}$$

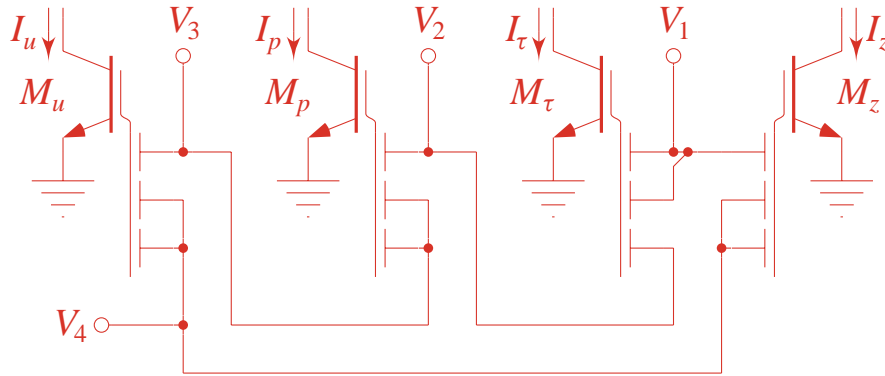
$$\Rightarrow -\frac{I_c}{I_\tau} + 1 = \frac{I_u^2}{I_z^2} - \frac{2I_u I_v}{I_z^2} + \frac{I_v^2}{I_z^2} \quad \Rightarrow \quad I_c - I_\tau = \underbrace{\frac{I_\tau I_u^2}{I_z^2}}_{I_p} - \underbrace{\frac{I_\tau I_u (2I_v)}{I_z^2}}_{I_q} + \underbrace{\frac{I_\tau I_v^2}{I_z^2}}_{I_r}$$

## Dynamic MITE Network Synthesis: **RMS-to-DC Converter**

$$\begin{aligned} \text{TLP: } I_p I_z^2 &= I_\tau I_u^2 & I_r I_z^2 &= I_\tau I_v^2 & \text{KCL: } I_p + I_r + I_c &= I_\tau + I_q \\ I_q I_z^2 &= I_\tau I_u I_{2v} \end{aligned}$$

# Dynamic MITE Network Synthesis: **RMS-to-DC Converter**

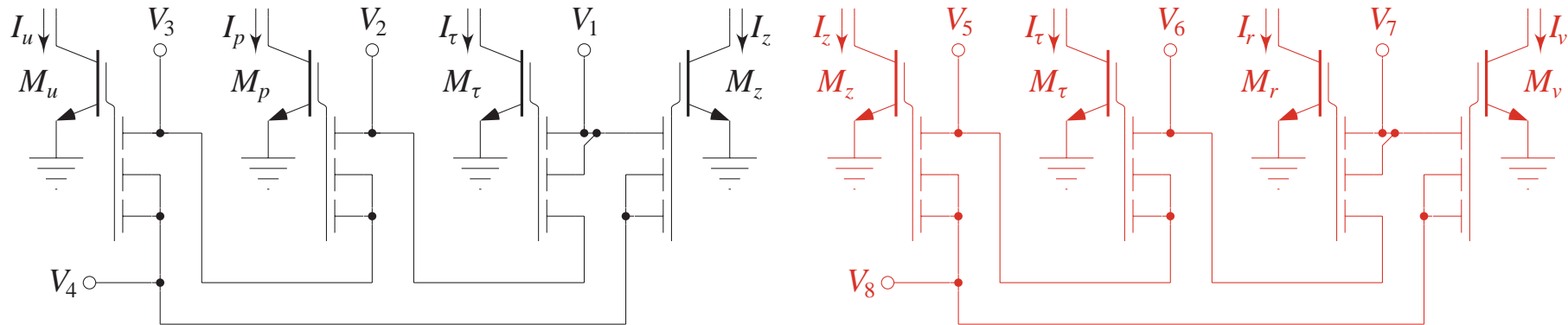
$$\text{TLP: } \begin{aligned} I_p I_z^2 &= I_\tau I_u^2 & I_r I_z^2 &= I_\tau I_v^2 \\ I_q I_z^2 &= I_\tau I_u I_{2v} \end{aligned} \quad \text{KCL: } I_p + I_r + I_c = I_\tau + I_q$$



# Dynamic MITE Network Synthesis: RMS-to-DC Converter

$$\text{TLP: } I_p I_z^2 = I_\tau I_u^2 \quad I_r I_z^2 = I_\tau I_v^2 \quad \text{KCL: } I_p + I_r + I_c = I_\tau + I_q$$

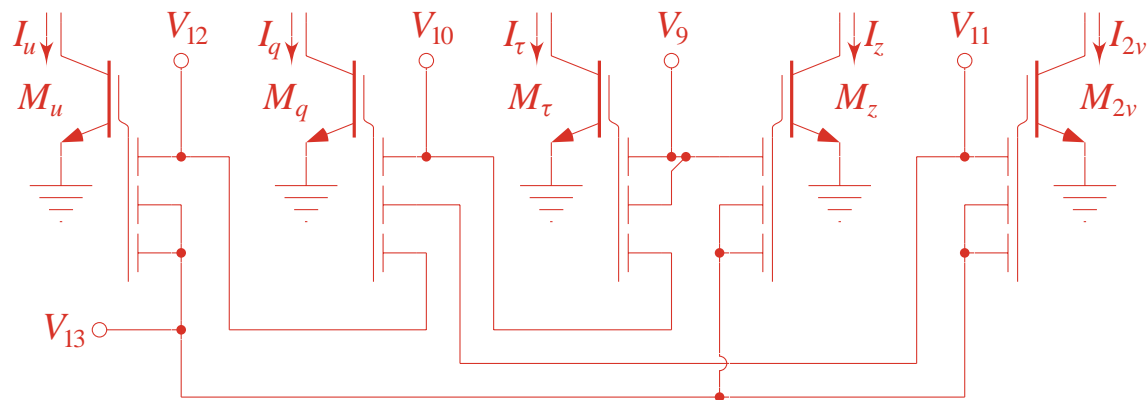
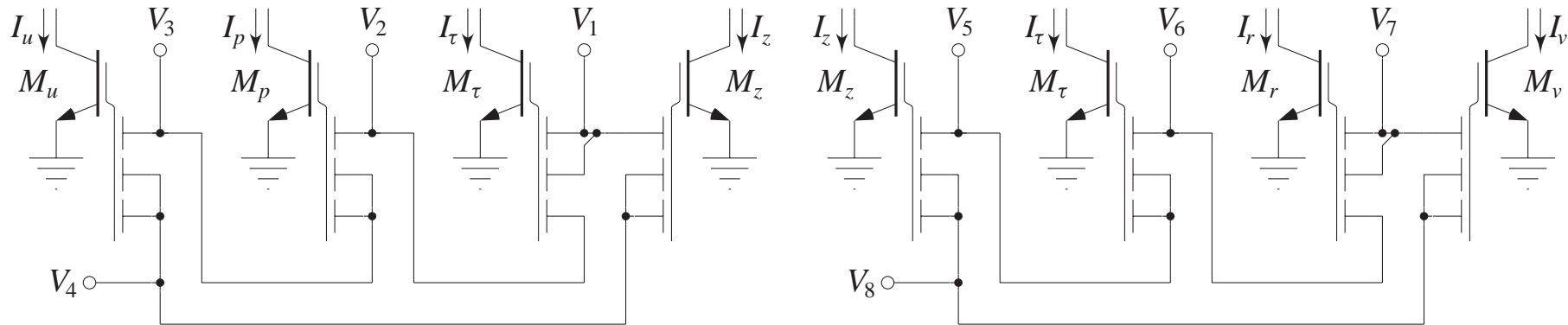
$$I_q I_z^2 = I_\tau I_u I_{2v}$$



# Dynamic MITE Network Synthesis: RMS-to-DC Converter

$$\text{TLP: } I_p I_z^2 = I_\tau I_u^2 \quad I_r I_z^2 = I_\tau I_v^2 \quad \text{KCL: } I_p + I_r + I_c = I_\tau + I_q$$

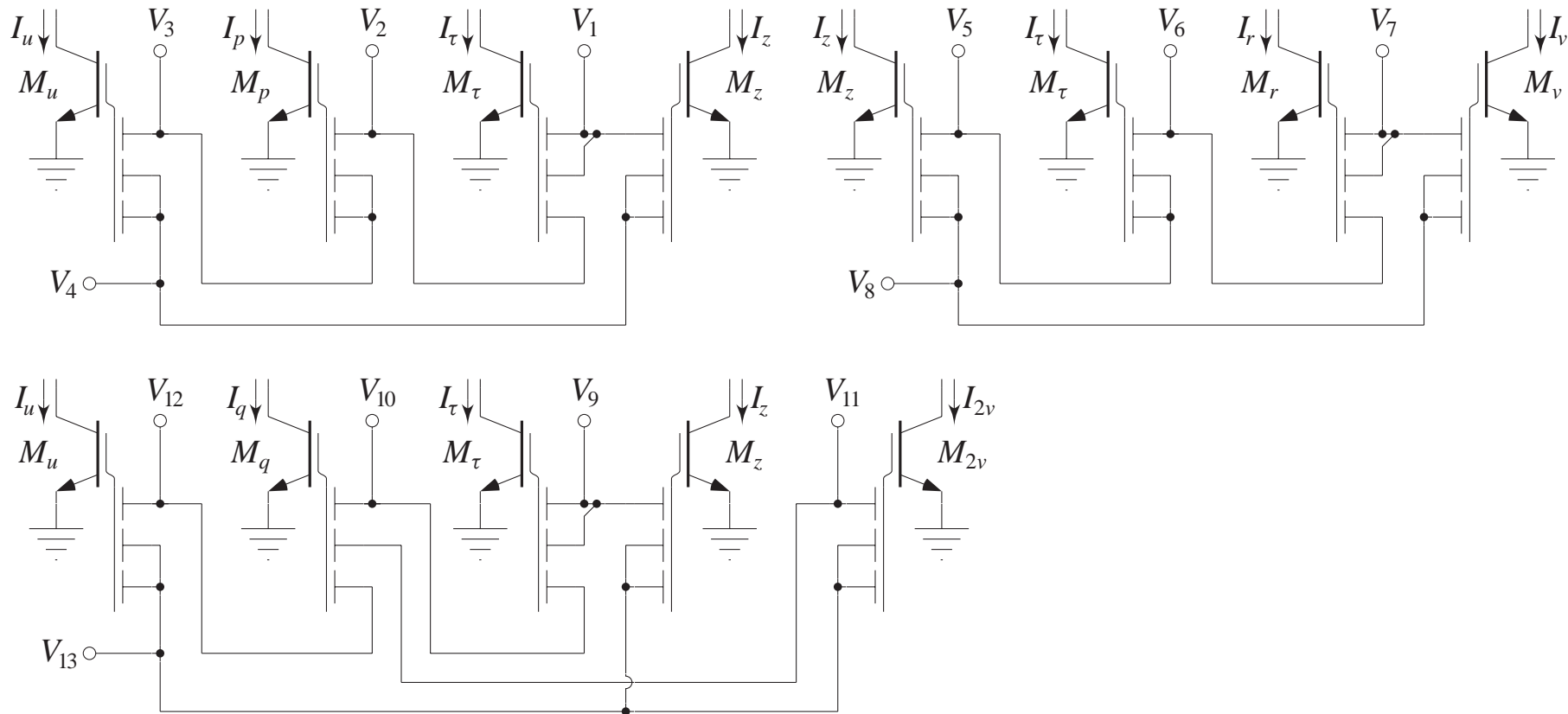
$$I_q I_z^2 = I_\tau I_u I_{2v}$$



# Dynamic MITE Network Synthesis: RMS-to-DC Converter

$$\text{TLP: } I_p I_z^2 = I_\tau I_u^2 \quad I_r I_z^2 = I_\tau I_v^2 \quad \text{KCL: } I_p + I_r + I_c = I_\tau + I_q$$

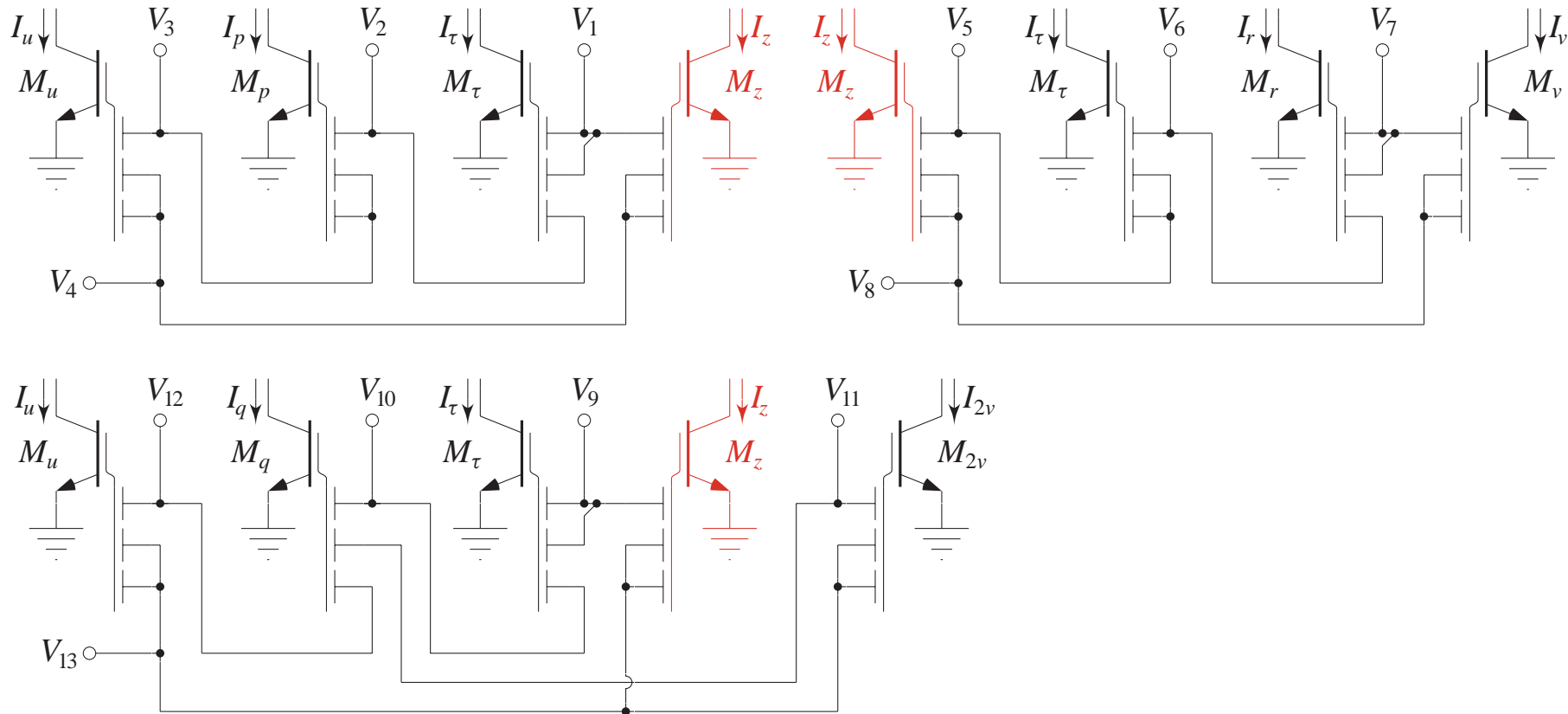
$$I_q I_z^2 = I_\tau I_u I_{2v}$$



# Dynamic MITE Network Synthesis: RMS-to-DC Converter

$$\text{TLP: } I_p I_z^2 = I_\tau I_u^2 \quad I_r I_z^2 = I_\tau I_v^2 \quad \text{KCL: } I_p + I_r + I_c = I_\tau + I_q$$

$$I_q I_z^2 = I_\tau I_u I_{2v}$$

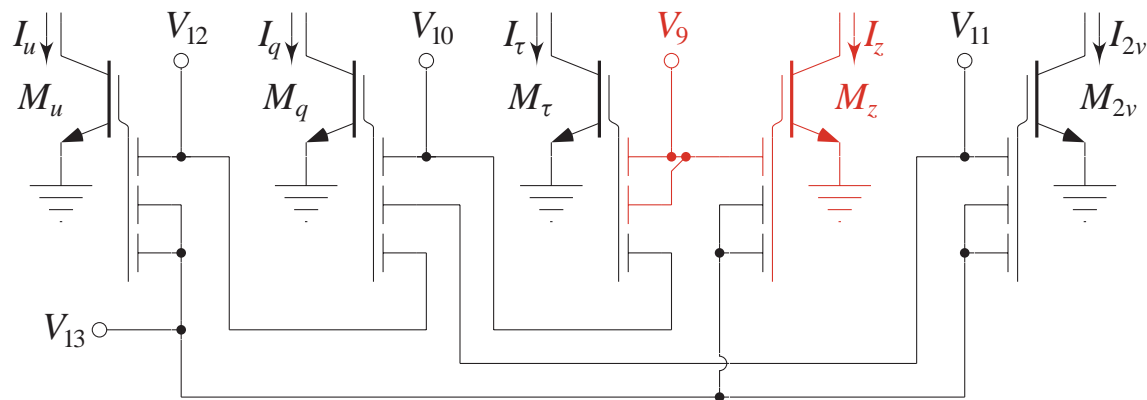
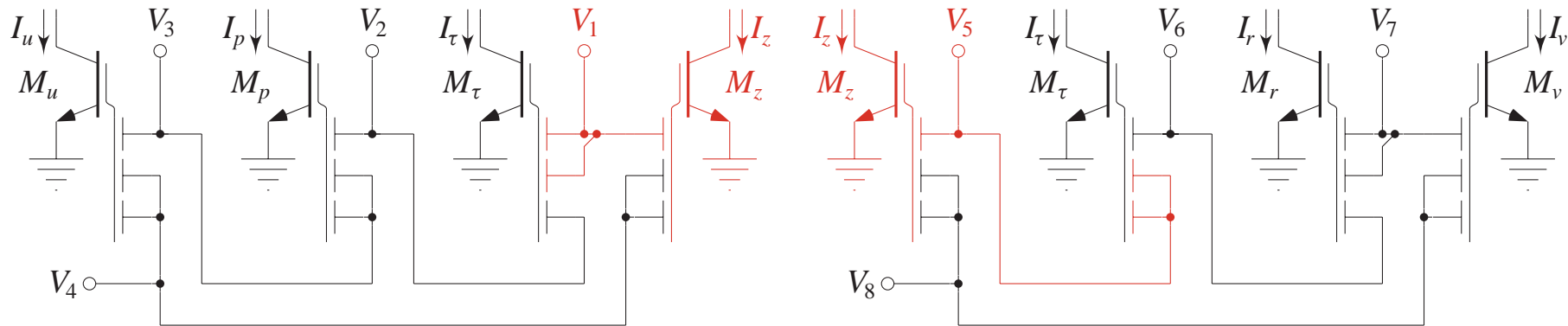




# Dynamic MITE Network Synthesis: RMS-to-DC Converter

$$\text{TLP: } I_p I_z^2 = I_\tau I_u^2 \quad I_r I_z^2 = I_\tau I_v^2 \quad \text{KCL: } I_p + I_r + I_c = I_\tau + I_q$$

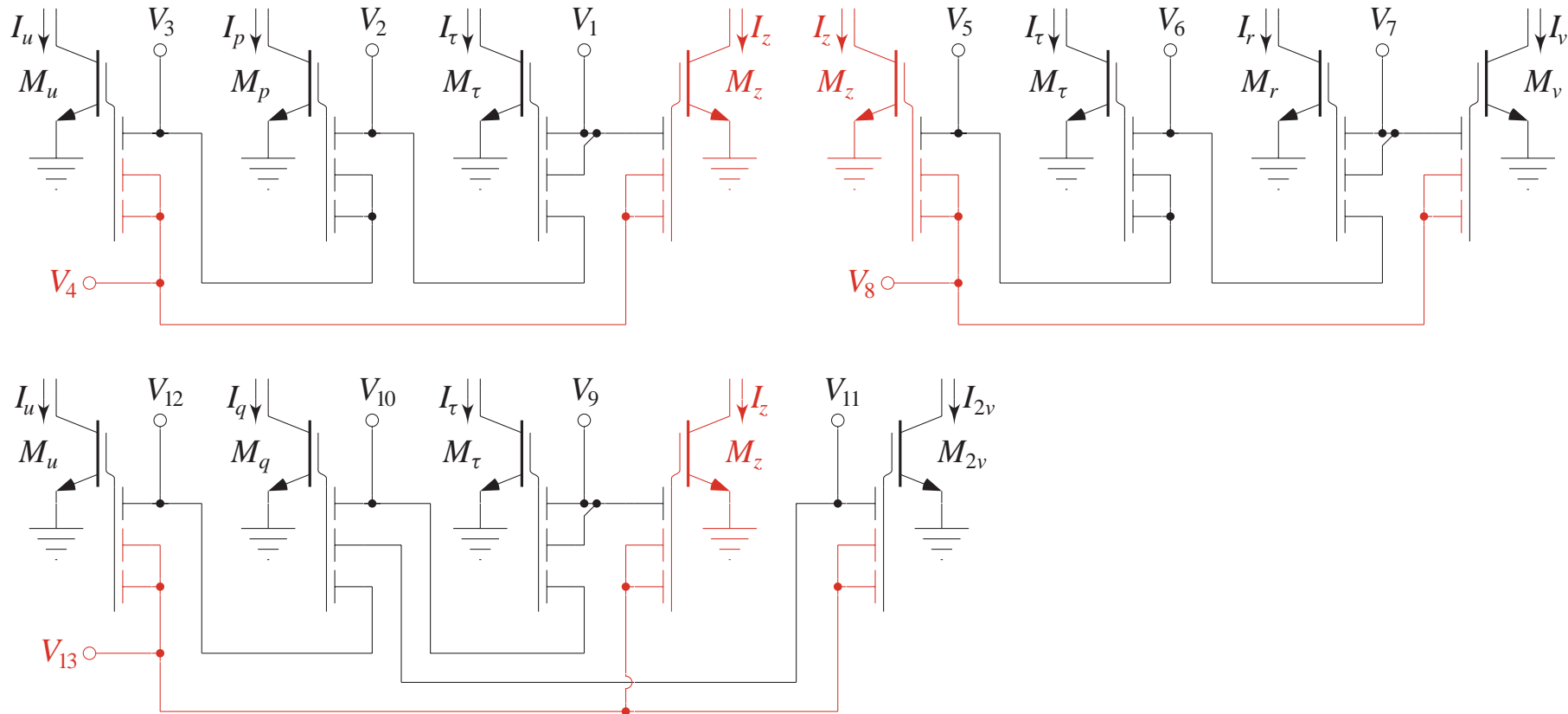
$$I_q I_z^2 = I_\tau I_u I_{2v}$$



# Dynamic MITE Network Synthesis: RMS-to-DC Converter

$$\text{TLP: } I_p I_z^2 = I_\tau I_u^2 \quad I_r I_z^2 = I_\tau I_v^2 \quad \text{KCL: } I_p + I_r + I_c = I_\tau + I_q$$

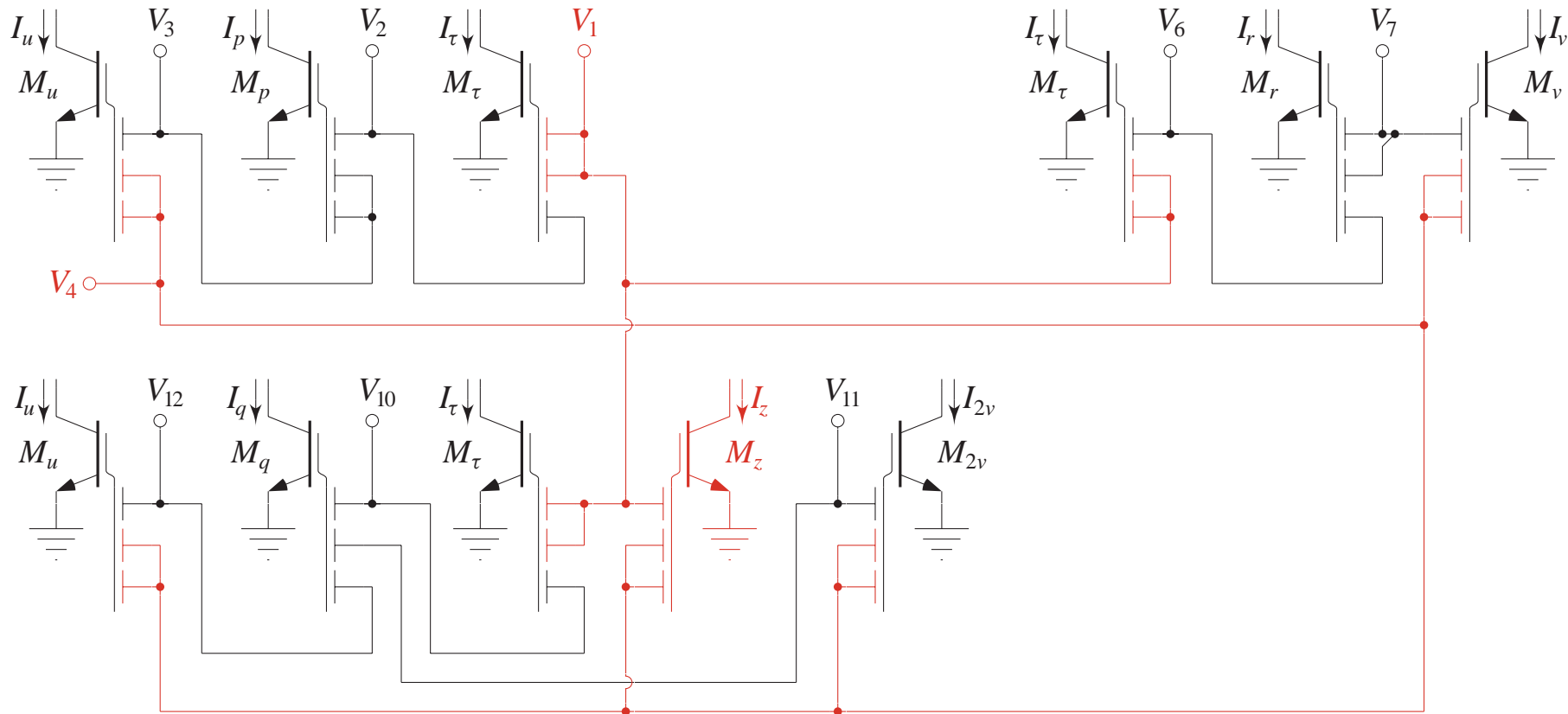
$$I_q I_z^2 = I_\tau I_u I_{2v}$$



# Dynamic MITE Network Synthesis: RMS-to-DC Converter

$$\text{TLP: } I_p I_z^2 = I_\tau I_u^2 \quad I_r I_z^2 = I_\tau I_v^2 \quad \text{KCL: } I_p + I_r + I_c = I_\tau + I_q$$

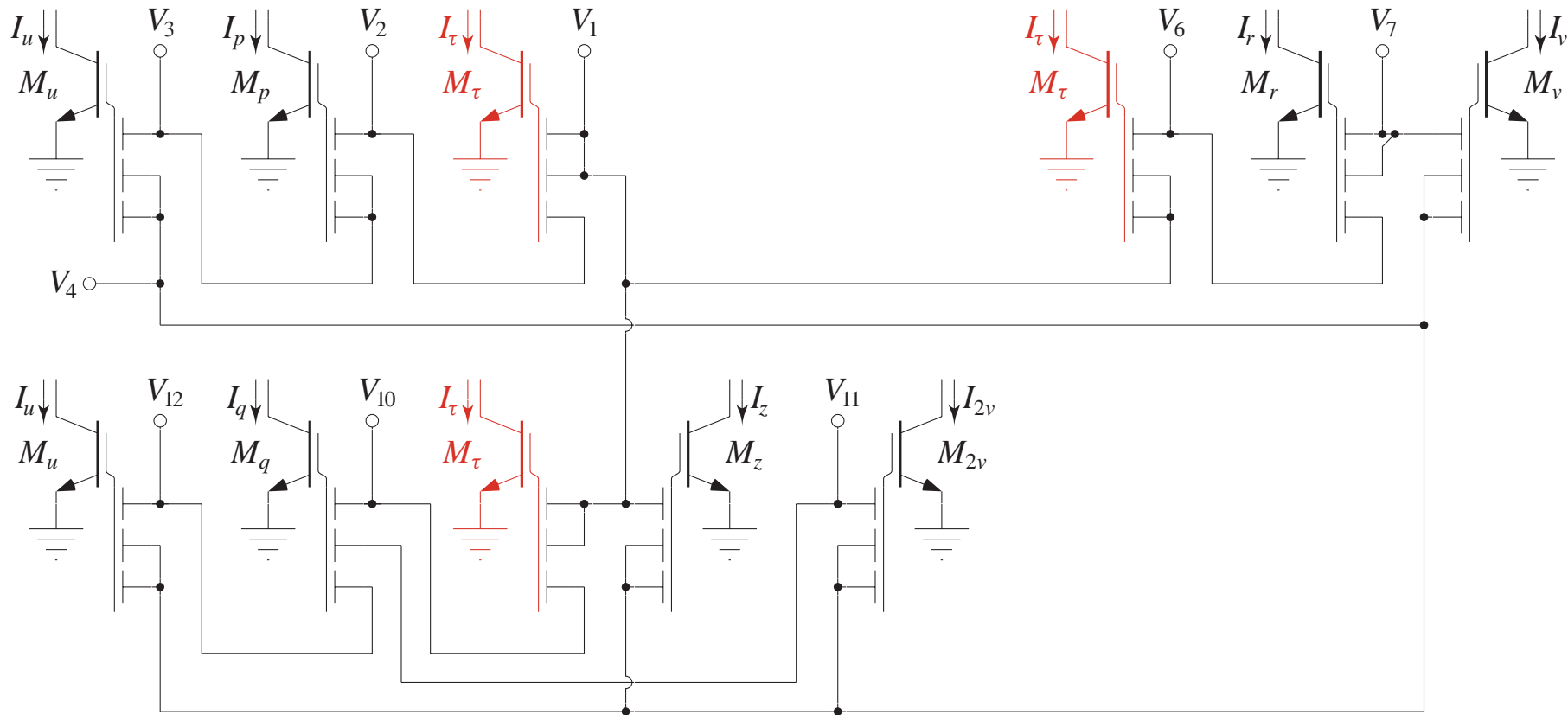
$$I_q I_z^2 = I_\tau I_u I_{2v}$$



# Dynamic MITE Network Synthesis: RMS-to-DC Converter

$$\text{TLP: } I_p I_z^2 = I_\tau I_u^2 \quad I_r I_z^2 = I_\tau I_v^2 \quad \text{KCL: } I_p + I_r + I_c = I_\tau + I_q$$

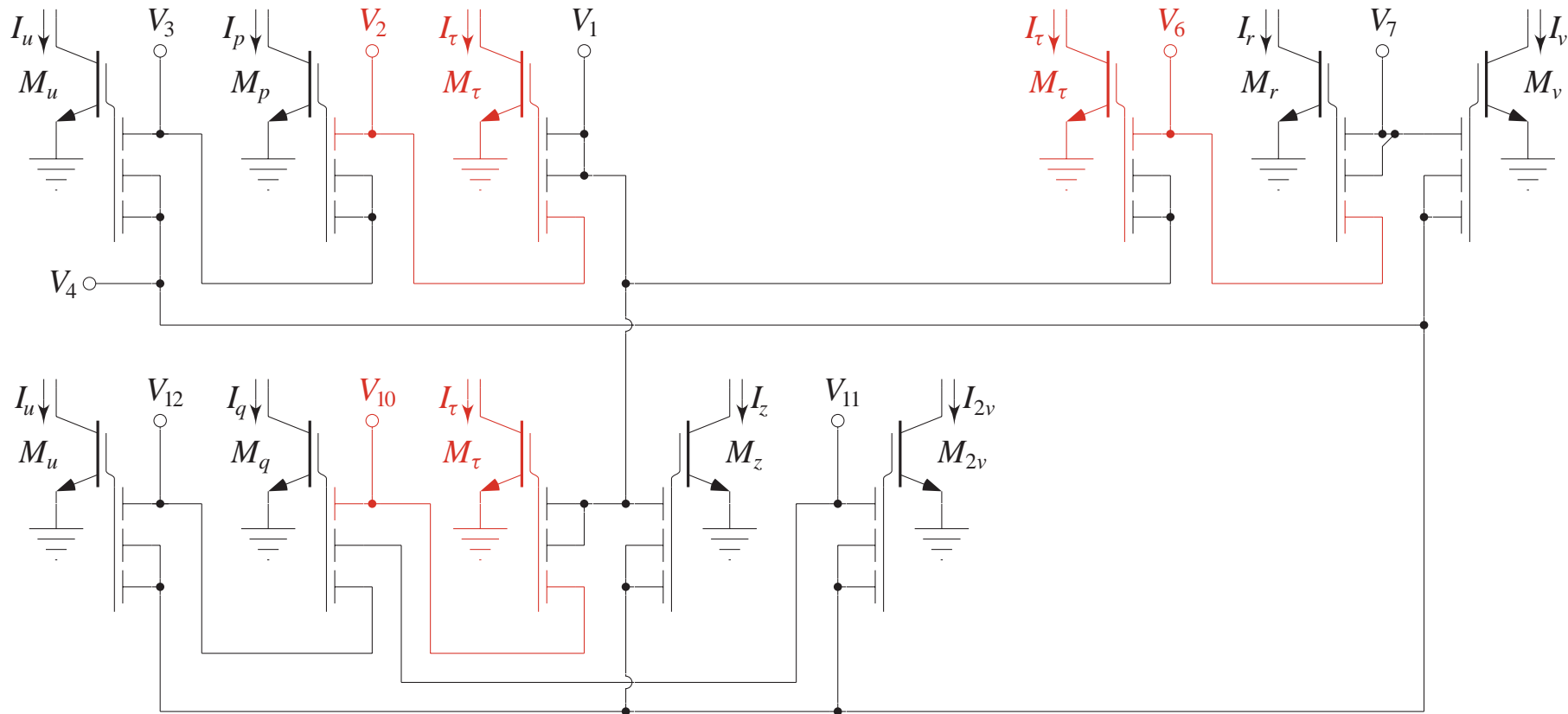
$$I_q I_z^2 = I_\tau I_u I_{2v}$$



# Dynamic MITE Network Synthesis: RMS-to-DC Converter

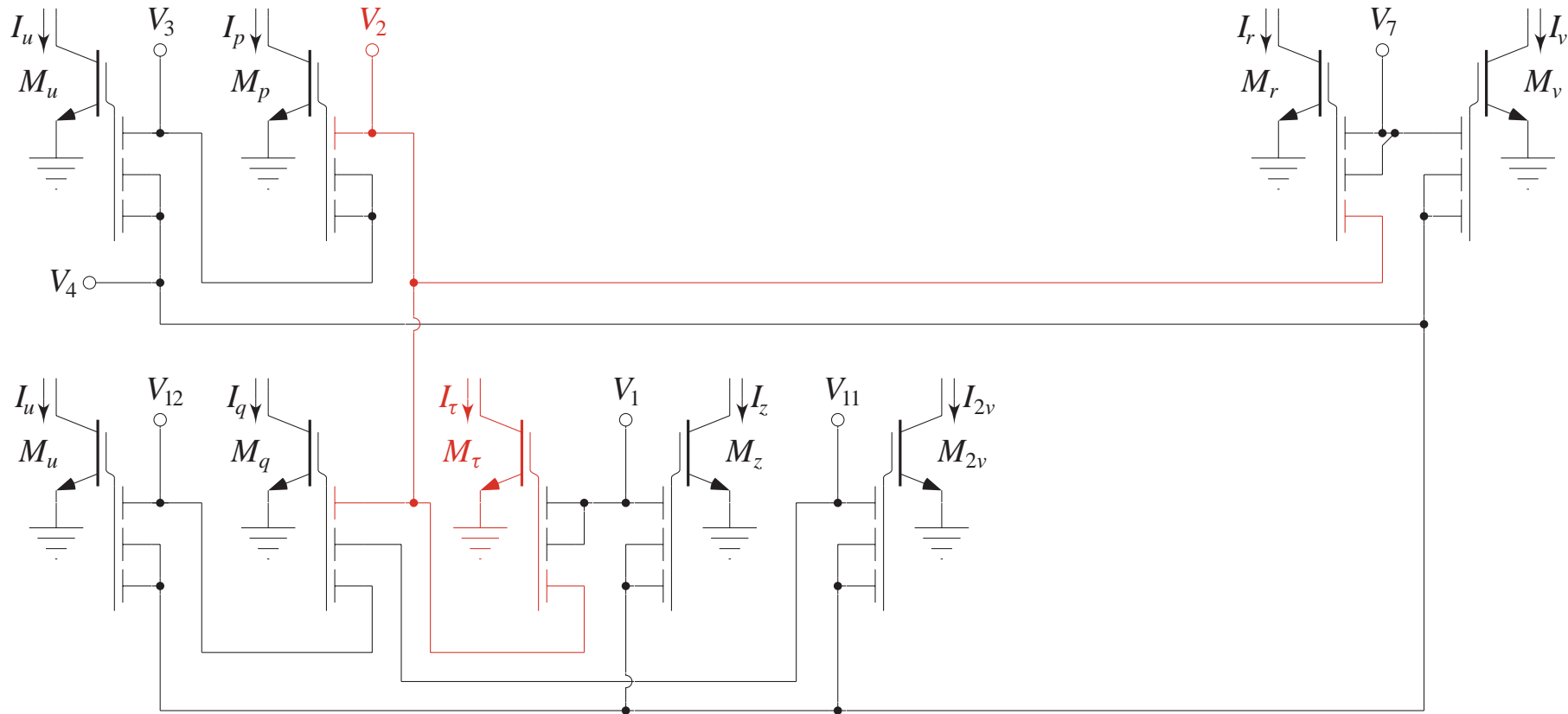
$$\text{TLP: } I_p I_z^2 = I_\tau I_u^2 \quad I_r I_z^2 = I_\tau I_v^2 \quad \text{KCL: } I_p + I_r + I_c = I_\tau + I_q$$

$$I_q I_z^2 = I_\tau I_u I_{2v}$$



# Dynamic MITE Network Synthesis: RMS-to-DC Converter

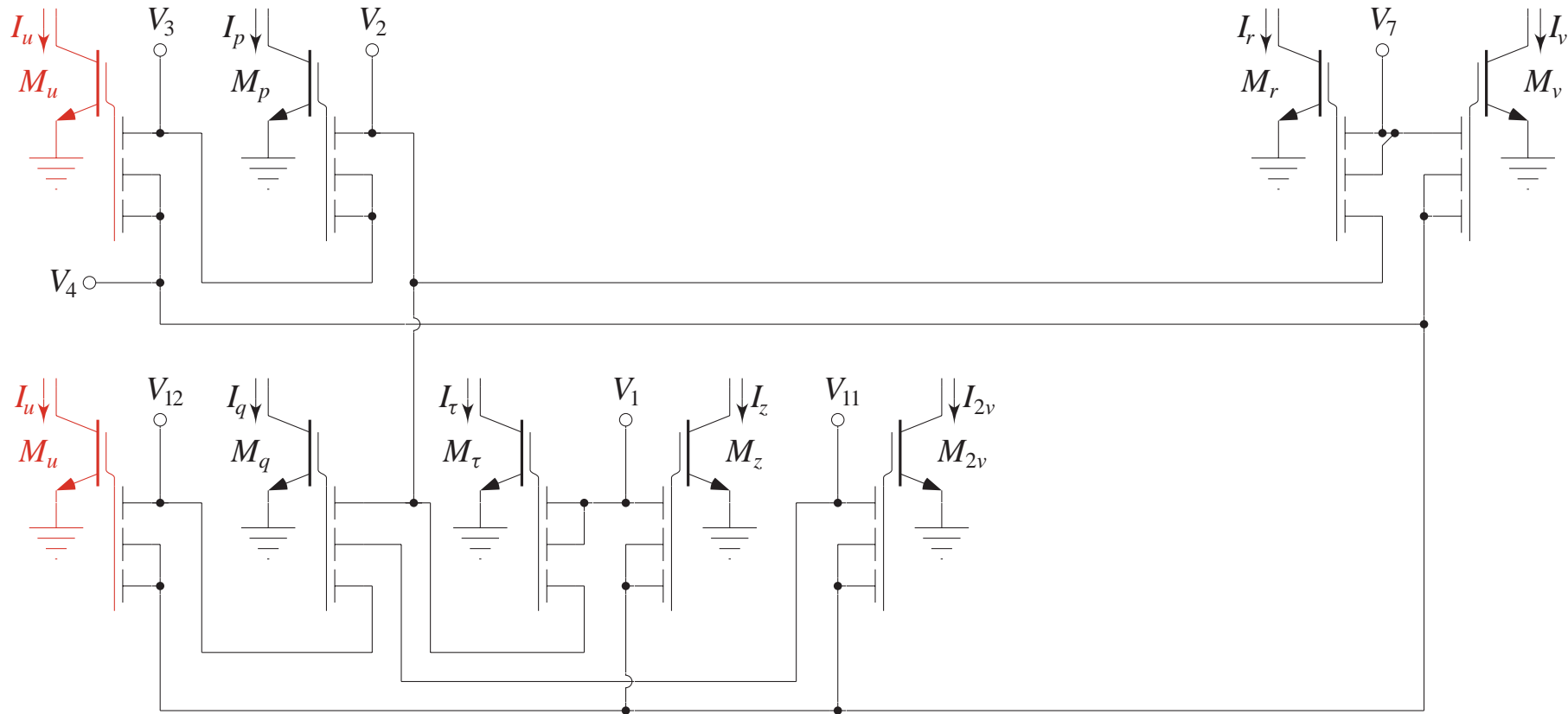
$$\begin{aligned} \text{TLP: } I_p I_z^2 &= I_\tau I_u^2 & I_r I_z^2 &= I_\tau I_v^2 & \text{KCL: } I_p + I_r + I_c &= I_\tau + I_q \\ I_q I_z^2 &= I_\tau I_u I_{2v} \end{aligned}$$



# Dynamic MITE Network Synthesis: RMS-to-DC Converter

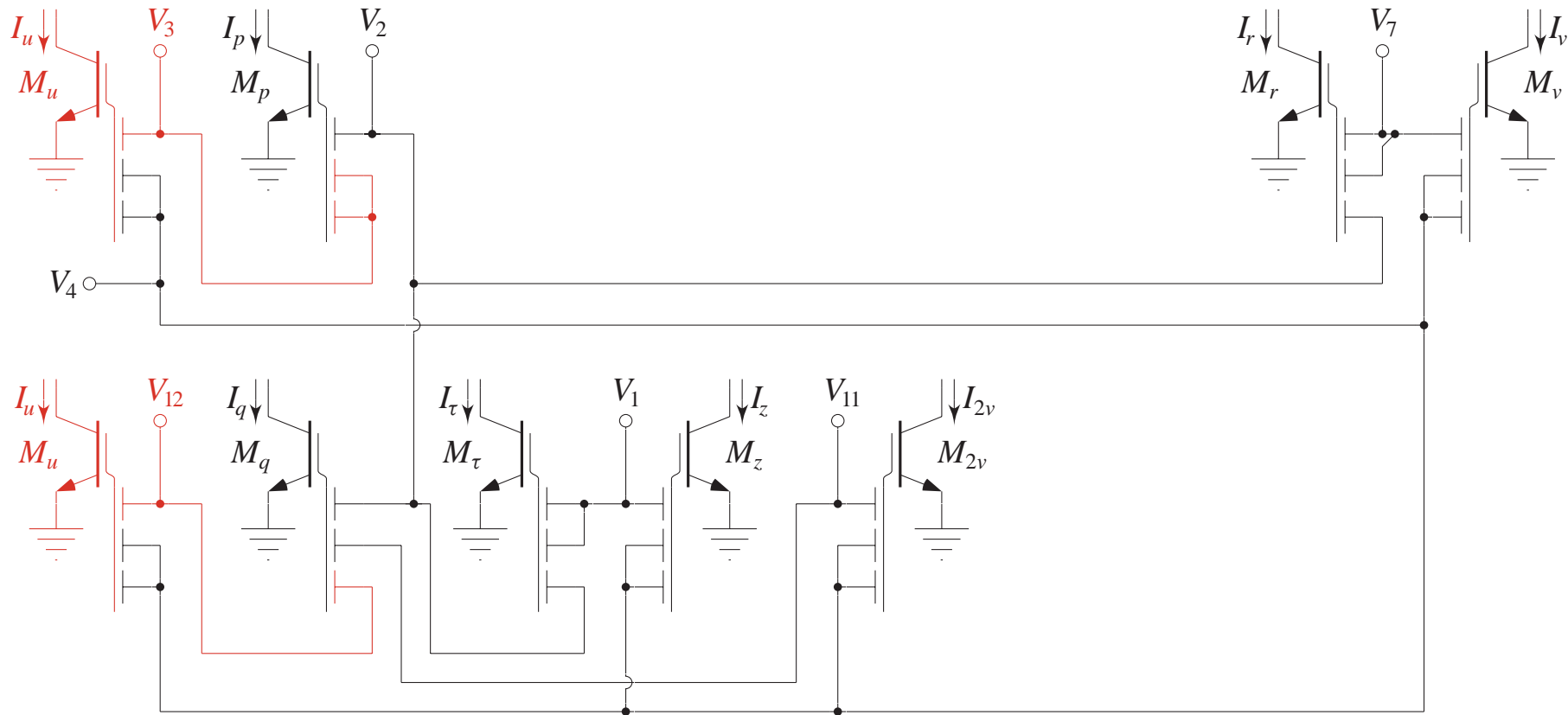
$$\text{TLP: } I_p I_z^2 = I_\tau I_u^2 \quad I_r I_z^2 = I_\tau I_v^2 \quad \text{KCL: } I_p + I_r + I_c = I_\tau + I_q$$

$$I_q I_z^2 = I_\tau I_u I_{2v}$$



# Dynamic MITE Network Synthesis: RMS-to-DC Converter

$$\begin{aligned} \text{TLP: } I_p I_z^2 &= I_\tau I_u^2 & I_r I_z^2 &= I_\tau I_v^2 & \text{KCL: } I_p + I_r + I_c &= I_\tau + I_q \\ I_q I_z^2 &= I_\tau I_u I_{2v} \end{aligned}$$

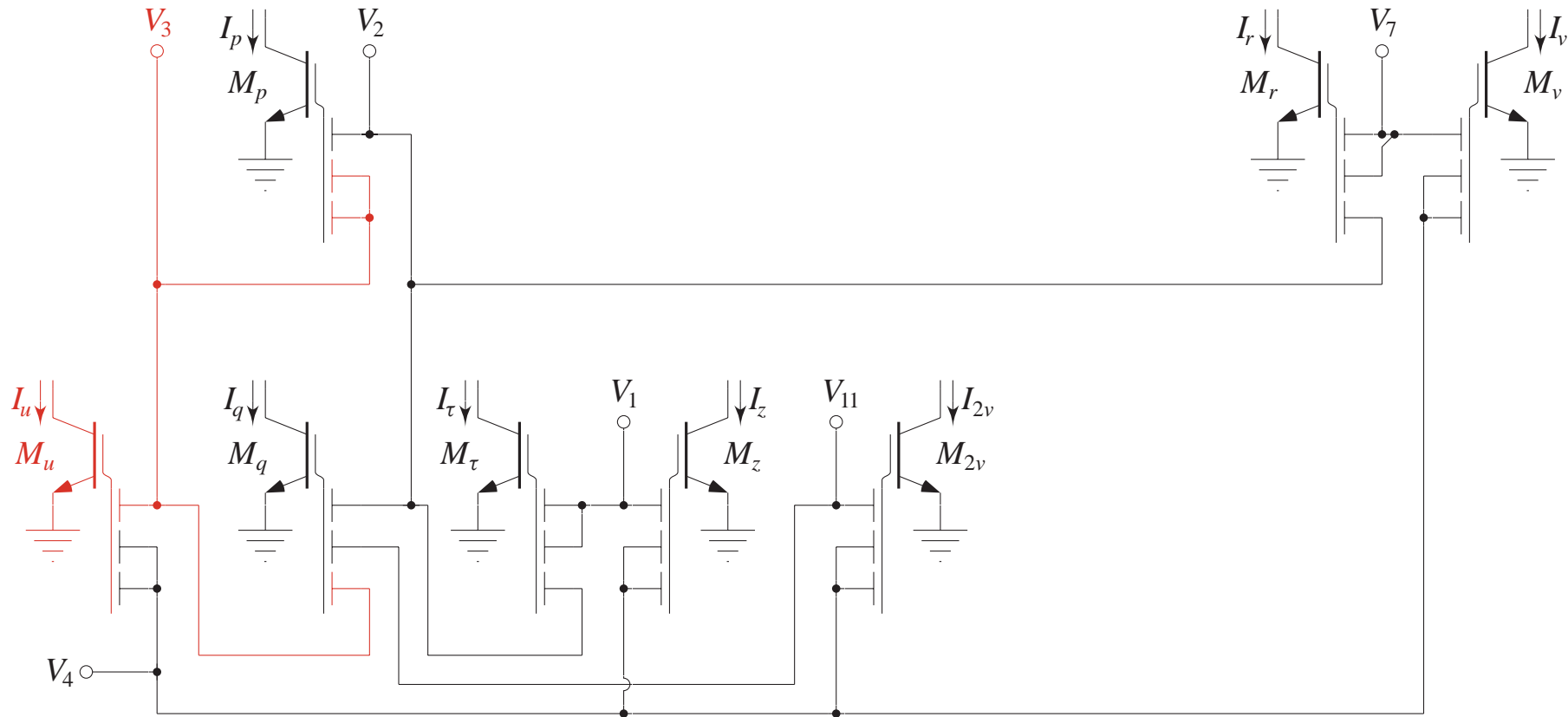




# Dynamic MITE Network Synthesis: RMS-to-DC Converter

$$\text{TLP: } I_p I_z^2 = I_\tau I_u^2 \quad I_r I_z^2 = I_\tau I_v^2 \quad \text{KCL: } I_p + I_r + I_c = I_\tau + I_q$$

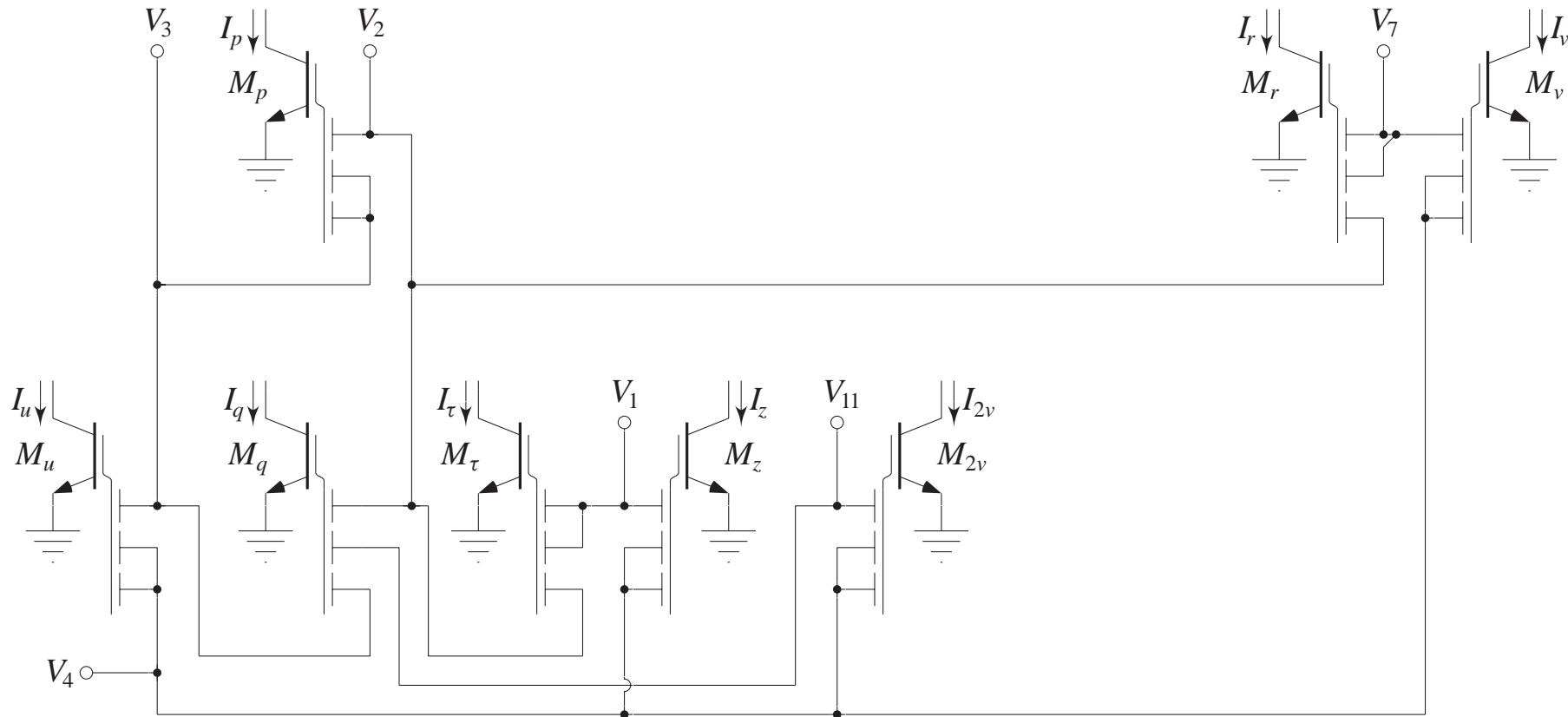
$$I_q I_z^2 = I_\tau I_u I_{2v}$$



# Dynamic MITE Network Synthesis: RMS-to-DC Converter

$$\text{TLP: } I_p I_z^2 = I_\tau I_u^2 \quad I_r I_z^2 = I_\tau I_v^2 \quad \text{KCL: } I_p + I_r + I_c = I_\tau + I_q$$

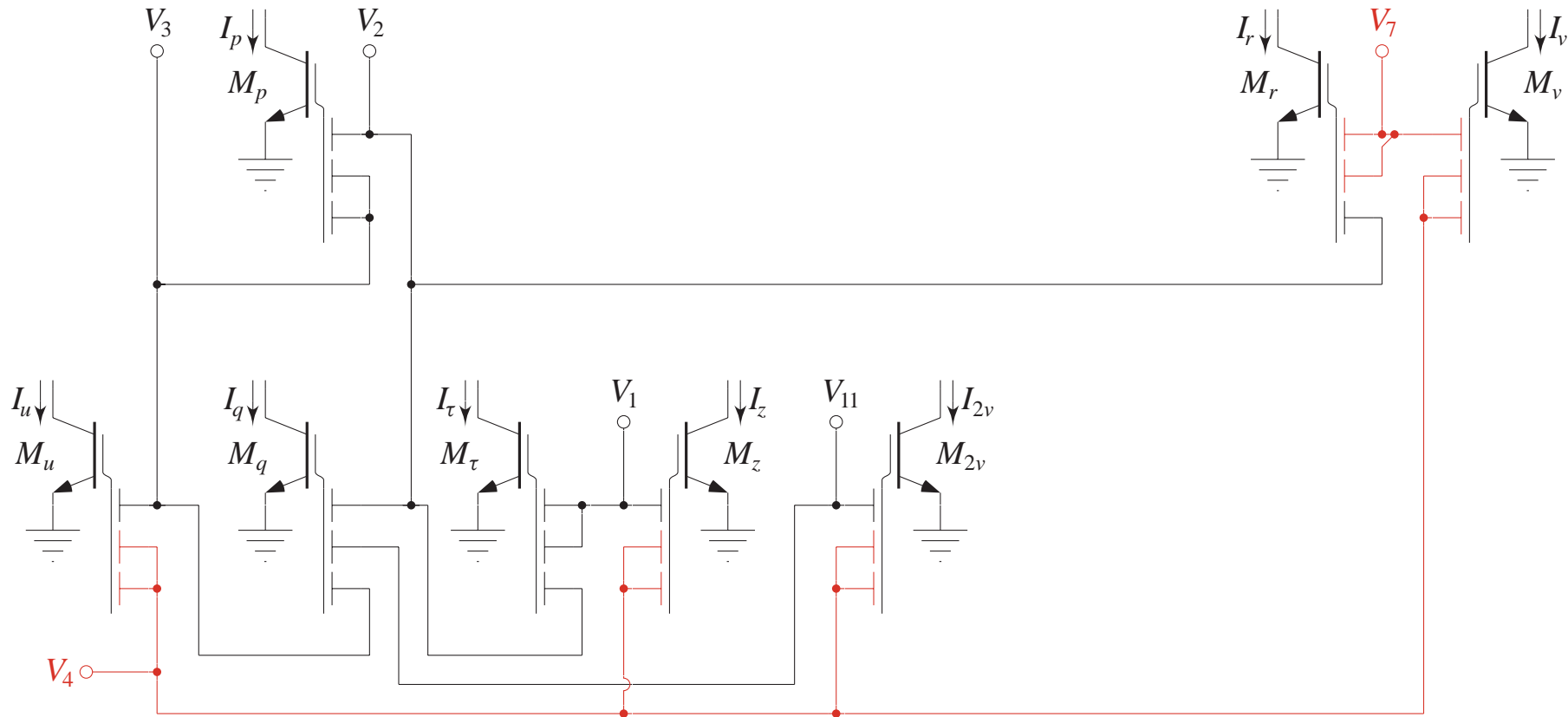
$$I_q I_z^2 = I_\tau I_u I_{2v}$$



# Dynamic MITE Network Synthesis: RMS-to-DC Converter

$$\text{TLP: } I_p I_z^2 = I_\tau I_u^2 \quad I_r I_z^2 = I_\tau I_v^2 \quad \text{KCL: } I_p + I_r + I_c = I_\tau + I_q$$

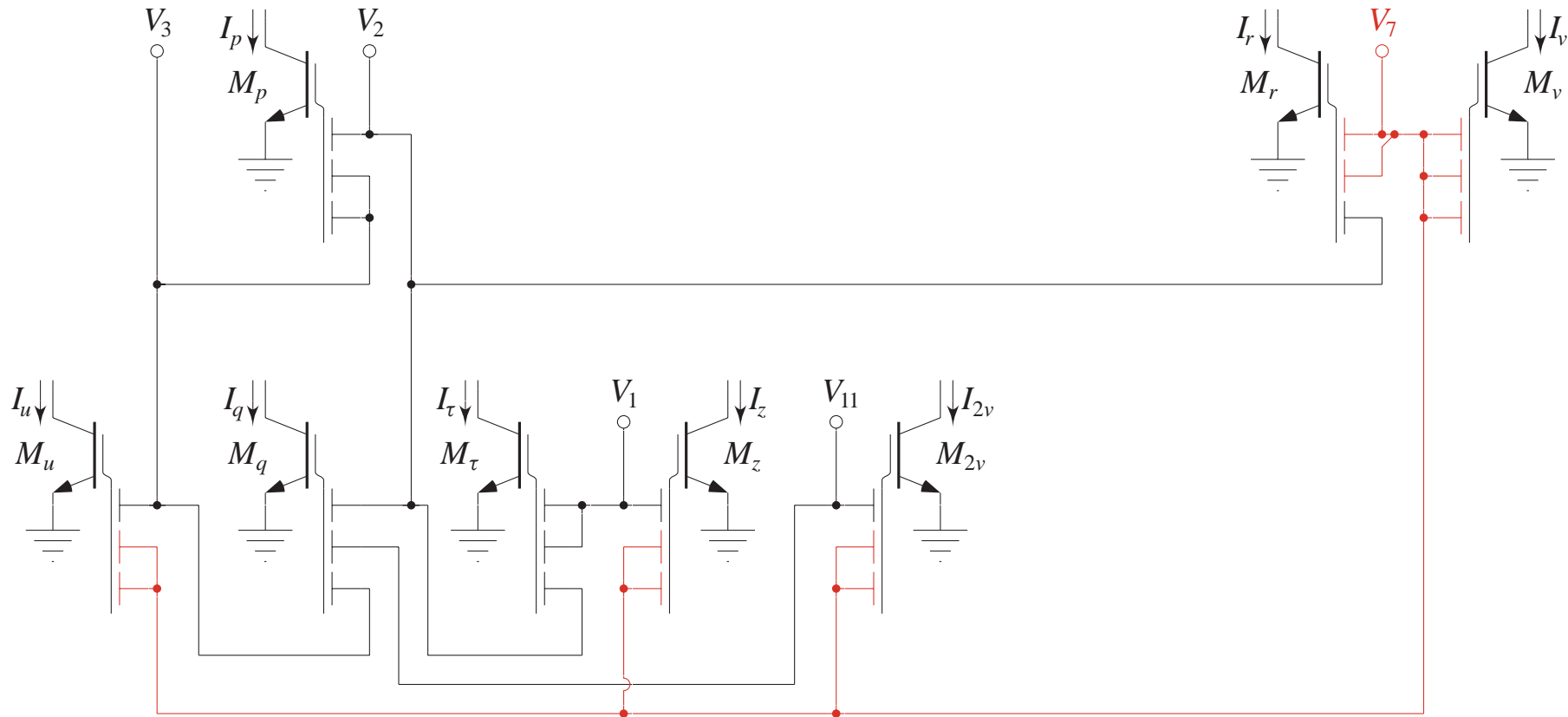
$$I_q I_z^2 = I_\tau I_u I_{2v}$$



# Dynamic MITE Network Synthesis: RMS-to-DC Converter

$$\text{TLP: } I_p I_z^2 = I_\tau I_u^2 \quad I_r I_z^2 = I_\tau I_v^2 \quad \text{KCL: } I_p + I_r + I_c = I_\tau + I_q$$

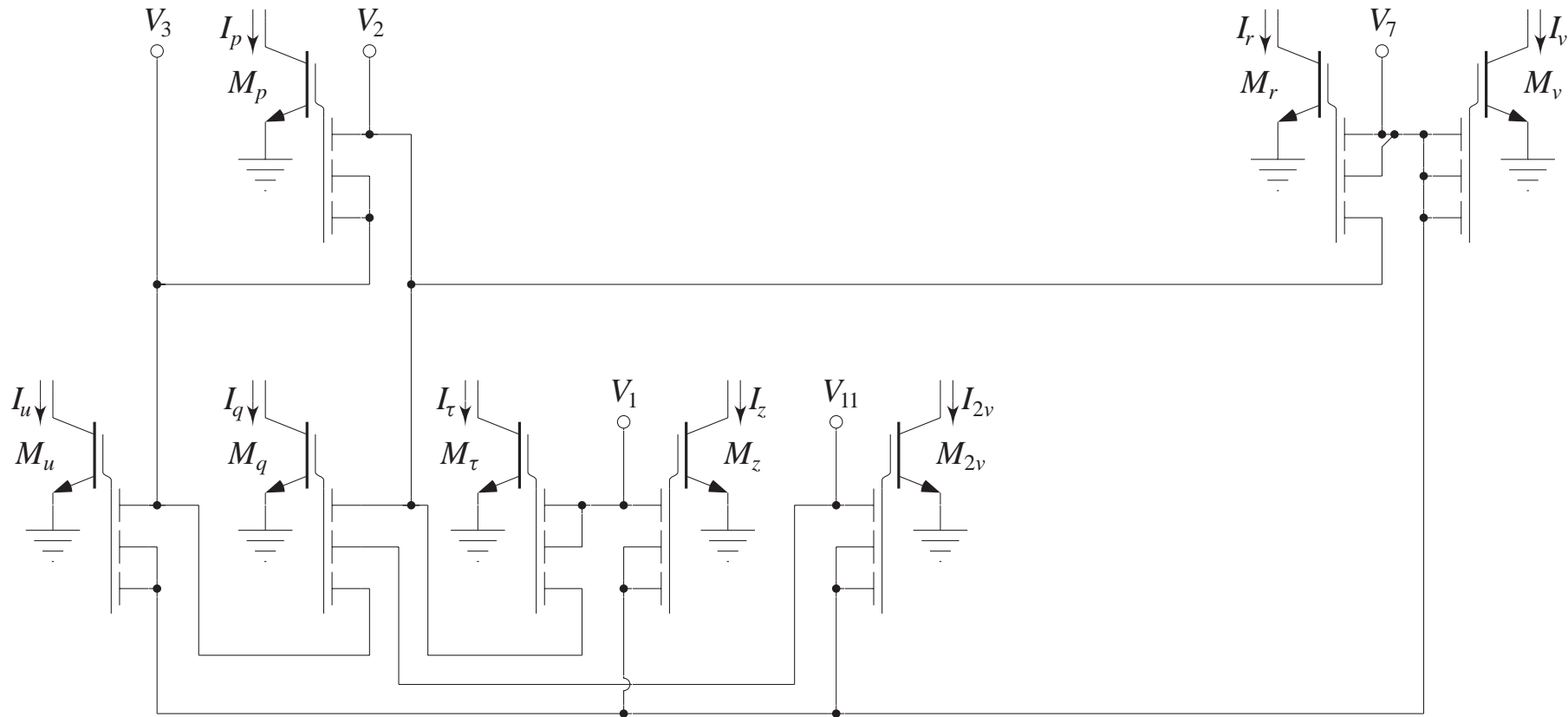
$$I_q I_z^2 = I_\tau I_u I_{2v}$$



# Dynamic MITE Network Synthesis: RMS-to-DC Converter

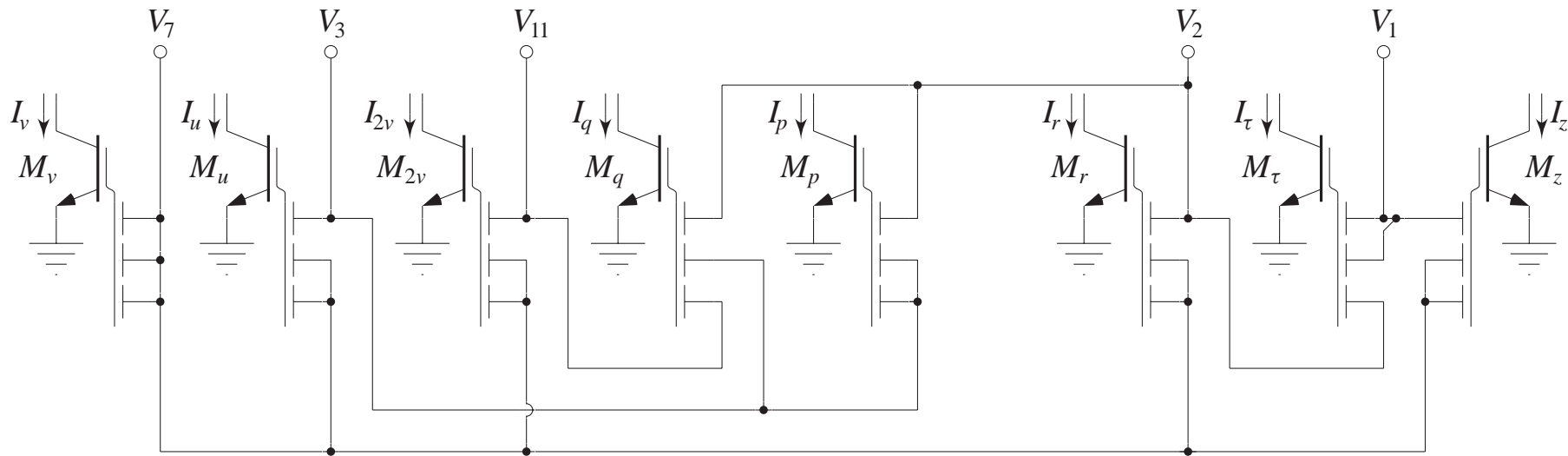
$$\text{TLP: } I_p I_z^2 = I_\tau I_u^2 \quad I_r I_z^2 = I_\tau I_v^2 \quad \text{KCL: } I_p + I_r + I_c = I_\tau + I_q$$

$$I_q I_z^2 = I_\tau I_u I_{2v}$$



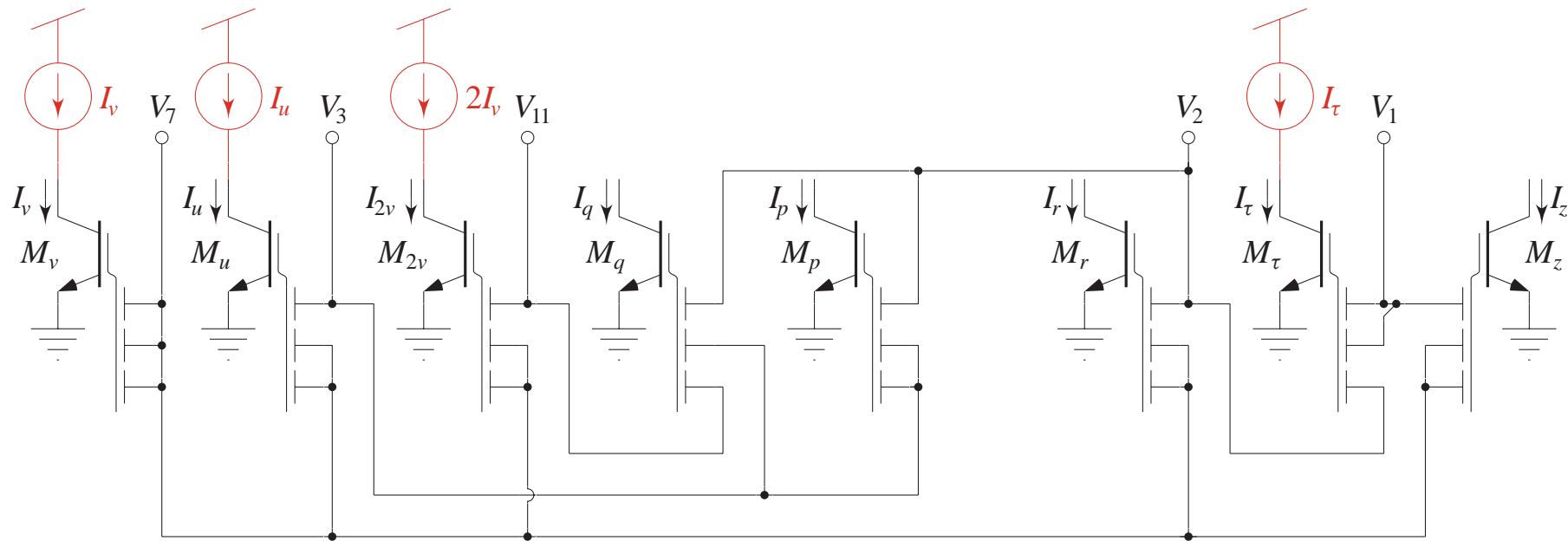
# Dynamic MITE Network Synthesis: RMS-to-DC Converter

$$\begin{aligned} \text{TLP: } I_p I_z^2 &= I_\tau I_u^2 & I_r I_z^2 &= I_\tau I_v^2 & \text{KCL: } I_p + I_r + I_c &= I_\tau + I_q \\ I_q I_z^2 &= I_\tau I_u I_{2v} \end{aligned}$$



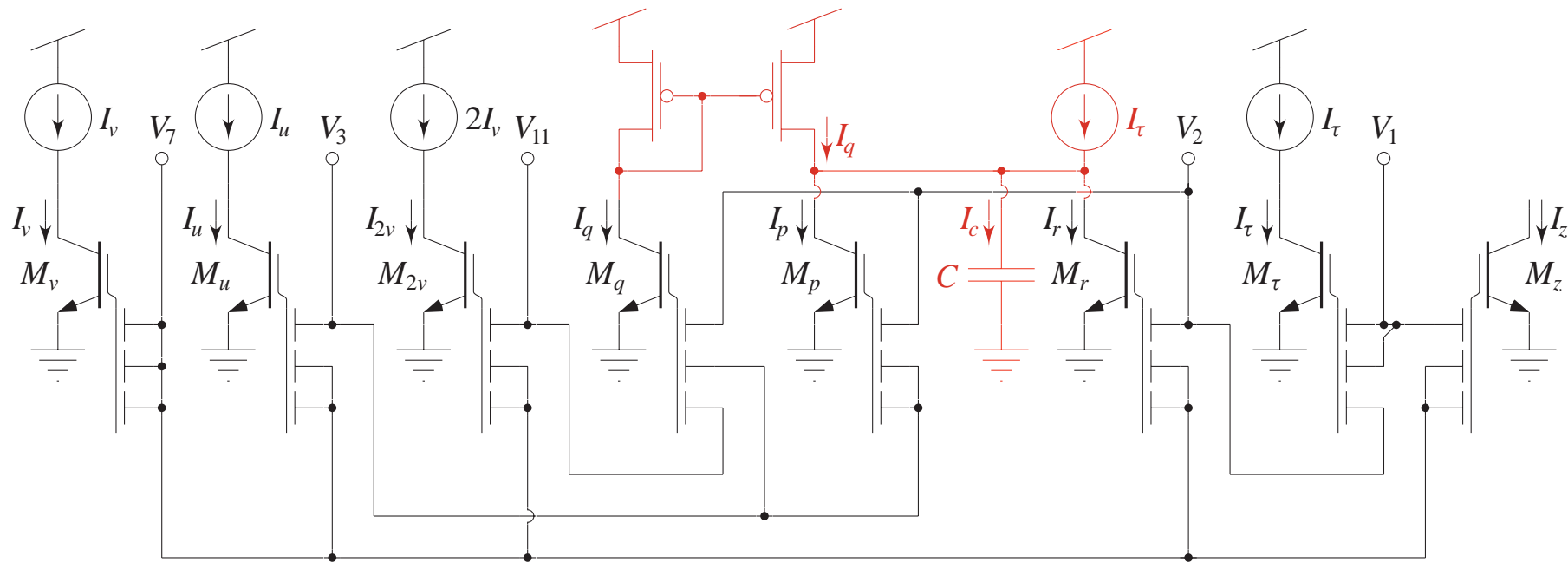
# Dynamic MITE Network Synthesis: **RMS-to-DC Converter**

$$\begin{aligned} \text{TLP: } I_p I_z^2 &= I_\tau I_u^2 & I_r I_z^2 &= I_\tau I_v^2 & \text{KCL: } I_p + I_r + I_c &= I_\tau + I_q \\ I_q I_z^2 &= I_\tau I_u I_{2v} \end{aligned}$$



# Dynamic MITE Network Synthesis: RMS-to-DC Converter

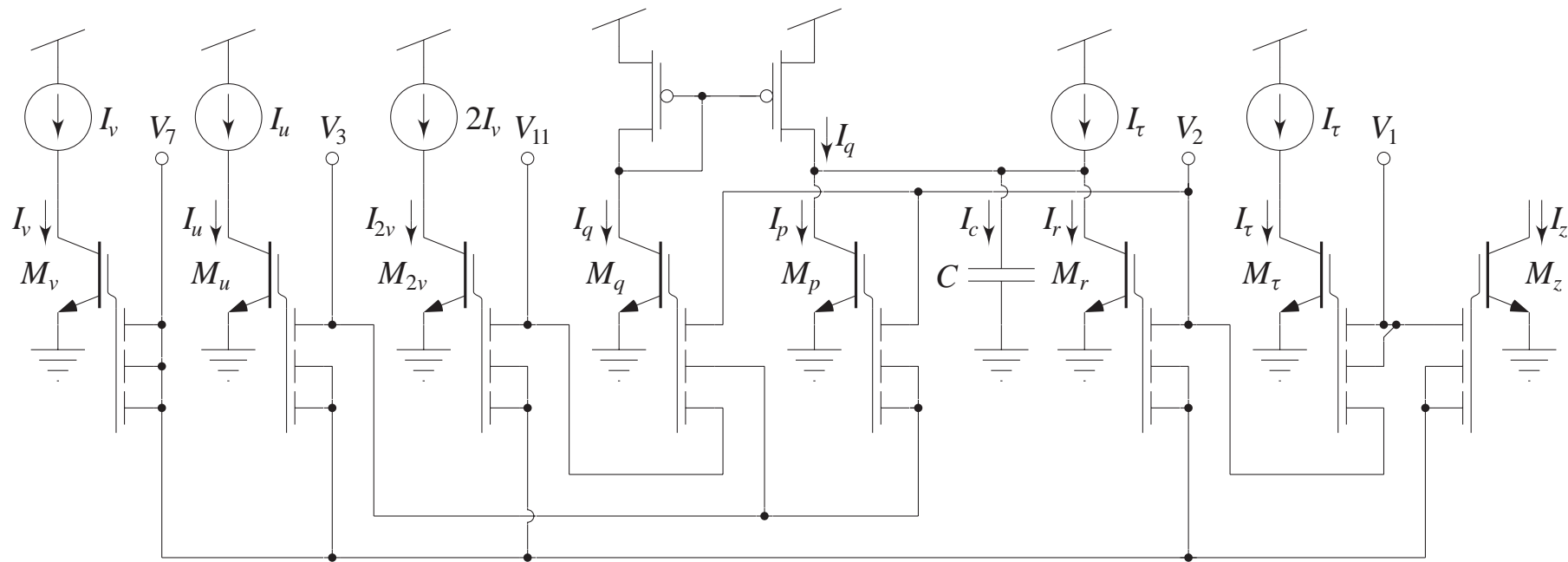
$$\begin{aligned} \text{TLP: } I_p I_z^2 &= I_\tau I_u^2 & I_r I_z^2 &= I_\tau I_v^2 & \text{KCL: } I_p + I_r + I_c &= I_\tau + I_q \\ I_q I_z^2 &= I_\tau I_u I_{2v} \end{aligned}$$





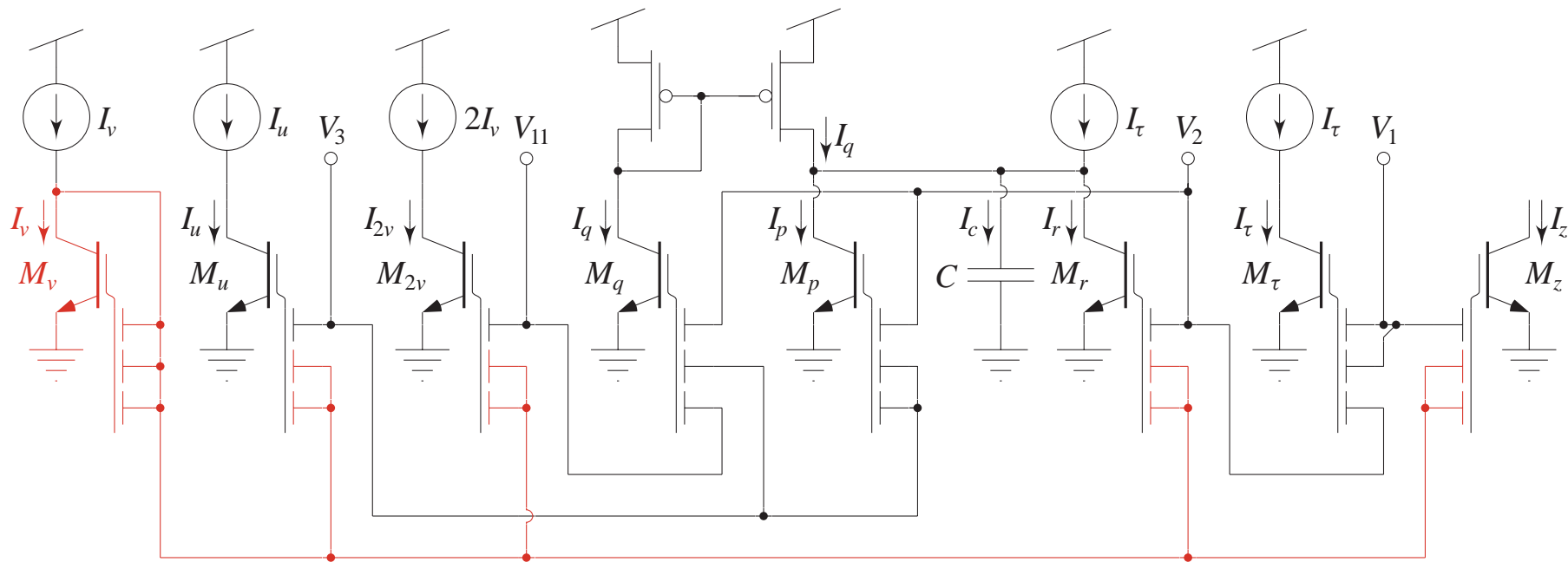
# Dynamic MITE Network Synthesis: RMS-to-DC Converter

$$\begin{aligned} \text{TLP: } I_p I_z^2 &= I_\tau I_u^2 & I_r I_z^2 &= I_\tau I_v^2 & \text{KCL: } I_p + I_r + I_c &= I_\tau + I_q \\ I_q I_z^2 &= I_\tau I_u I_{2v} & & & & \end{aligned}$$



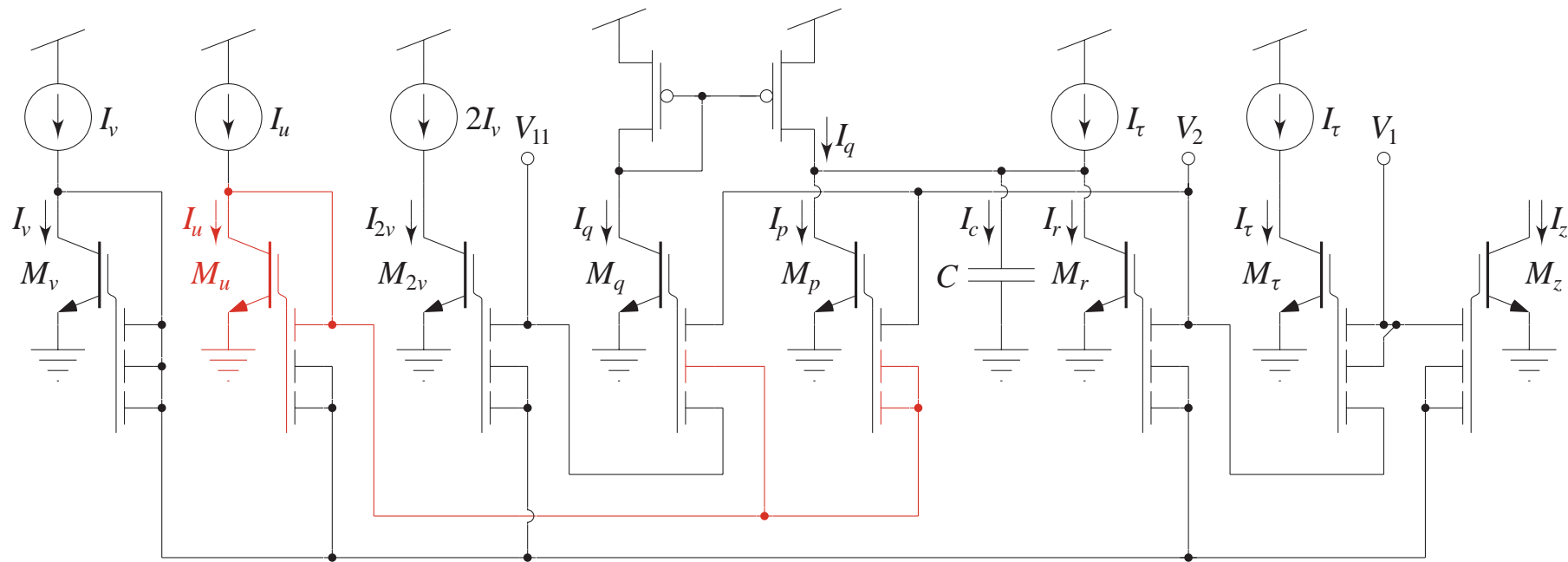
# Dynamic MITE Network Synthesis: **RMS-to-DC Converter**

$$\begin{aligned} \text{TLP: } I_p I_z^2 &= I_\tau I_u^2 & I_r I_z^2 &= I_\tau I_v^2 & \text{KCL: } I_p + I_r + I_c &= I_\tau + I_q \\ I_q I_z^2 &= I_\tau I_u I_{2v} \end{aligned}$$



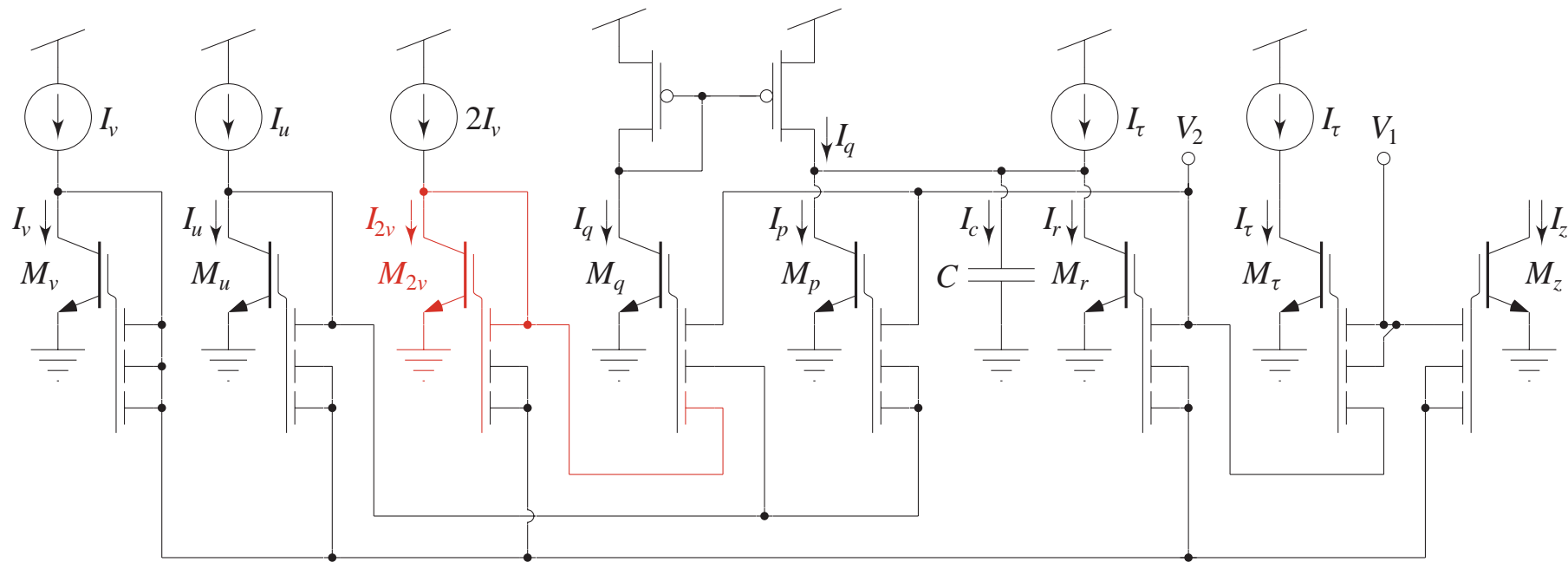
# Dynamic MITE Network Synthesis: **RMS-to-DC Converter**

$$\begin{aligned} \text{TLP: } I_p I_z^2 &= I_\tau I_u^2 & I_r I_z^2 &= I_\tau I_v^2 & \text{KCL: } I_p + I_r + I_c &= I_\tau + I_q \\ I_q I_z^2 &= I_\tau I_u I_{2v} \end{aligned}$$



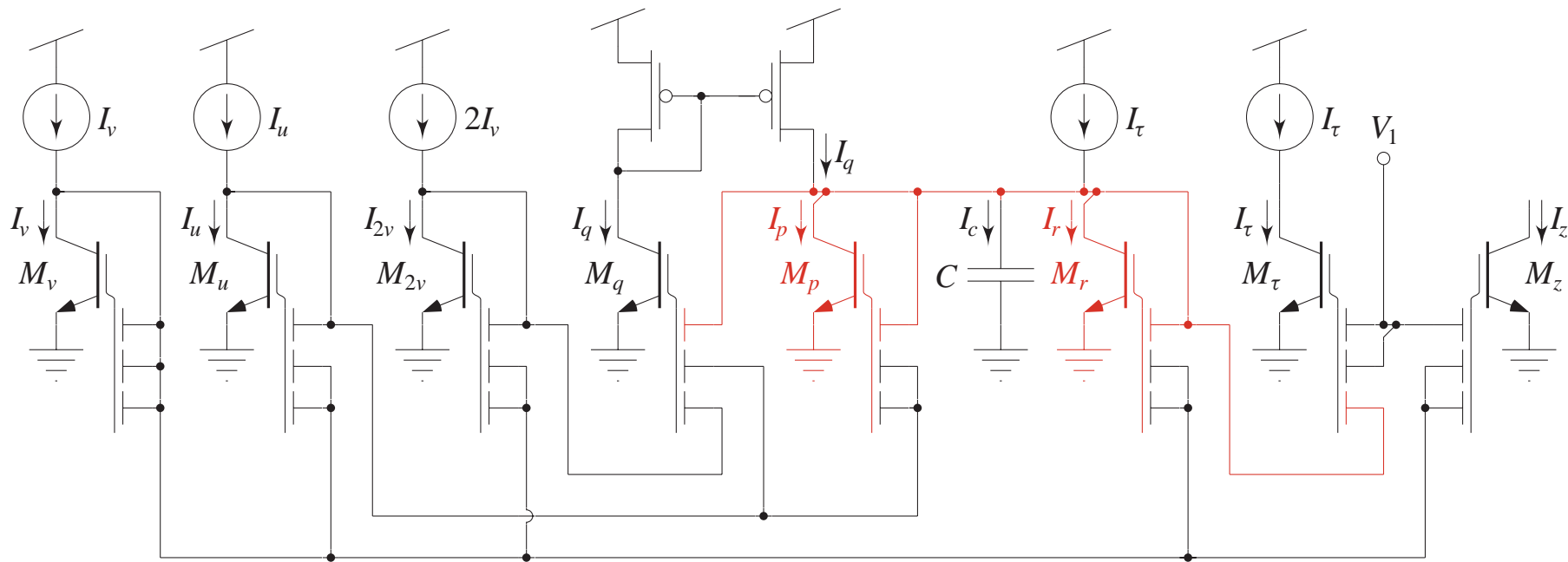
# Dynamic MITE Network Synthesis: RMS-to-DC Converter

$$\begin{aligned} \text{TLP: } I_p I_z^2 &= I_\tau I_u^2 & I_r I_z^2 &= I_\tau I_v^2 & \text{KCL: } I_p + I_r + I_c &= I_\tau + I_q \\ I_q I_z^2 &= I_\tau I_u I_{2v} \end{aligned}$$



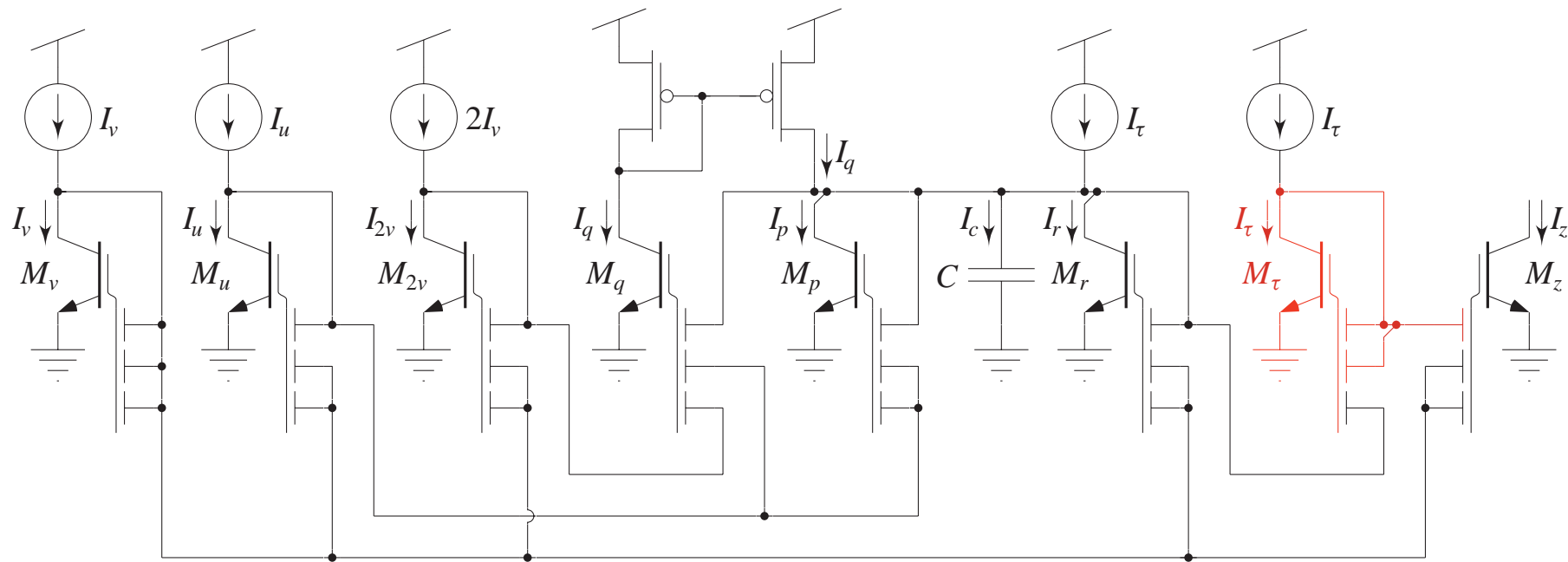
# Dynamic MITE Network Synthesis: RMS-to-DC Converter

$$\begin{aligned} \text{TLP: } I_p I_z^2 &= I_\tau I_u^2 & I_r I_z^2 &= I_\tau I_v^2 & \text{KCL: } I_p + I_r + I_c &= I_\tau + I_q \\ I_q I_z^2 &= I_\tau I_u I_{2v} \end{aligned}$$



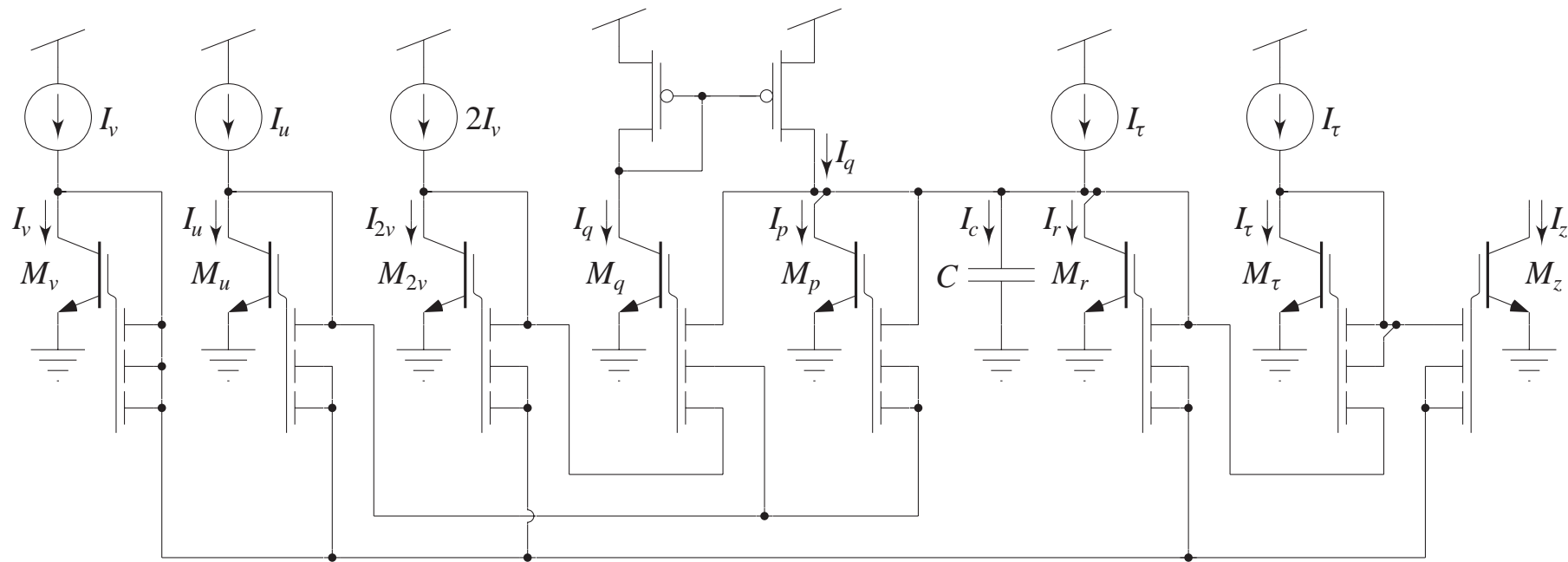
# Dynamic MITE Network Synthesis: **RMS-to-DC Converter**

$$\begin{aligned} \text{TLP: } I_p I_z^2 &= I_\tau I_u^2 & I_r I_z^2 &= I_\tau I_v^2 & \text{KCL: } I_p + I_r + I_c &= I_\tau + I_q \\ I_q I_z^2 &= I_\tau I_u I_{2v} \end{aligned}$$



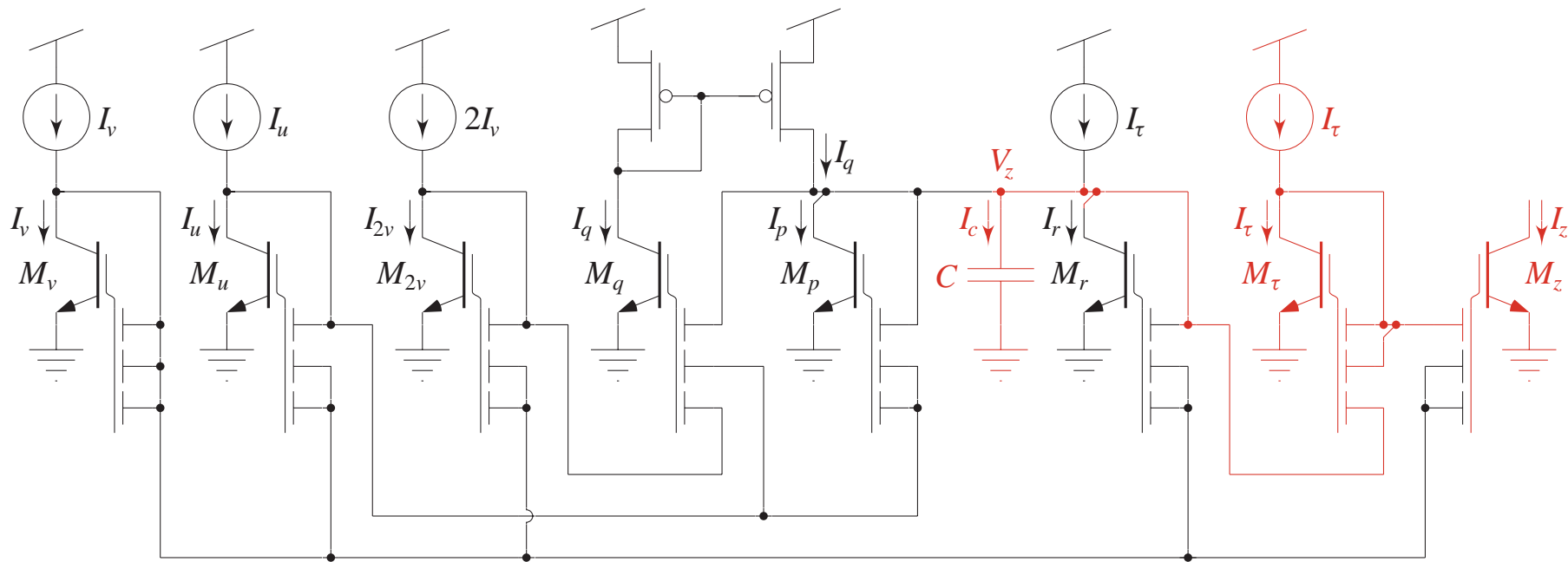
# Dynamic MITE Network Synthesis: RMS-to-DC Converter

$$\begin{aligned} \text{TLP: } I_p I_z^2 &= I_\tau I_u^2 & I_r I_z^2 &= I_\tau I_v^2 & \text{KCL: } I_p + I_r + I_c &= I_\tau + I_q \\ I_q I_z^2 &= I_\tau I_u I_{2v} \end{aligned}$$



# Dynamic MITE Network Synthesis: RMS-to-DC Converter

$$\begin{aligned} \text{TLP: } I_p I_z^2 &= I_\tau I_u^2 & I_r I_z^2 &= I_\tau I_v^2 & \text{KCL: } I_p + I_r + I_c &= I_\tau + I_q \\ I_q I_z^2 &= I_\tau I_u I_{2v} \end{aligned}$$





# Dynamic MITE Network Synthesis: **RMS-to-DC Converter**

$$\begin{aligned} \text{TLP: } I_p I_z^2 &= I_\tau I_u^2 & I_r I_z^2 &= I_\tau I_v^2 & \text{KCL: } I_p + I_r + I_c &= I_\tau + I_q \\ I_q I_z^2 &= I_\tau I_u I_{2v} \end{aligned}$$

