Teaching Microelectronics at Olin College

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Mixed Analog-Digital VLSI Circuits & Systems Lab
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Olin College of Engineering

- Location: Needham, Massachusetts
- Recent: Chartered in 1997, Inaugural class 2006
- Small: 350 students total, 43 full-time faculty
- 3 degree programs: ECE, ME, and E (undergrad only)
- 80% of curriculum common among the 3 programs
- Faculty not in departments and not tenured

Olin is dedicated to discovering and developing new and effective approaches to educating engineers for the future and aspires to serve as a model for other institutions around the world.

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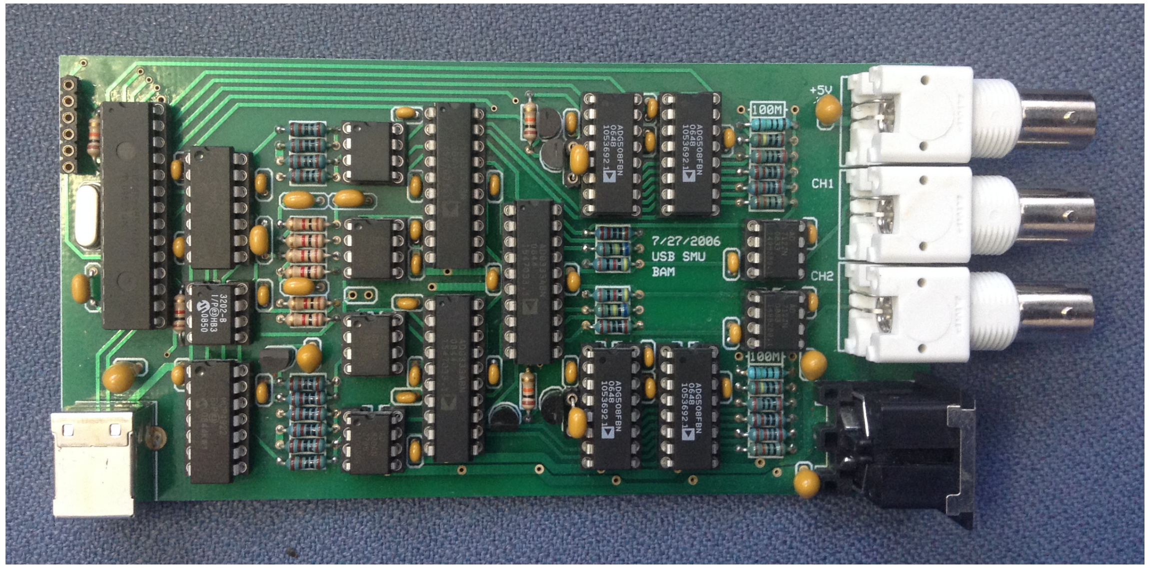
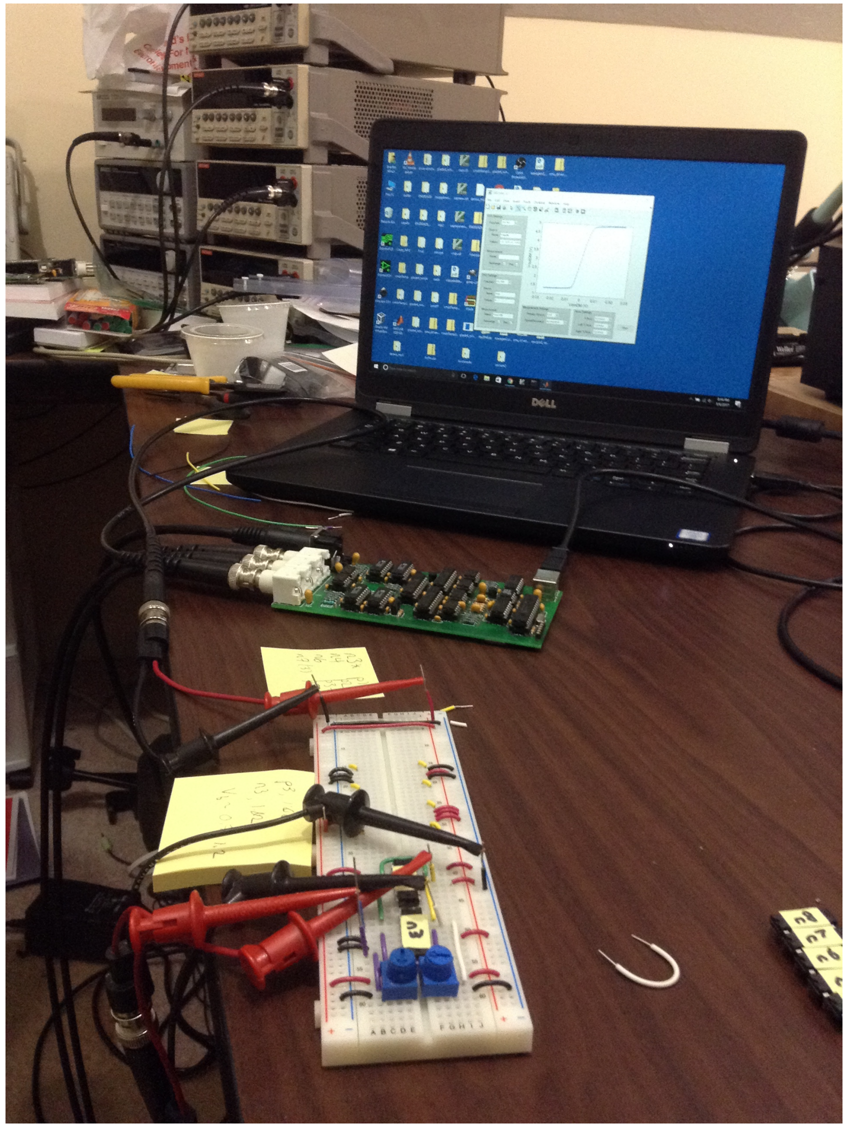
DIY Two-Channel USB Source/Measure Unit

Features:

- 2 channels, SV/MI or SI/MV functions
- 3 voltage ranges: $\pm 10\,\mathrm{V}$, $\pm 4\,\mathrm{V}$, $\pm 2\,\mathrm{V}$
- 6 current ranges: $\pm 20\,\mathrm{mA}$, $\pm 2\,\mathrm{mA}$, $\ldots$, $\pm 2\,\mu\mathrm{A}$, $\pm 200\,\mathrm{nA}$
- Accuracy: 1%
- Resolution: 13 bits (source), 12 bits (measure), $1\,\mathrm{nA}$ (current)
- Speed: 60 readings/second
- Software: Matlab GUIs, Matlab and Python APIs





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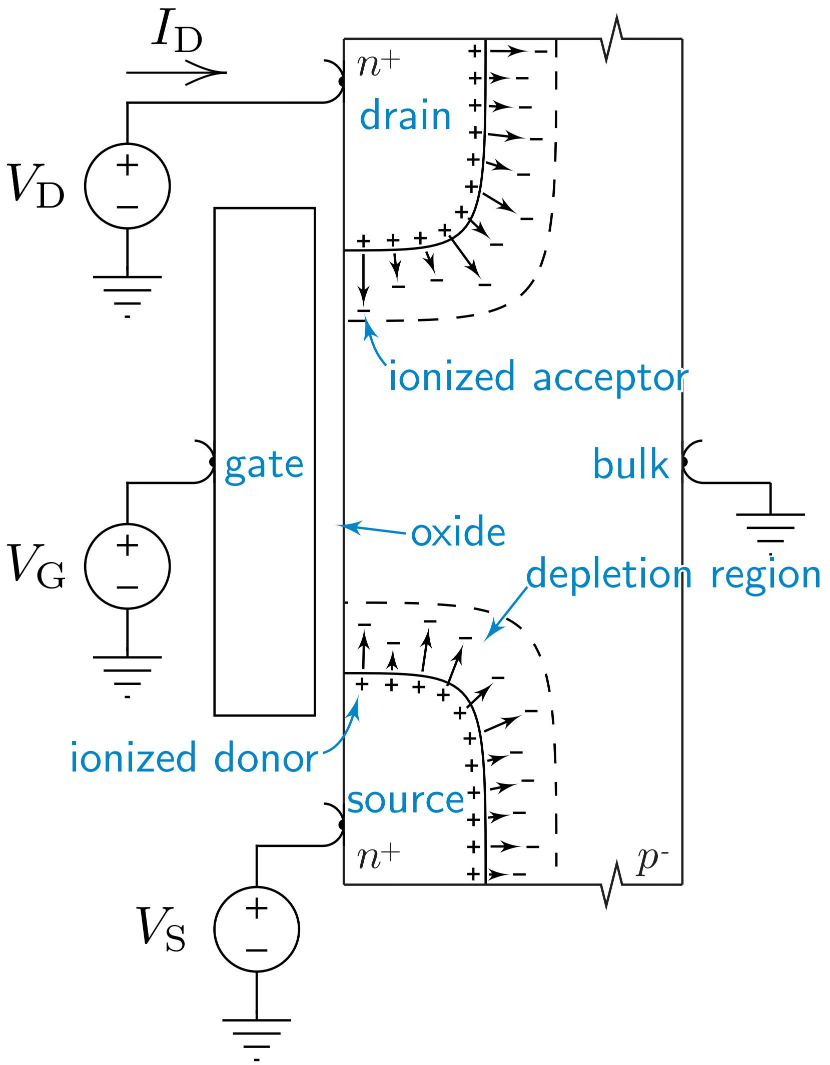
Treatment of MOS Transistors/Circuits

Guiding principles:

- Weak inversion (WI) and moderate inversion (MI) have become as important as strong inversion (SI)—we should treat them right from the start.
- MOS transistors have ohmic and saturation regions at all inversion levels—our terminology should reflect that fact.
- MOS transistors are (quantitatively) source/drain symmetric devices—our models and circuit symbols should be too.
- In bulk CMOS, the body effect is *never* negligible.
- Simplified MOS transistor model should connect directly to SPICE models.
- Whenever possible, develop principles that hold across all inversion levels (e.g., the MOS current divider principle).
- Compare/contrast the behavior of circuits across inversion levels.



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MOS Transistor: Long-Channel EKV Model

We model the drain current of an *n*MOS transistor as the difference between a *forward current* and a *reverse current*,

$$I_{\mathrm{D}} = I_{\mathrm{F}} - I_{\mathrm{R}},$$

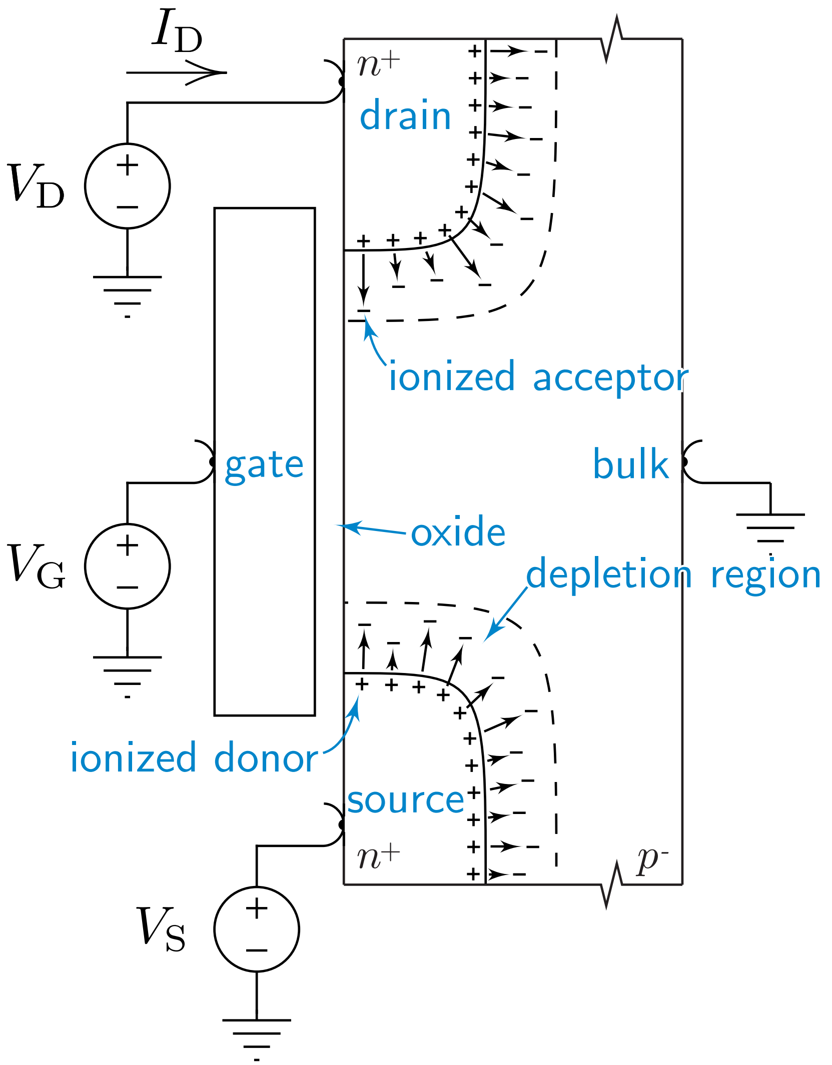
which are given by

$$I_{\mathrm{F(R)}} = SI_{\mathrm{s}} \log^2\left(1 + e^{\left(\kappa(V_{\mathrm{G}} - V_{\mathrm{T0}}) - V_{\mathrm{S(D)}}\right)/2U_{\mathrm{T}}}\right),$$

where

$$S = \frac{W}{L},\ I_{\mathrm{s}} = \frac{2\mu C_{\mathrm{ox}} U_{\mathrm{T}}^2}{\kappa},\ U_{\mathrm{T}} = \frac{kT}{q},\ \text{and}\ \kappa = \frac{C_{\mathrm{ox}}}{C_{\mathrm{ox}} + C_{\mathrm{dep}}}.$$

Note that SI_{s} is about twice the saturation current at threshold. This simple model covers all normal regions of MOS transistor operation.



MOS Transistor: Saturation Current

Saturation: $I_\mathrm{F} \gg I_\mathrm{R}$ so $I_\mathrm{D} \approx I_\mathrm{F} = I_\mathrm{sat}$

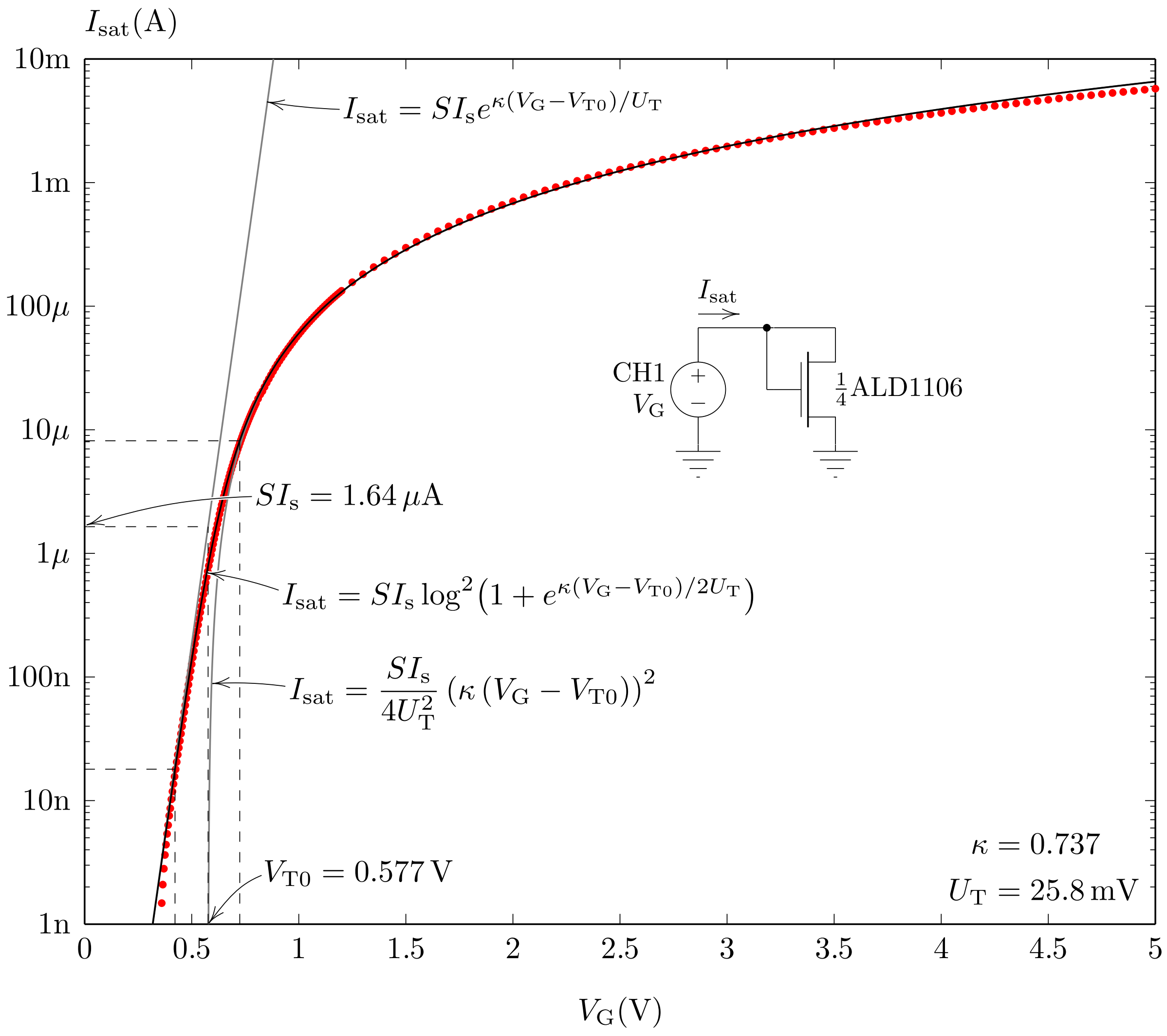
$$I_\mathrm{sat} = SI_\mathrm{s} \log^2\left(1 + e^{(\kappa(V_\mathrm{G} - V_\mathrm{T0}) - V_\mathrm{S})/2U_\mathrm{T}}\right)$$

$$\approx \begin{cases} SI_\mathrm{s} e^{(\kappa(V_\mathrm{G} - V_\mathrm{T0}) - V_\mathrm{S})/U_\mathrm{T}} & \text{in WI} \\ \dfrac{SI_\mathrm{s}}{4U_\mathrm{T}^2}\left(\kappa\left(V_\mathrm{G} - V_\mathrm{T0}\right) - V_\mathrm{S}\right)^2 & \text{in SI} \end{cases}$$

WI: $\kappa\left(V_\mathrm{G} - V_\mathrm{T0}\right) - V_\mathrm{S} < -8U_\mathrm{T}$ and $I_\mathrm{sat} \ll SI_\mathrm{s}$

MI: $\left|\kappa\left(V_\mathrm{G} - V_\mathrm{T0}\right) - V_\mathrm{S}\right| \leq 8U_\mathrm{T}$ and $I_\mathrm{sat} \approx SI_\mathrm{s}$

SI: $\kappa\left(V_\mathrm{G} - V_\mathrm{T0}\right) - V_\mathrm{S} > 8U_\mathrm{T}$ and $I_\mathrm{sat} \gg SI_\mathrm{s}$



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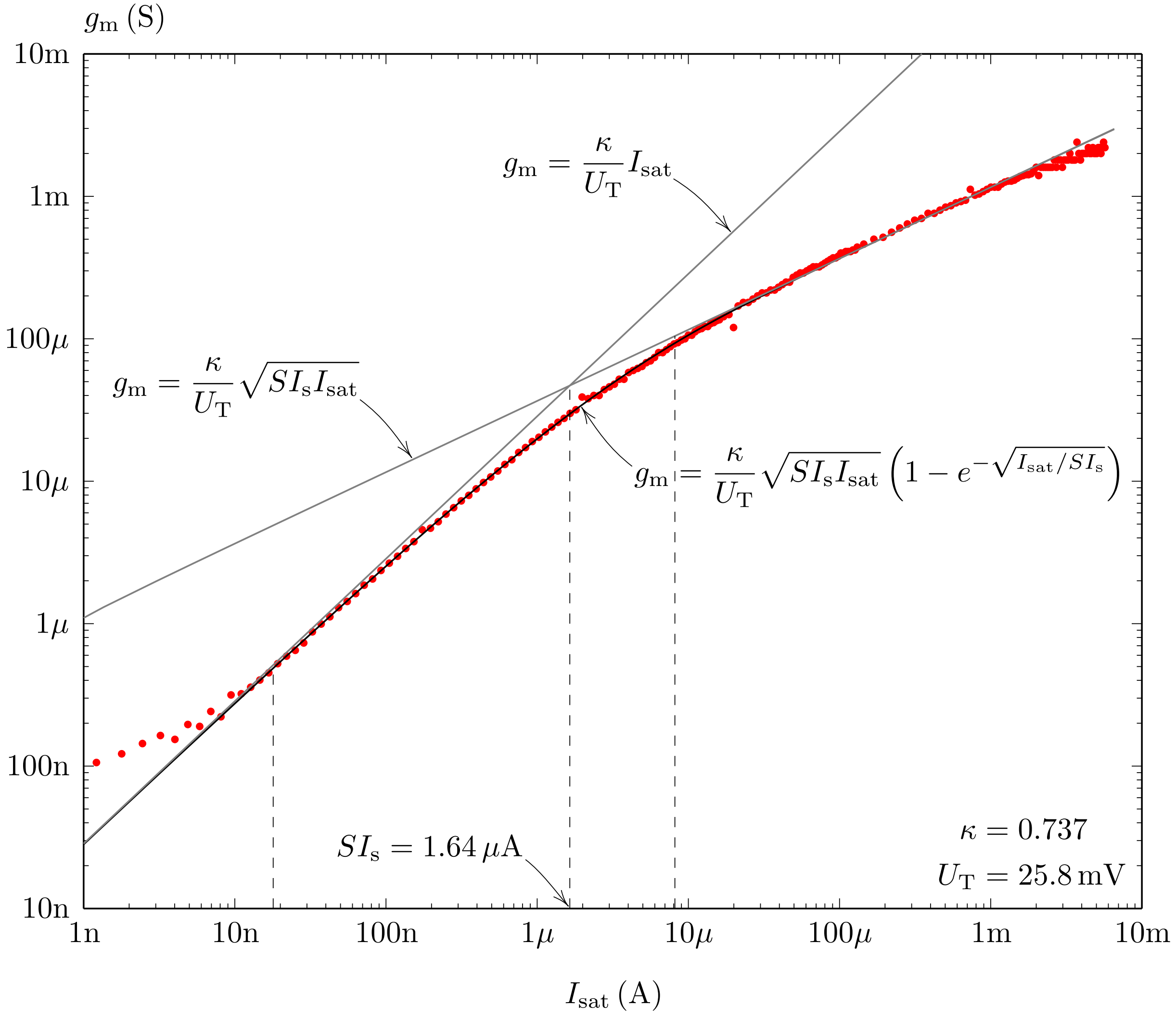
MOS Transistor: Transconductance Gain

Transconductance:

$$\begin{aligned} g_{\mathrm{m}} &= \frac{\partial I_{\mathrm{sat}}}{\partial V_{\mathrm{G}}} \\ &= \frac{\kappa}{U_{\mathrm{T}}}\sqrt{SI_{\mathrm{s}}I_{\mathrm{sat}}}\left(1 - e^{-\sqrt{I_{\mathrm{sat}}/SI_{\mathrm{s}}}}\right) \\ &\approx \begin{cases} \dfrac{\kappa}{U_{\mathrm{T}}} I_{\mathrm{sat}} & \text{in WI (i.e., } I_{\mathrm{sat}} \ll SI_{\mathrm{s}}) \\ \dfrac{\kappa}{U_{\mathrm{T}}}\sqrt{SI_{\mathrm{s}}I_{\mathrm{sat}}} & \text{in SI (i.e., } I_{\mathrm{sat}} \gg SI_{\mathrm{s}}) \end{cases} \end{aligned}$$

Transconductance/source conductance relation at all inversion levels:

$$g_{\mathrm{m}} = \kappa g_{\mathrm{s}}$$



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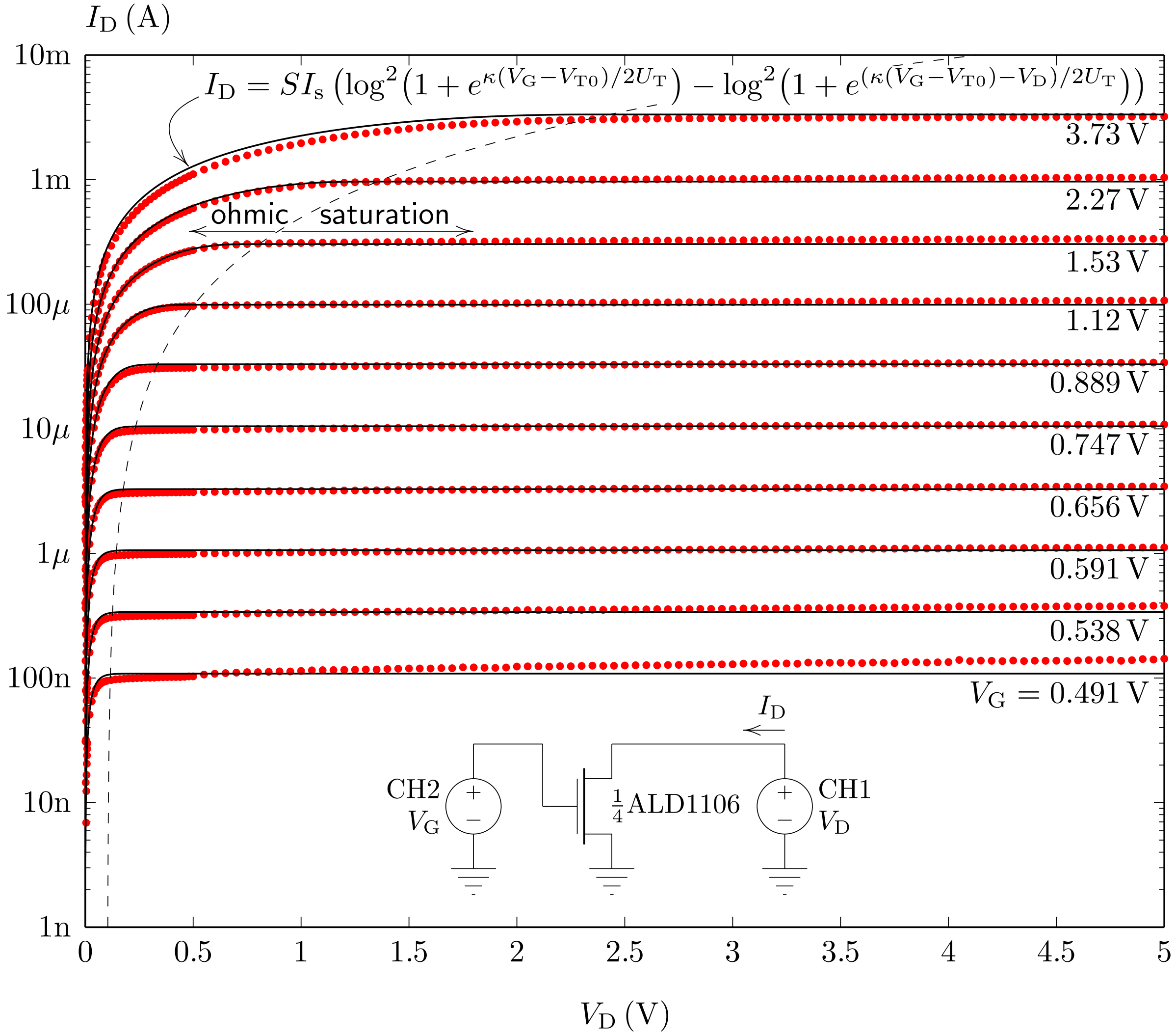
MOS Transistor: Drain Characterisitcs

Ohmic/saturation boundary:

$$V_{\mathrm{DSsat}} = 2U_{\mathrm{T}}\left(2 + \sqrt{\frac{I_{\mathrm{sat}}}{SI_{\mathrm{s}}}}\right)$$

$$\approx \begin{cases} 4U_{\mathrm{T}} & \text{in WI (i.e., } I_{\mathrm{sat}} \ll SI_{\mathrm{s}}) \\ 2U_{\mathrm{T}}\sqrt{\dfrac{I_{\mathrm{sat}}}{SI_{\mathrm{s}}}} & \text{in SI (i.e., } I_{\mathrm{sat}} \gg SI_{\mathrm{s}}) \end{cases}$$



Deep ohmic region: $V_{\mathrm{DS}} \approx 0\,\mathrm{V}$ and $I_{\mathrm{F}} \approx I_{\mathrm{R}}$

$$I_{\mathrm{D}} \approx \frac{V_{\mathrm{DS}}}{r_{\mathrm{on}}} = g_{\mathrm{s}} V_{\mathrm{DS}}$$

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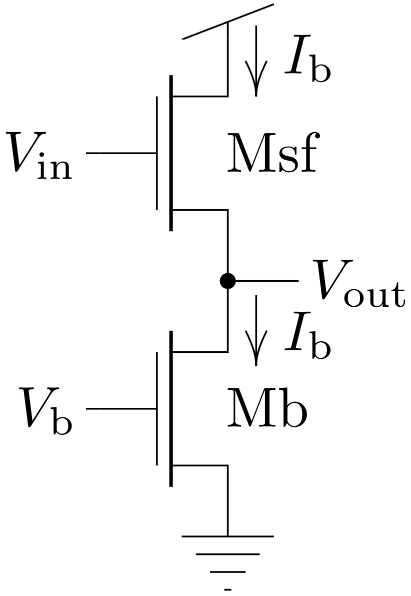
Source Follower Characteristics

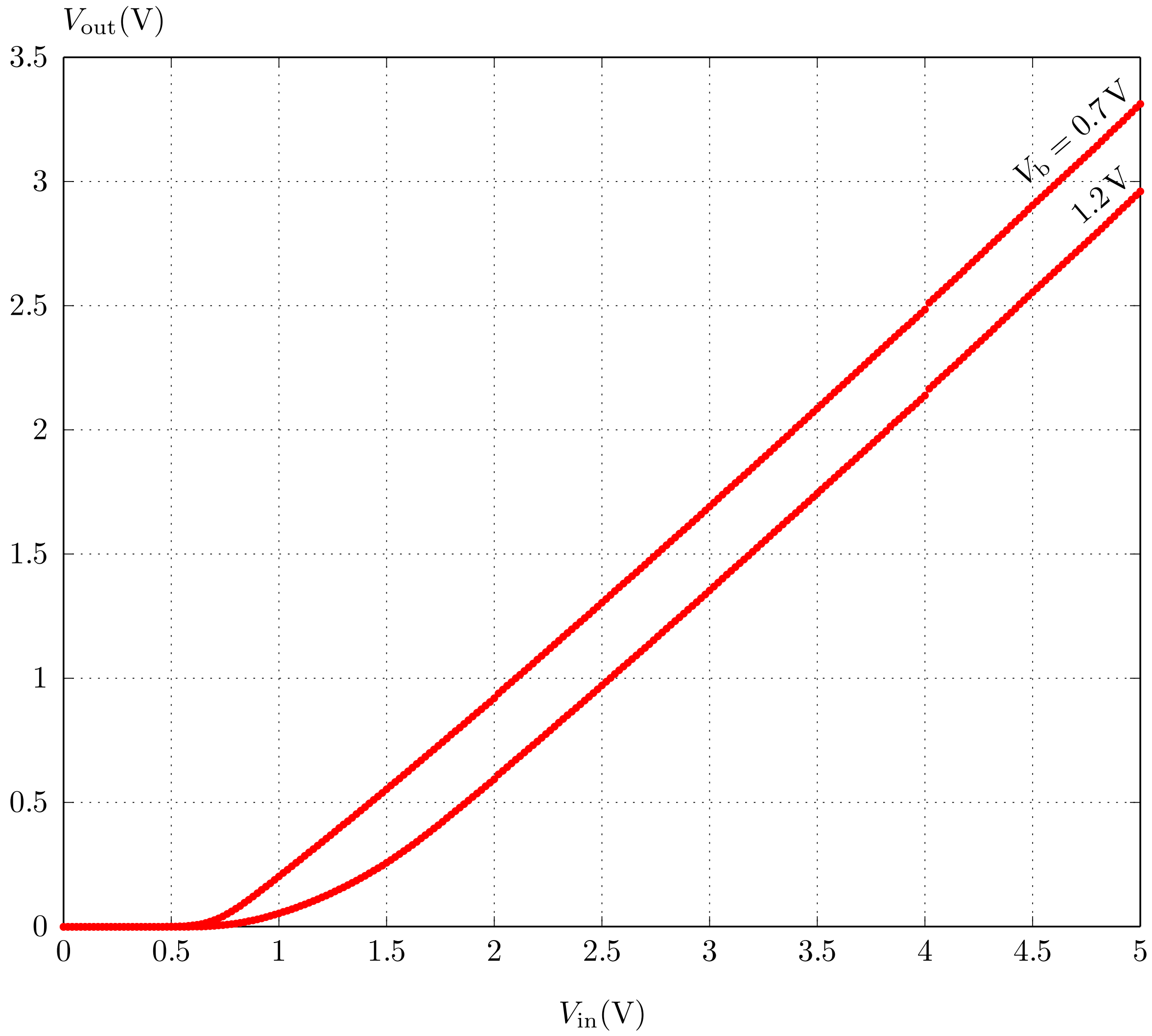


KCL $\Rightarrow I_\mathrm{b} = SI_\mathrm{s}\log^2\left(1+e^{(\kappa(V_\mathrm{in}-V_\mathrm{T0})-V_\mathrm{out})/2U_\mathrm{T}}\right)$

$$= SI_\mathrm{s}\log^2\left(1+e^{\kappa(V_\mathrm{b}-V_\mathrm{T0})/2U_\mathrm{T}}\right)$$

$$\Rightarrow \kappa\left(V_\mathrm{in}-V_\mathrm{T0}\right)-V_\mathrm{out}=\kappa\left(V_\mathrm{b}-V_\mathrm{T0}\right)$$

$$\boxed{V_\mathrm{out}=\kappa\left(V_\mathrm{in}-V_\mathrm{b}\right)\ \text{for } V_\mathrm{in}\geq V_\mathrm{b}+\frac{V_\mathrm{DSsat}}{\kappa}}$$



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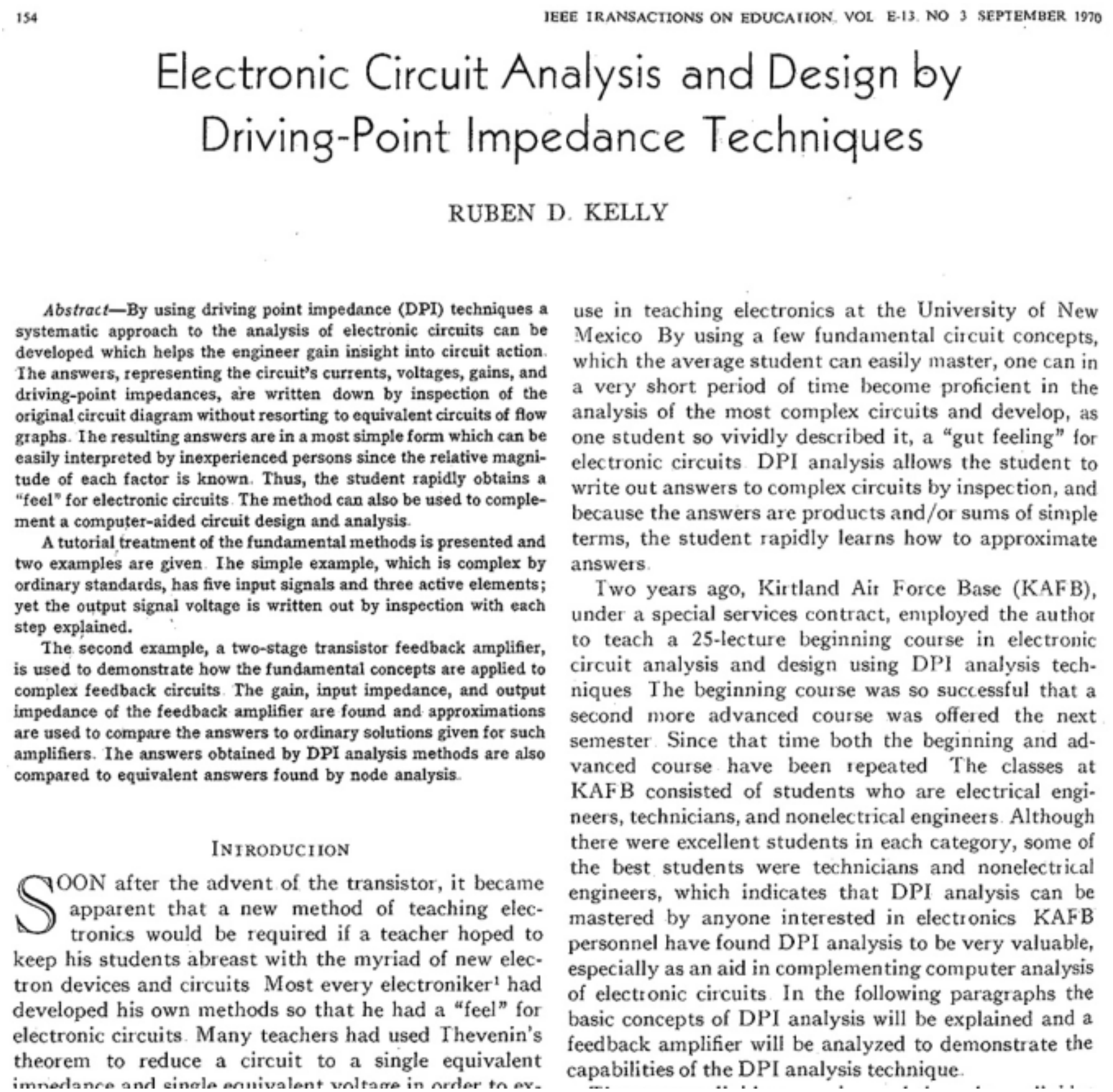
Kelly's Driving-Point Impedance Techniques

Key ideas:

- Apply superposition to divide complex circuit analysis problems into many simpler subproblems.
- Create opportunities to apply superposition where none apparently exist by splitting existing sources (voltage/current source splitting) or by strategically adding new sources (node fixing).

Voltage gain by node fixing: $\boxed{A_\mathrm{v} = G_\mathrm{m} R_\mathrm{out}}$

- G_m and R_out are computed *separately*.
- G_m is computed with the output voltage held fixed, which often disables feedback.
- Finding R_out is often necessary anyhow, saving work.



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Electronic Circuit Analysis and Design by Driving-Point Impedance Techniques

RUBEN D. KELLY

Abstract—By using driving point impedance (DPI) techniques a systematic approach to the analysis of electronic circuits can be developed which helps the engineer gain insight into circuit action. The answers, representing the circuit's currents, voltages, gains, and driving-point impedances, are written down by inspection of the original circuit diagram without resorting to equivalent circuits of flow graphs. The resulting answers are in a most simple form which can be easily interpreted by inexperienced persons since the relative magnitude of each factor is known. Thus, the student rapidly obtains a "feel" for electronic circuits. The method can also be used to complement a computer-aided circuit design and analysis.

A tutorial treatment of the fundamental methods is presented and two examples are given. The simple example, which is complex by ordinary standards, has five input signals and three active elements; yet the output signal voltage is written out by inspection with each step explained.

The second example, a two-stage transistor feedback amplifier, is used to demonstrate how the fundamental concepts are applied to complex feedback circuits. The gain, input impedance, and output impedance of the feedback amplifier are found and approximations are used to compare the answers to ordinary solutions given for such amplifiers. The answers obtained by DPI analysis methods are also compared to equivalent answers found by node analysis.

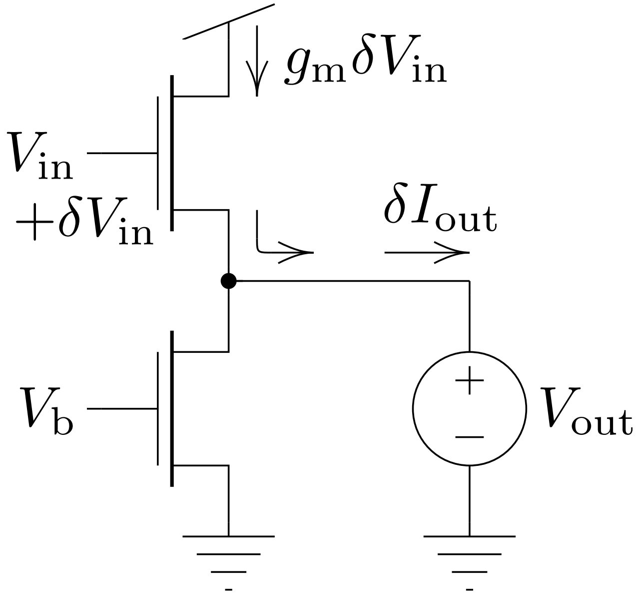
INTRODUCTION

SOON after the advent of the transistor, it became apparent that a new method of teaching electronics would be required if a teacher hoped to keep his students abreast with the myriad of new electron devices and circuits. Most every electroniker[1] had developed his own methods so that he had a "feel" for electronic circuits. Many teachers had used Thevenin's theorem to reduce a circuit to a single equivalent impedance and single equivalent voltage in order to ex-

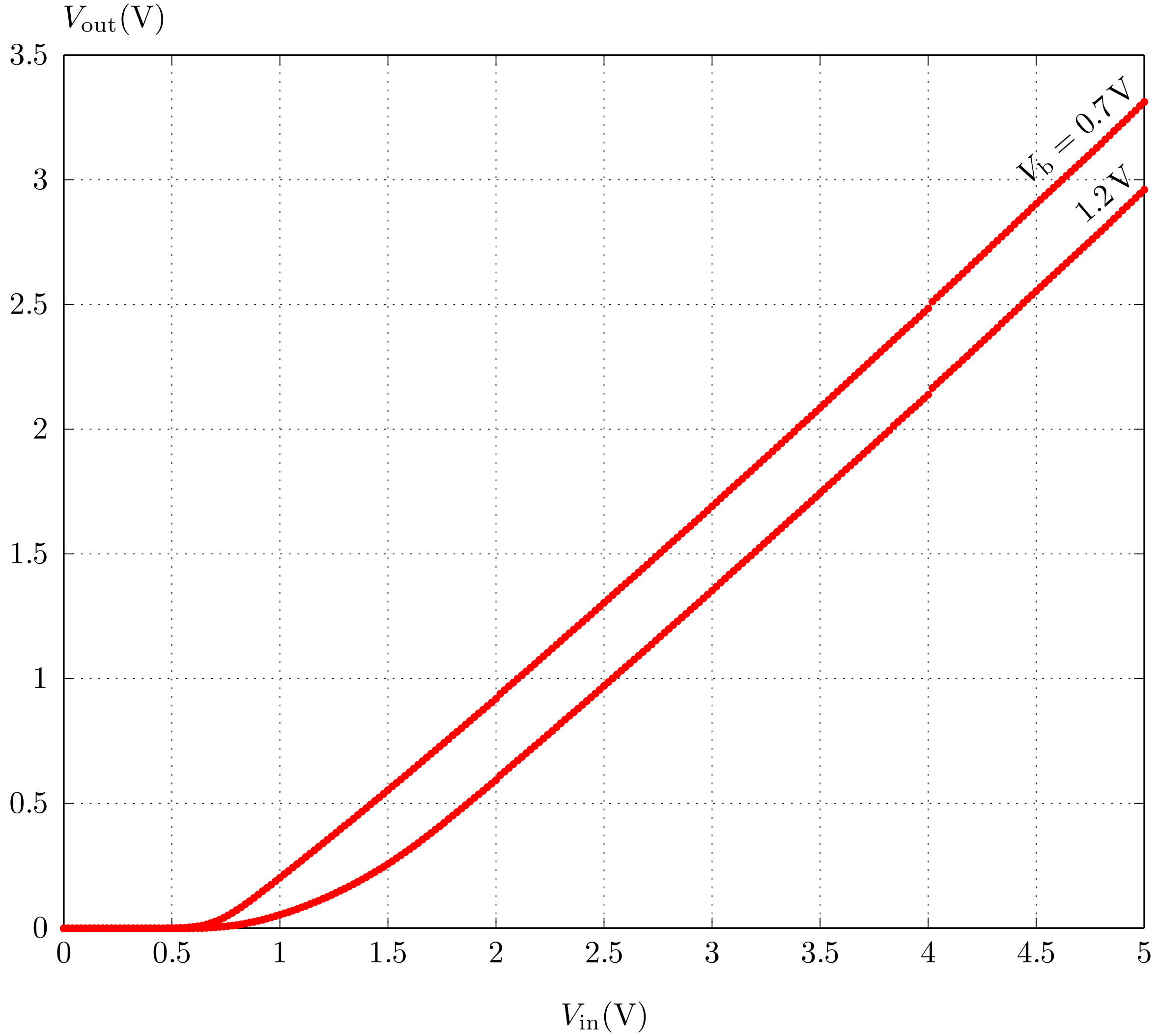
use in teaching electronics at the University of New Mexico. By using a few fundamental circuit concepts, which the average student can easily master, one can in a very short period of time become proficient in the analysis of the most complex circuits and develop, as one student so vividly described it, a "gut feeling" for electronic circuits. DPI analysis allows the student to write out answers to complex circuits by inspection, and because the answers are products and/or sums of simple terms, the student rapidly learns how to approximate answers.

Two years ago, Kirtland Air Force Base (KAFB), under a special services contract, employed the author to teach a 25-lecture beginning course in electronic circuit analysis and design using DPI analysis techniques. The beginning course was so successful that a second more advanced course was offered the next semester. Since that time both the beginning and advanced course have been repeated. The classes at KAFB consisted of students who are electrical engineers, technicians, and nonelectrical engineers. Although there were excellent students in each category, some of the best students were technicians and nonelectrical engineers, which indicates that DPI analysis can be mastered by anyone interested in electronics. KAFB personnel have found DPI analysis to be very valuable, especially as an aid in complementing computer analysis of electronic circuits. In the following paragraphs the basic concepts of DPI analysis will be explained and a feedback amplifier will be analyzed to demonstrate the capabilities of the DPI analysis technique.

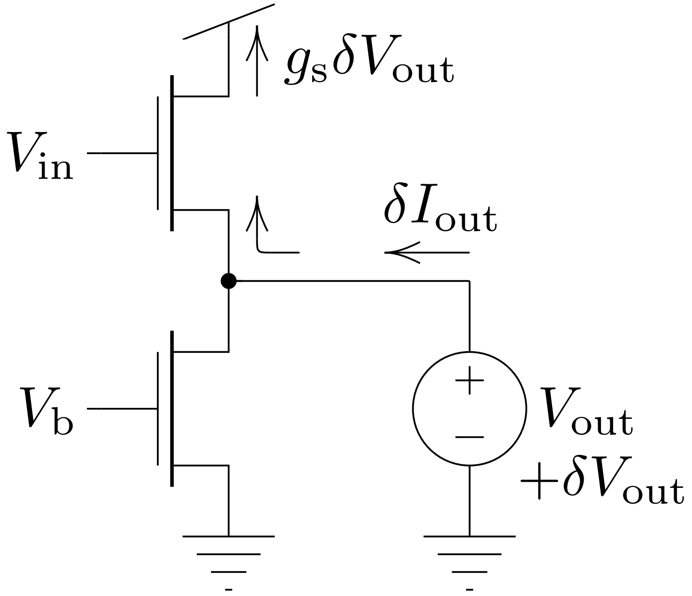
Node Fixing: Source Follower Voltage Gain



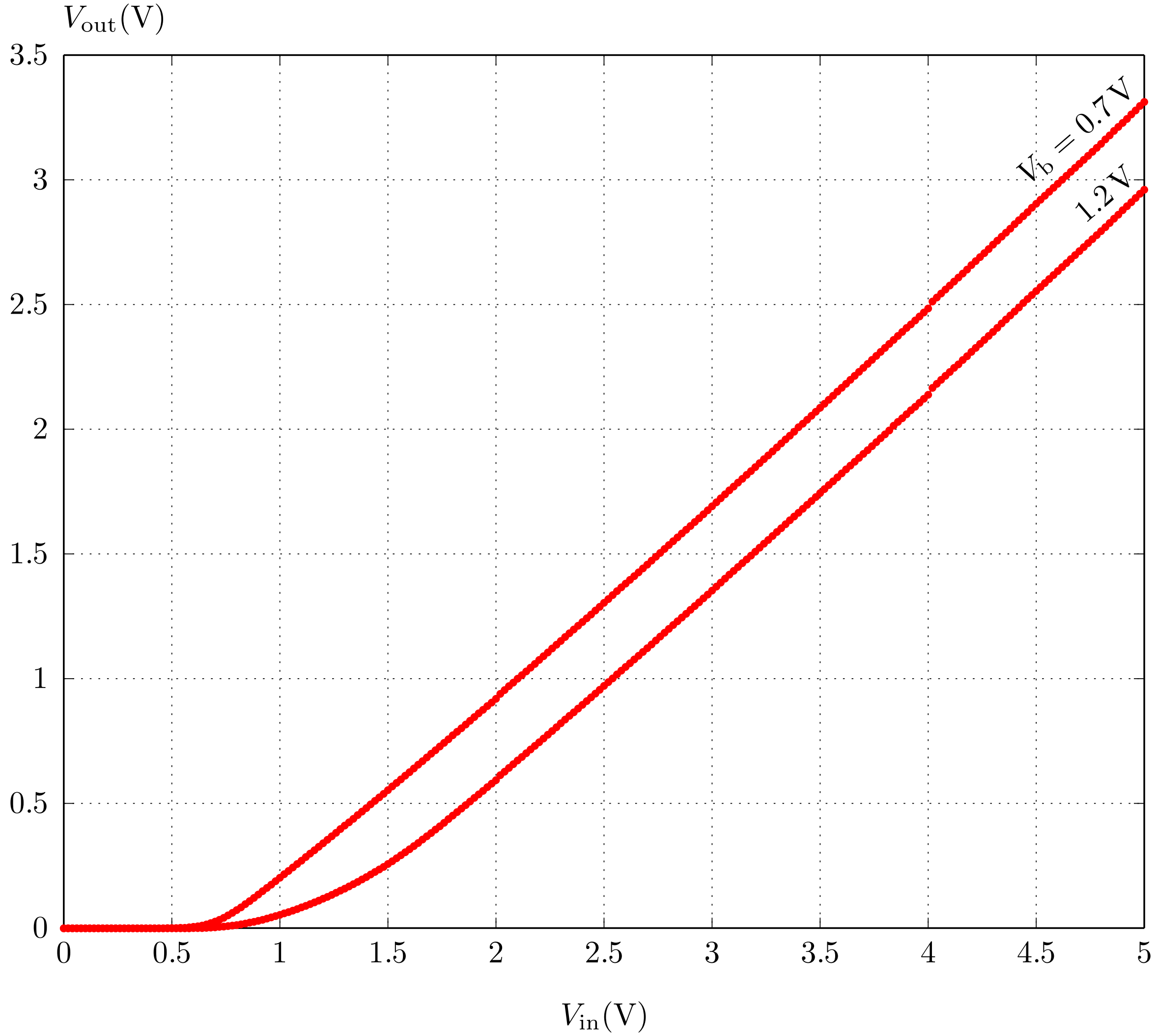
$$G_m = \frac{\delta I_{out}}{\delta V_{in}} = g_m$$



Node Fixing: Source Follower Voltage Gain



$$G_m = \frac{\delta I_{out}}{\delta V_{in}} = g_m \qquad R_{out} = \frac{\delta V_{out}}{\delta I_{out}} = \frac{1}{g_s}$$



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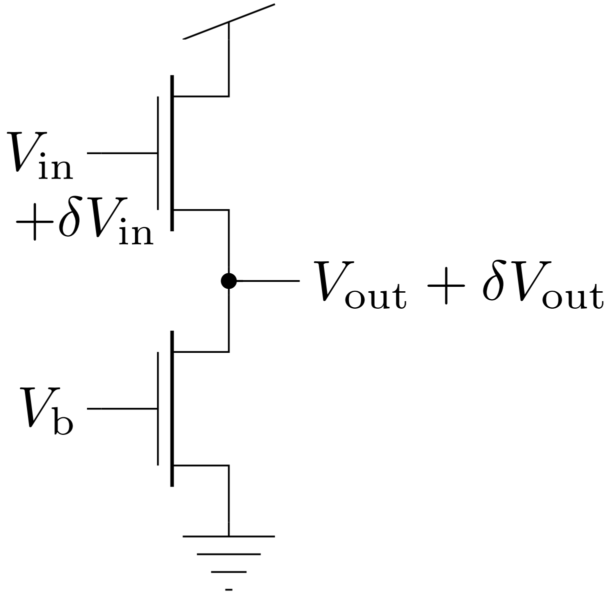
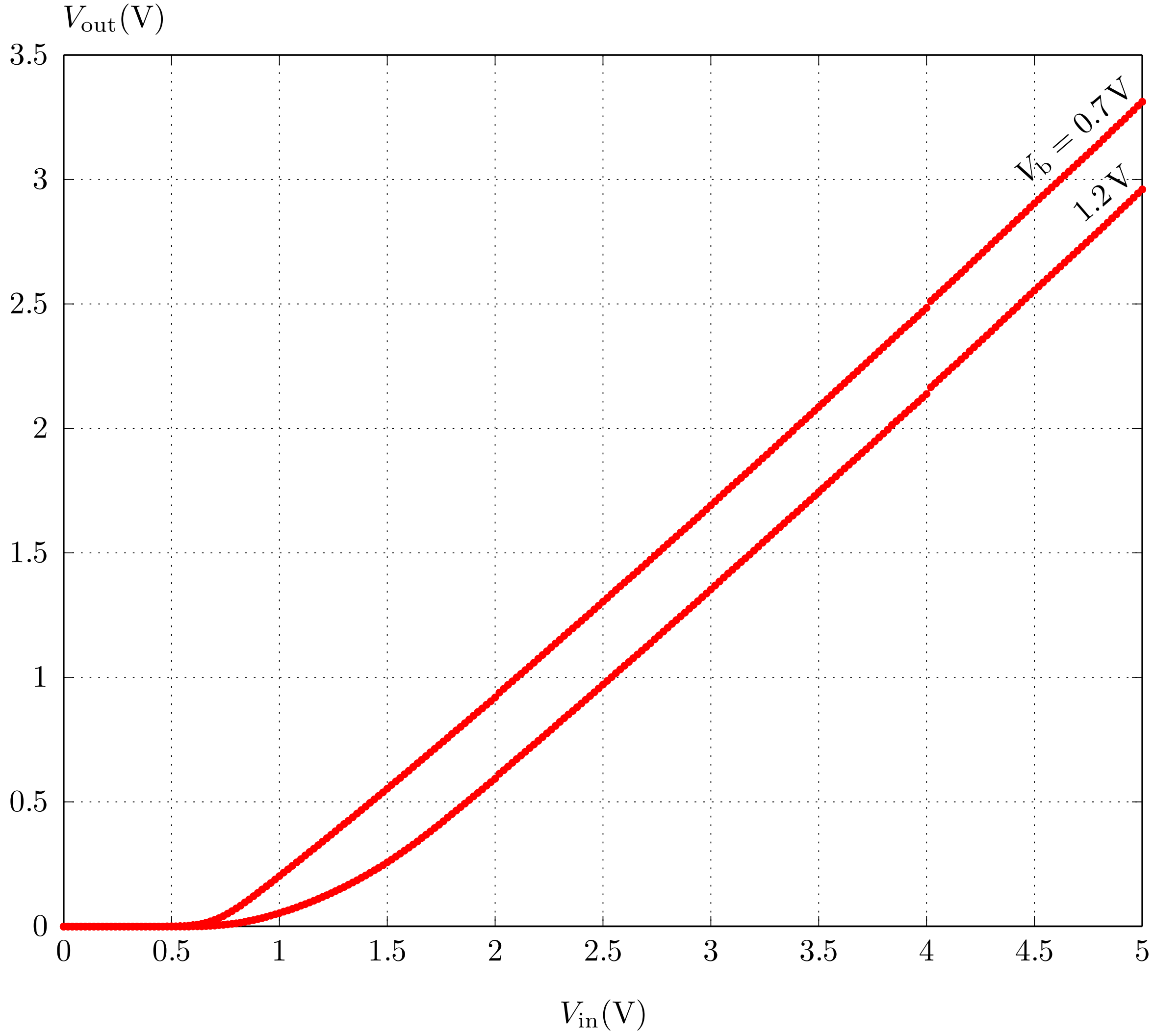
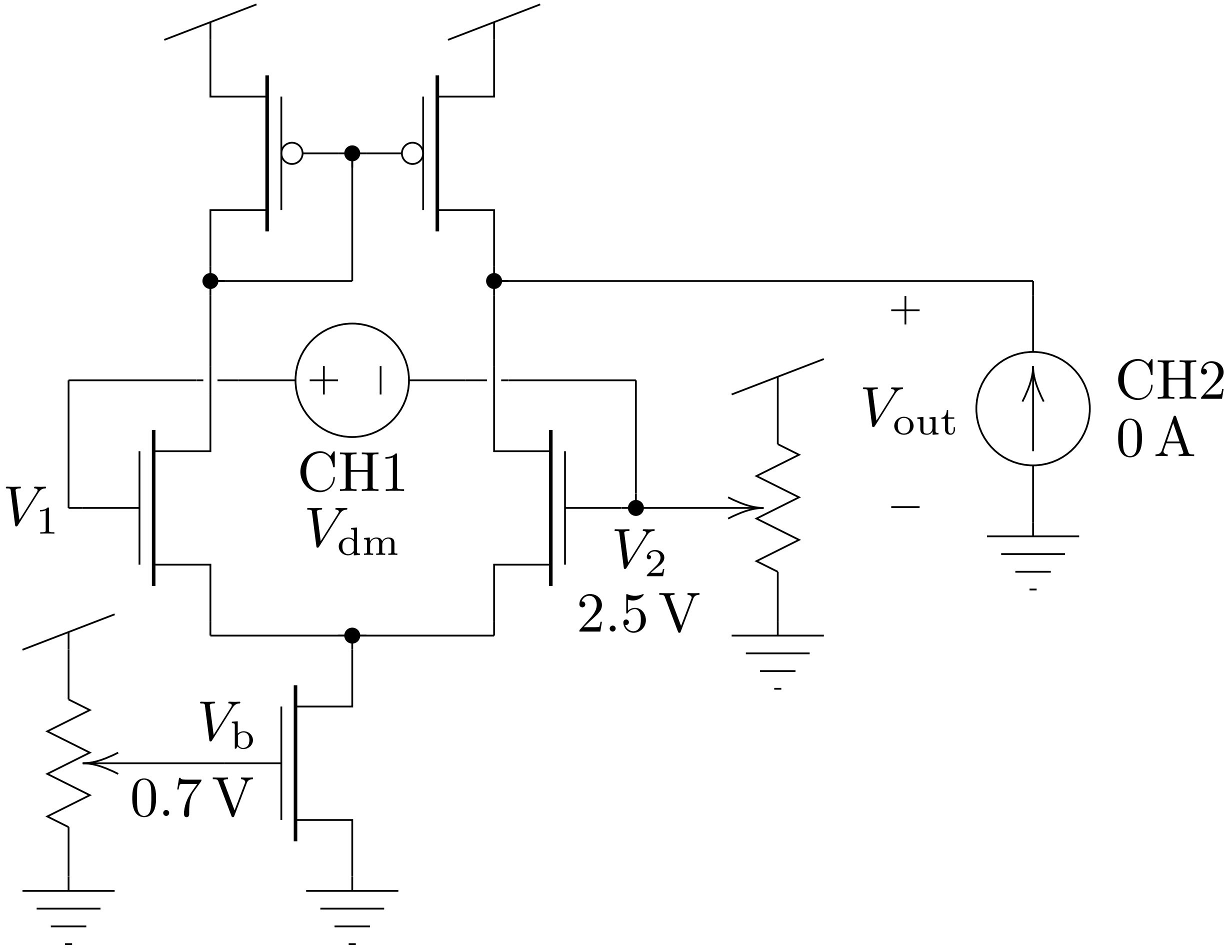


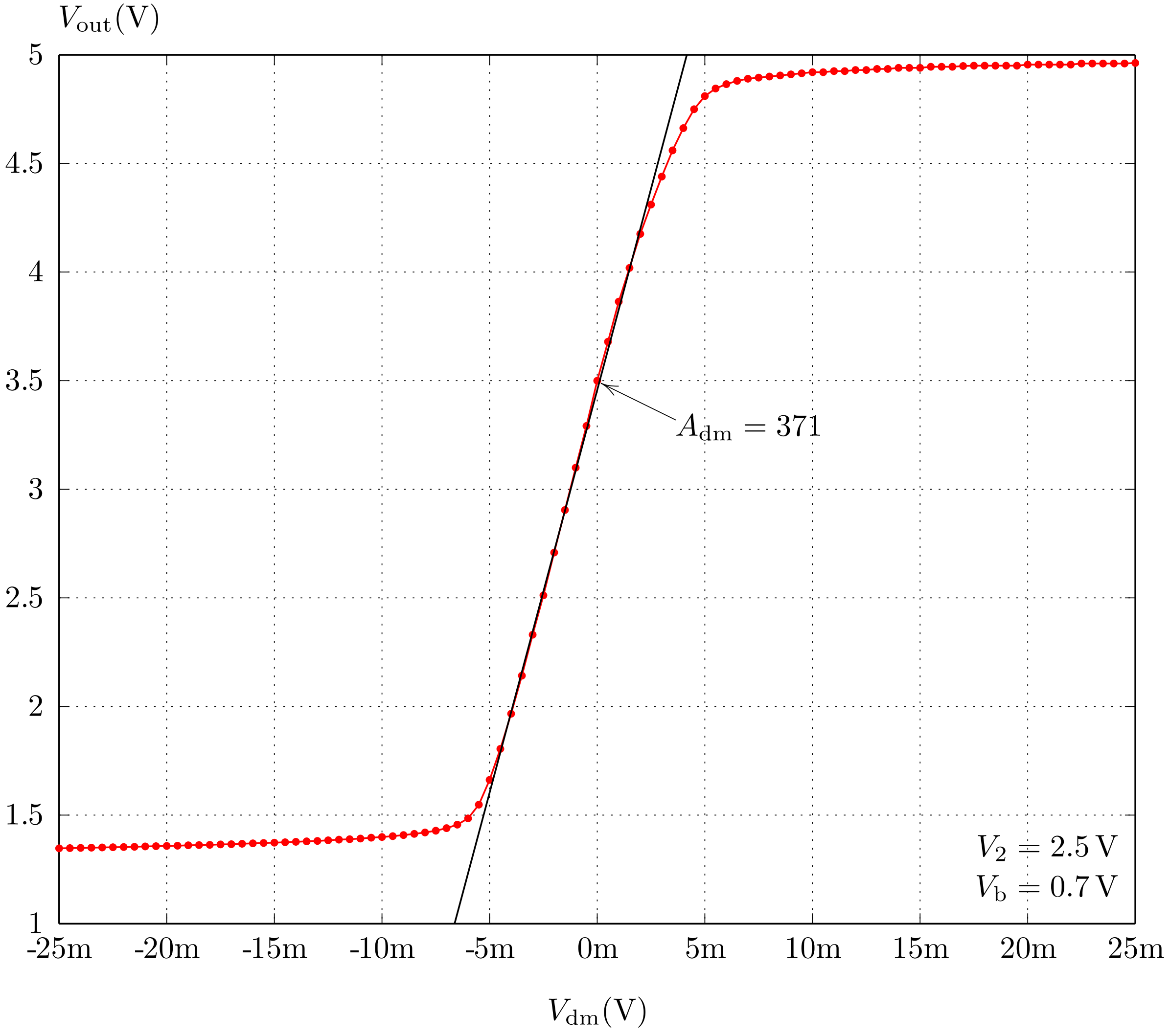
$$G_{\text{m}} = \frac{\delta I_{\text{out}}}{\delta V_{\text{in}}} = g_{\text{m}} \qquad R_{\text{out}} = \frac{\delta V_{\text{out}}}{\delta I_{\text{out}}} = \frac{1}{g_{\text{s}}}$$

$$\boxed{A_{\text{v}} = \frac{\delta V_{\text{out}}}{\delta V_{\text{in}}} = G_{\text{m}} R_{\text{out}} = \frac{g_{\text{m}}}{g_{\text{s}}} = \kappa}$$



Differential Amplifier: Voltage Gain from VTC





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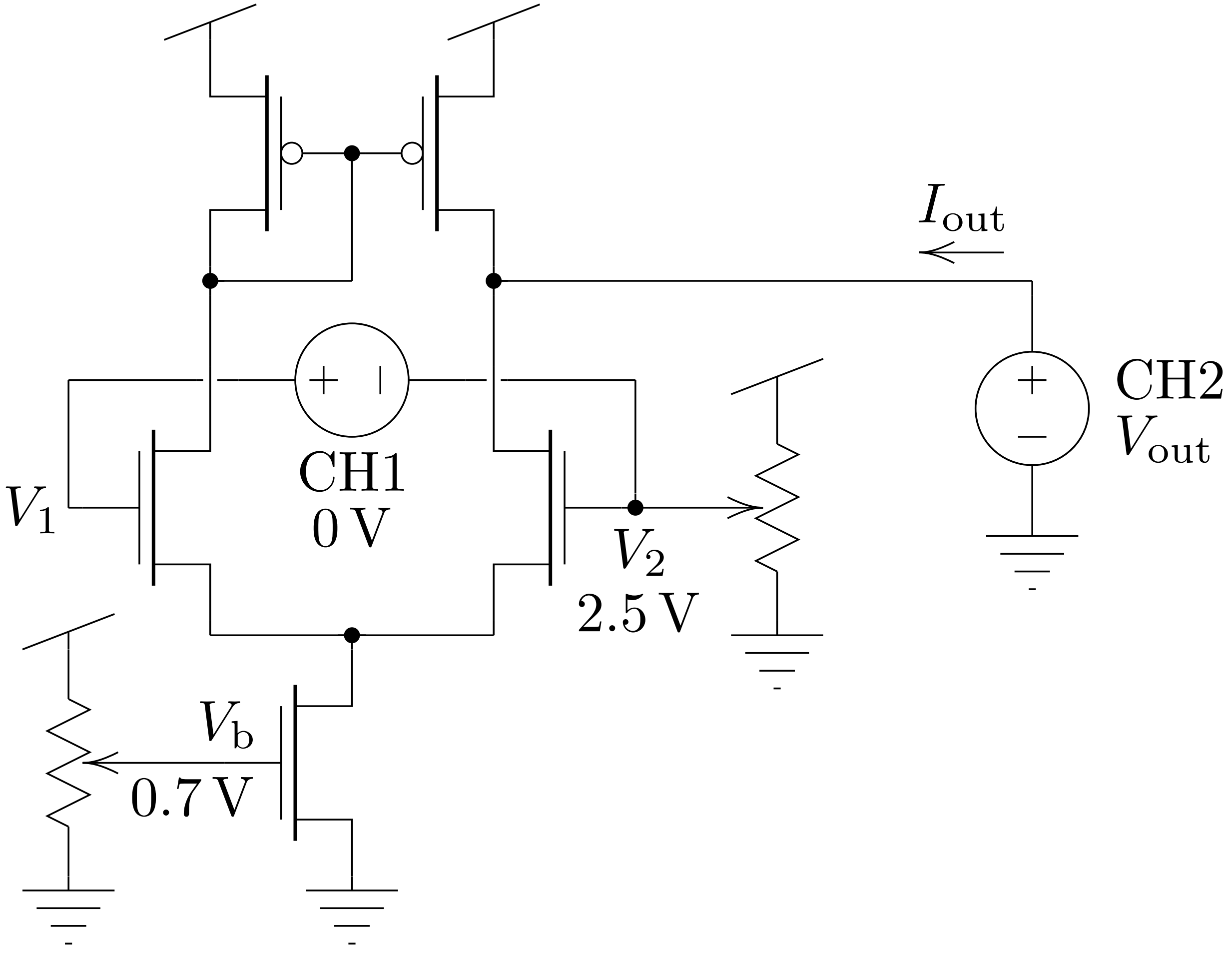
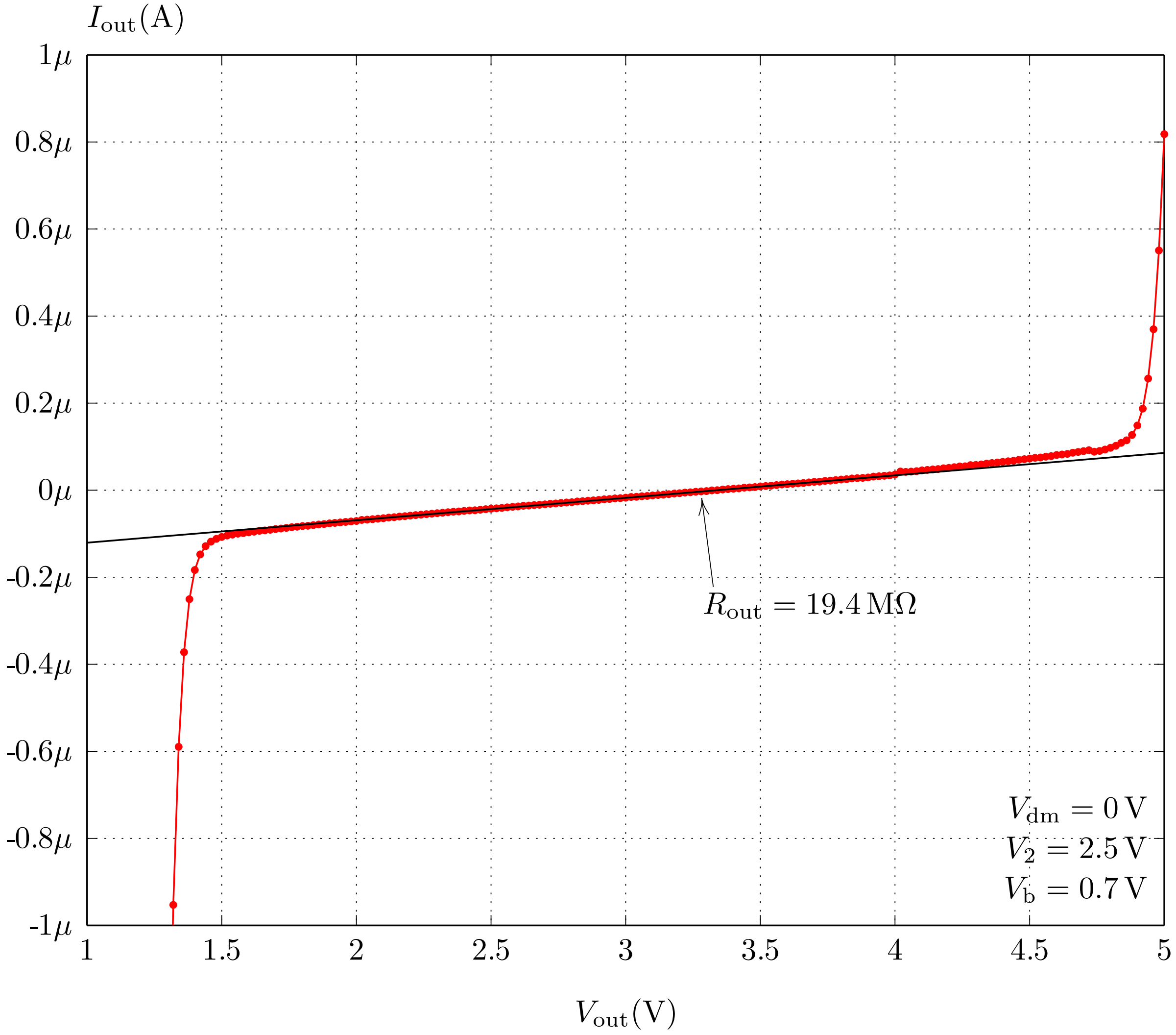
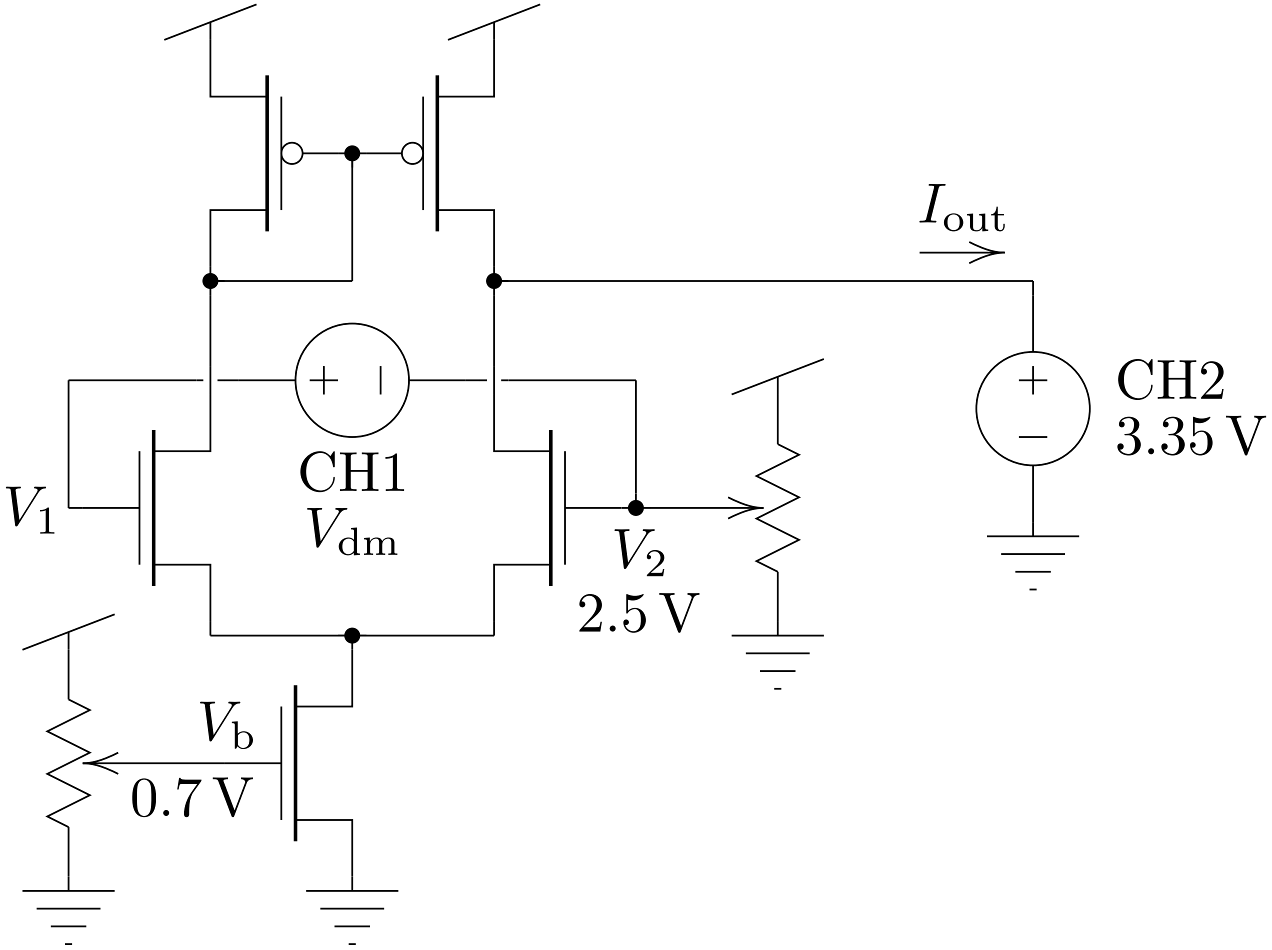
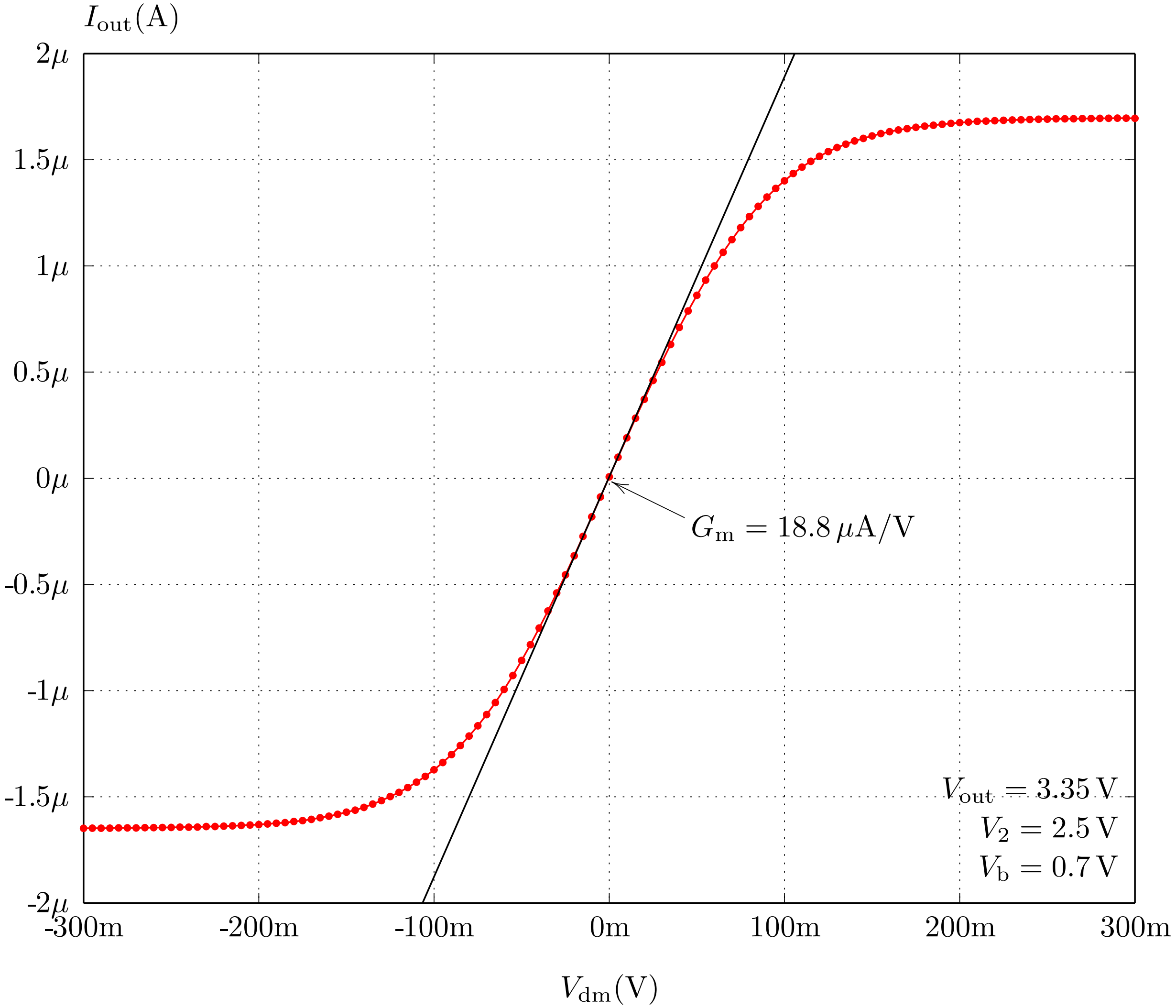
Differential Amplifier: Output Resistance





Differential Amplifier: Transconductance Gain





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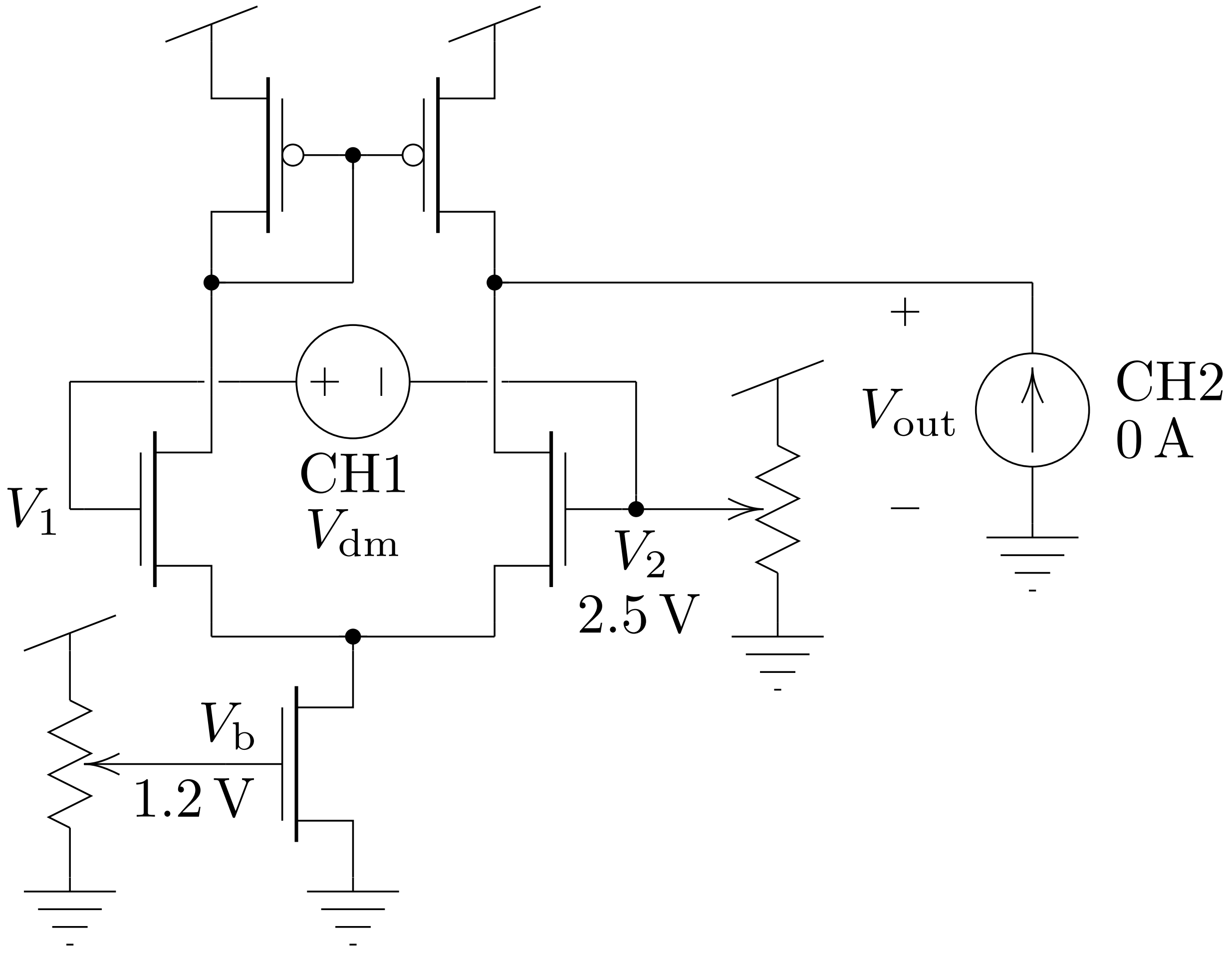
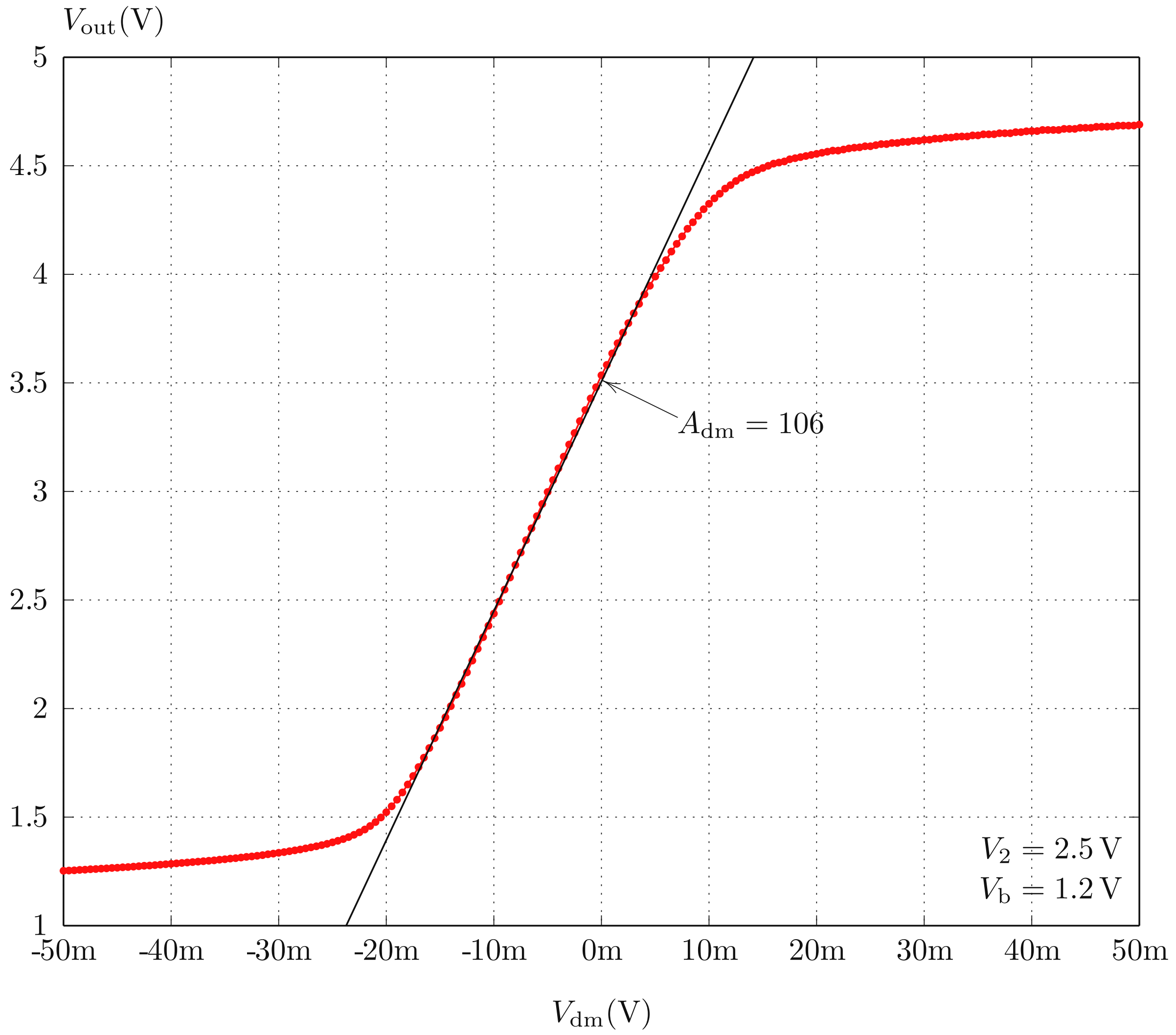
Differential Amplifier: Voltage Gain from VTC





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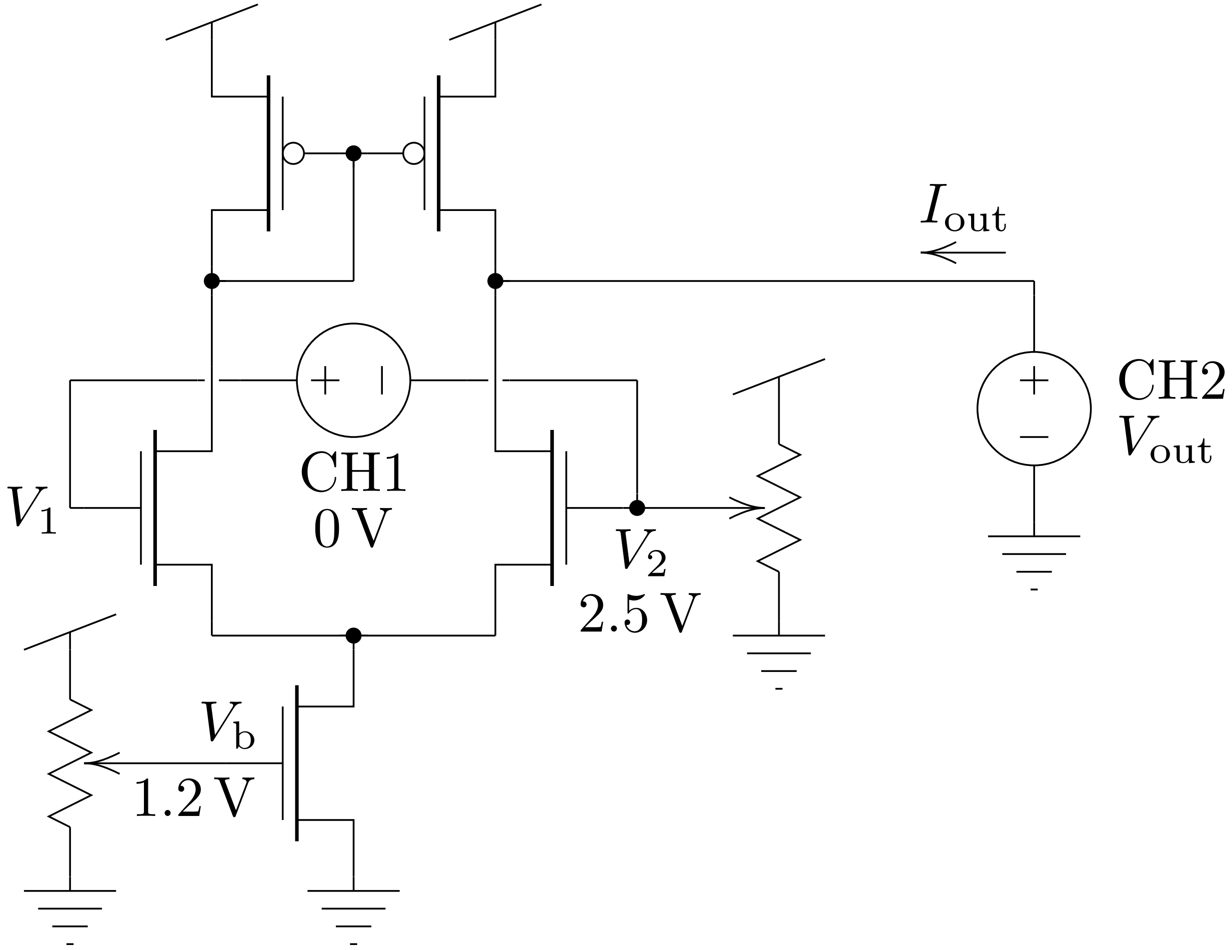
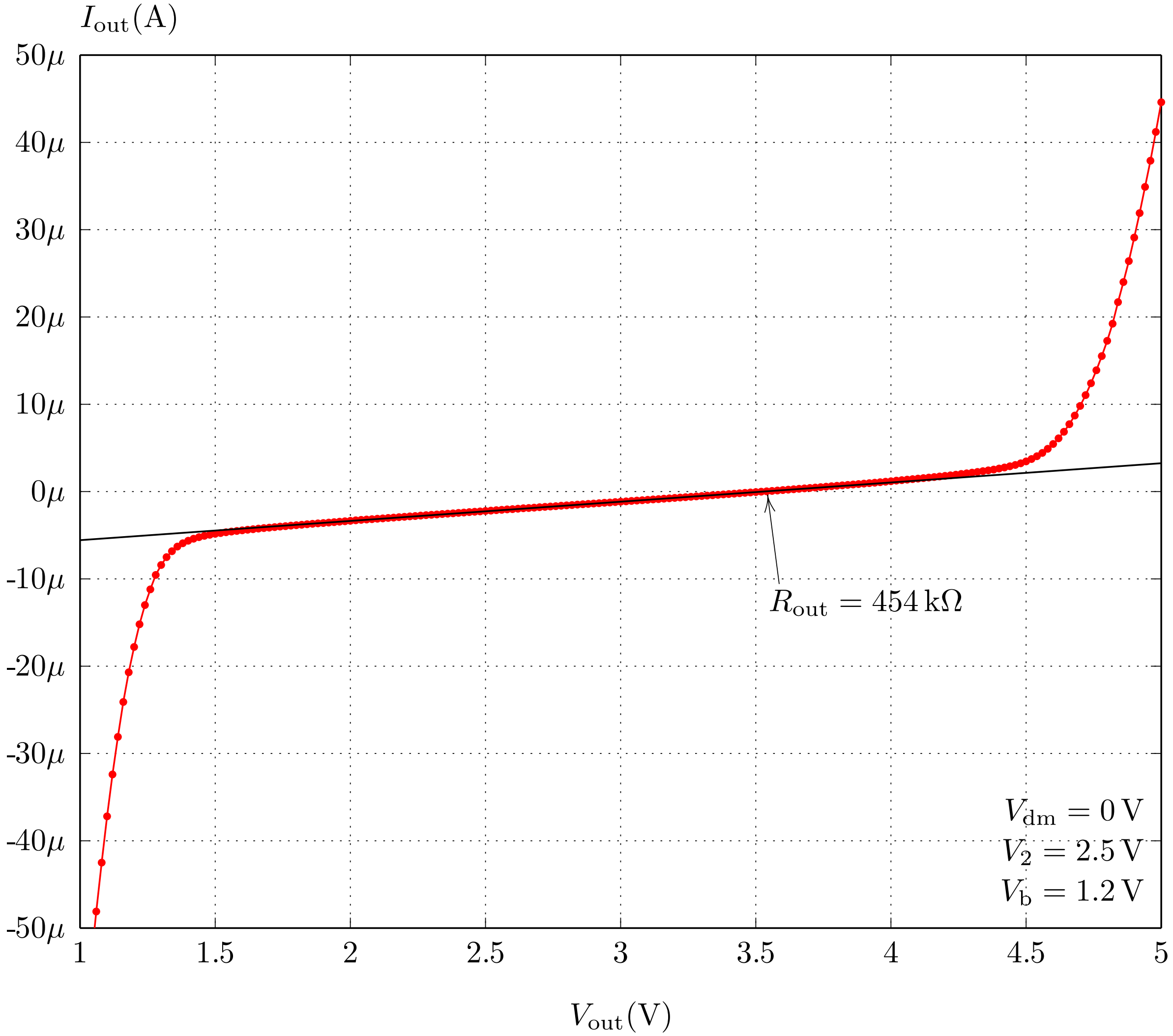
Differential Amplifier: Output Resistance





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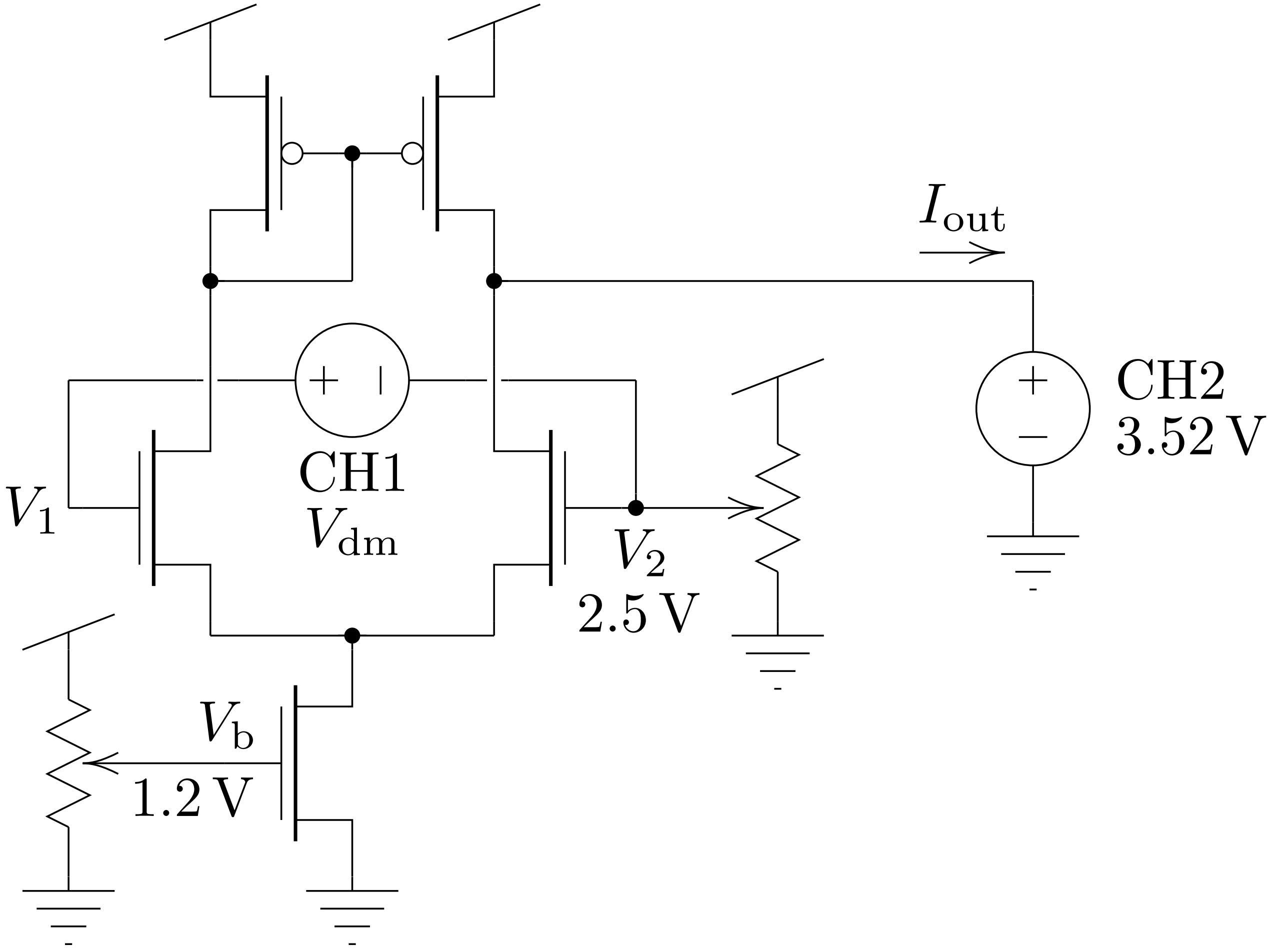
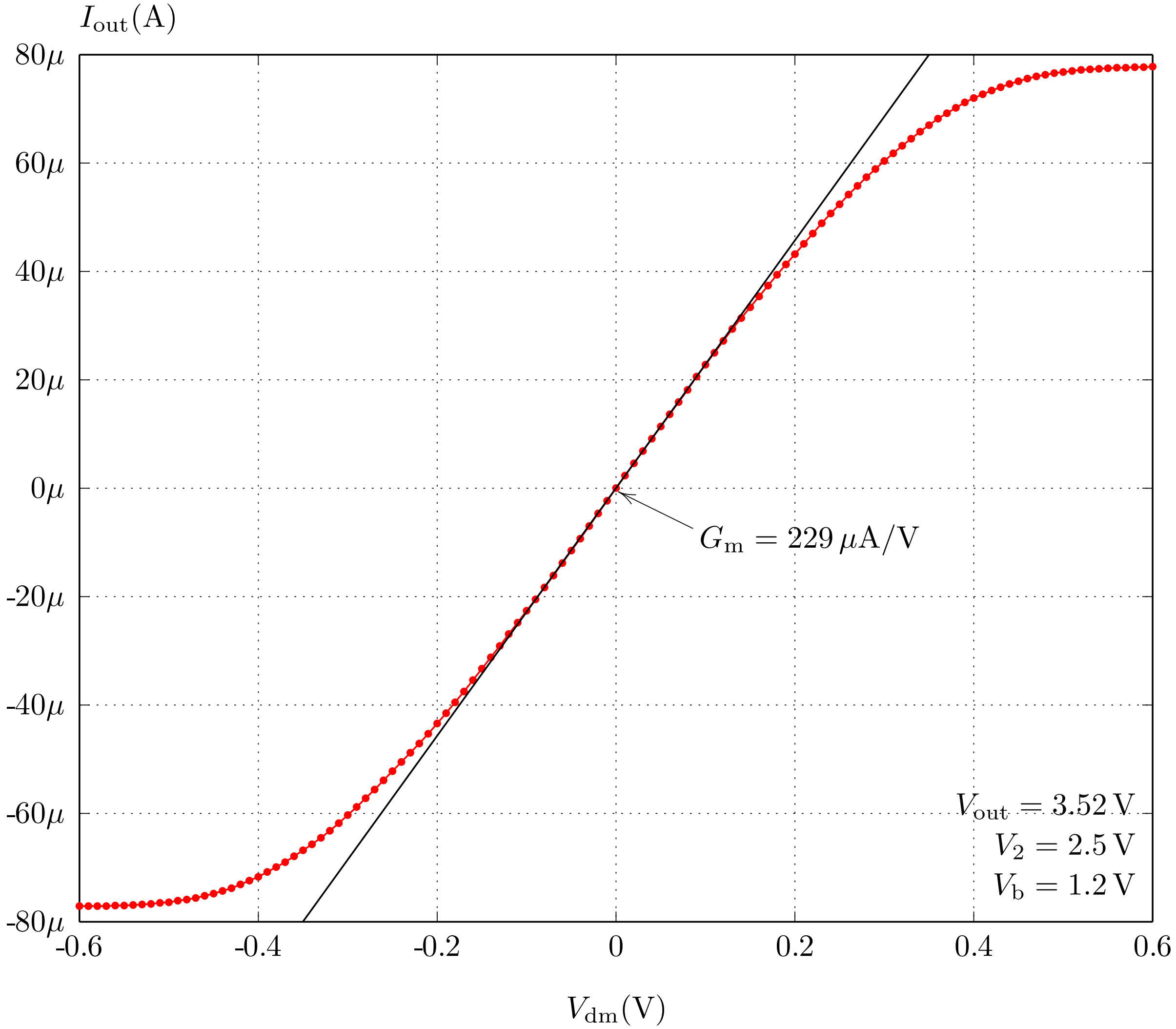
Differential Amplifier: Transconductance Gain





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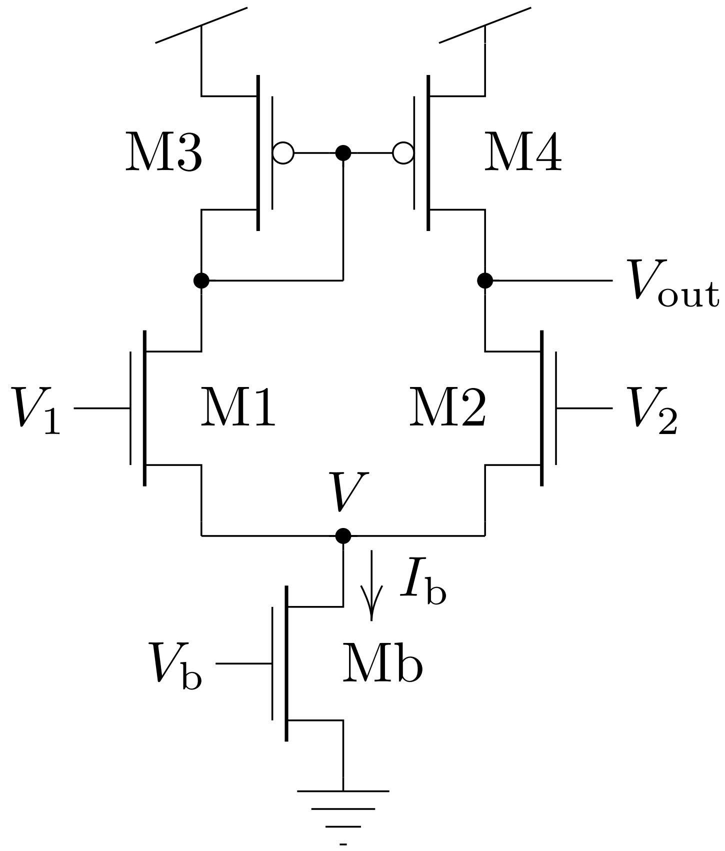
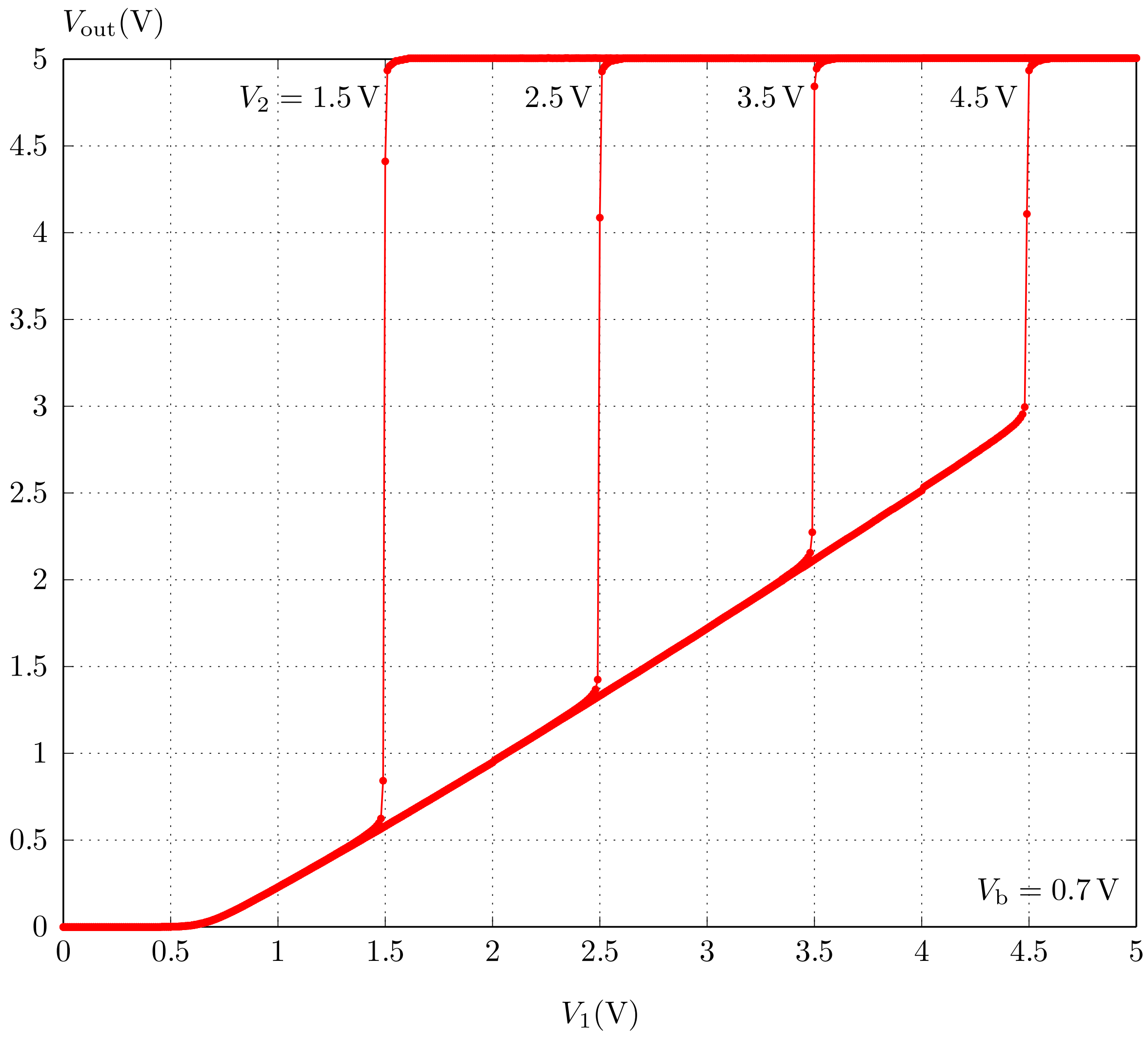
Circuit Reasoning: Explain the Diffamp's VTC





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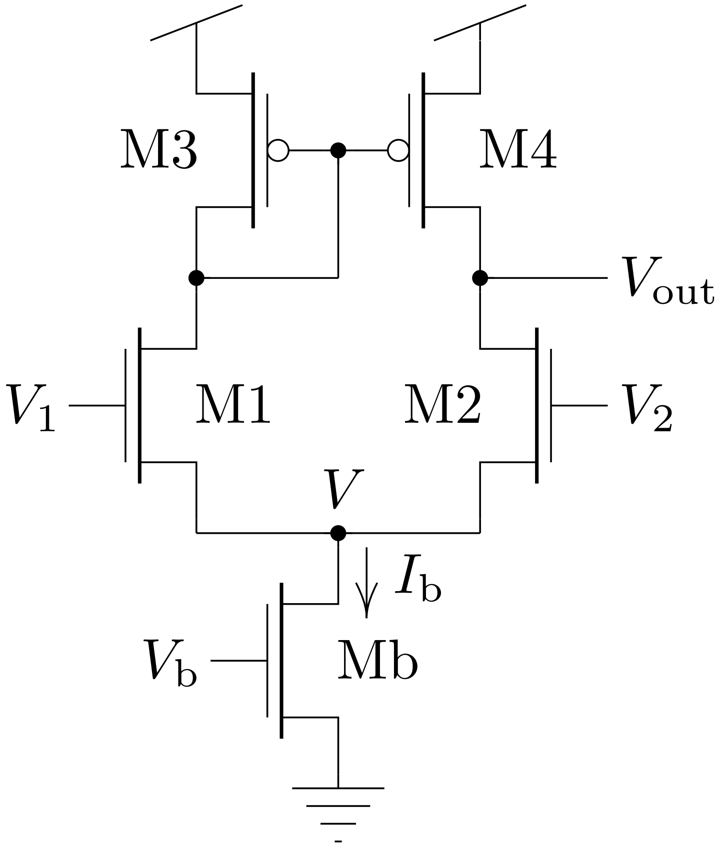

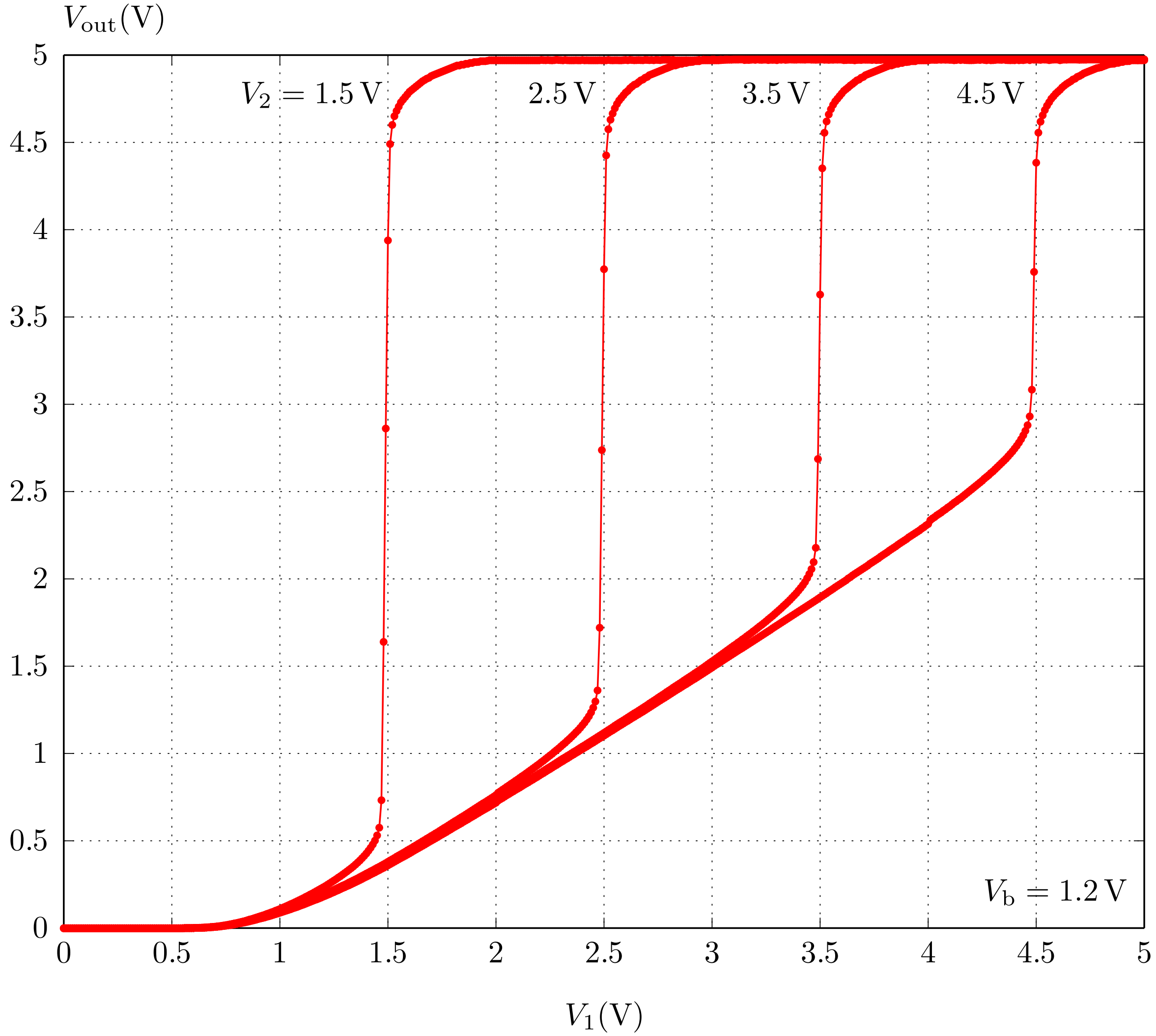

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