Translinear Circuits

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Translinear Circuits: What's in a Name?

In 1975, Barrie Gilbert coined the term *translinear* to describe a class of circuits whose large-signal behavior hinges both on the precise exponential I/V relationship of the bipolar transistor and on the intimate thermal contact and close matching of monolithically integrated devices.

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The word translinear refers to the exponential I/V characteristic of the bipolar transistor—its transconductance is linear in its collector current:

$$I_{\mathrm{C}} = I_{\mathrm{s}} e^{V_{\mathrm{BE}}/U_{\mathrm{T}}} \quad \Longrightarrow \quad g_{\mathrm{m}} = \frac{\partial I_{\mathrm{C}}}{\partial V_{\mathrm{B}}} = \underbrace{I_{\mathrm{s}} e^{V_{\mathrm{BE}}/U_{\mathrm{T}}}}_{I_{\mathrm{C}}} \cdot \frac{1}{U_{\mathrm{T}}} = \frac{I_{\mathrm{C}}}{U_{\mathrm{T}}}.$$

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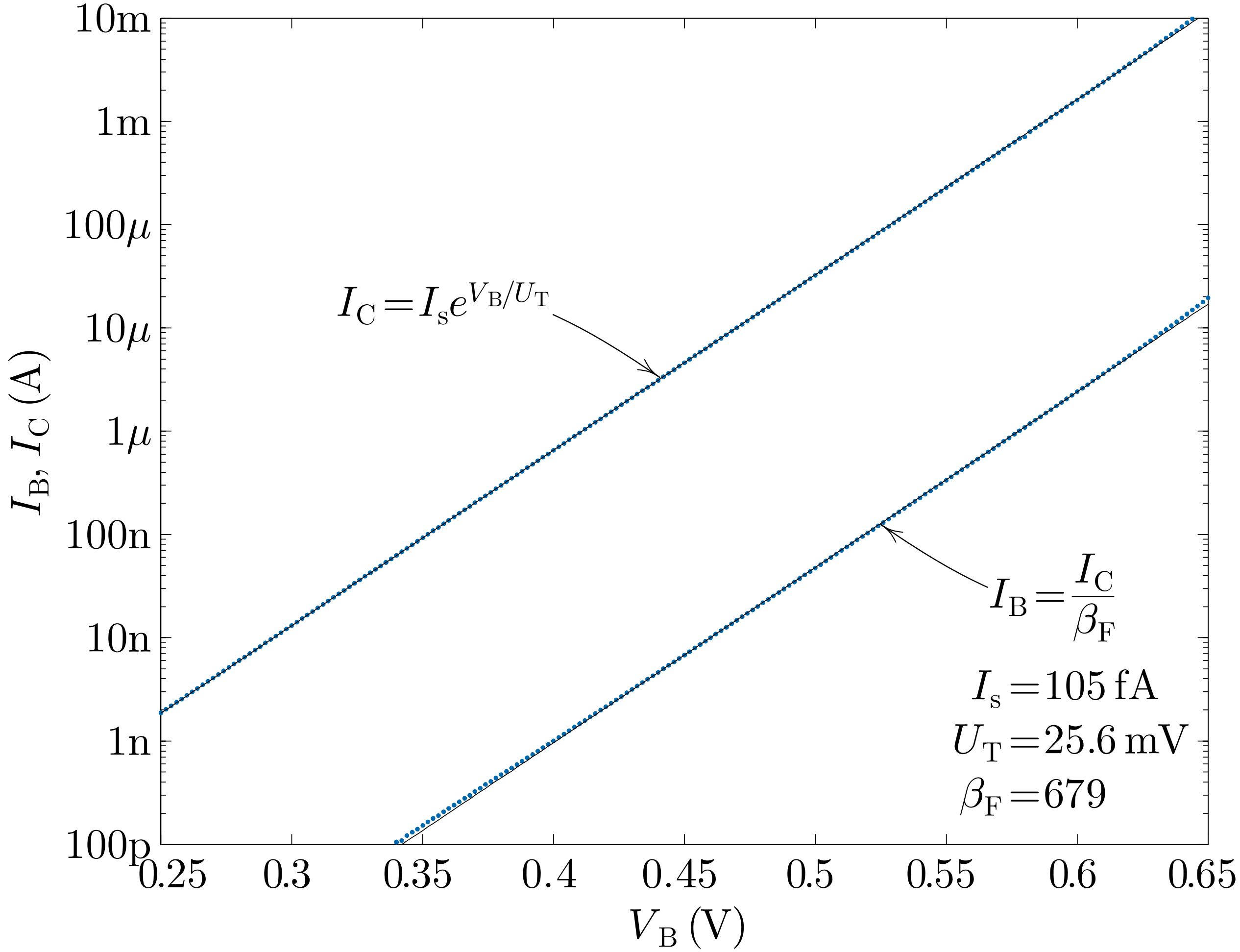
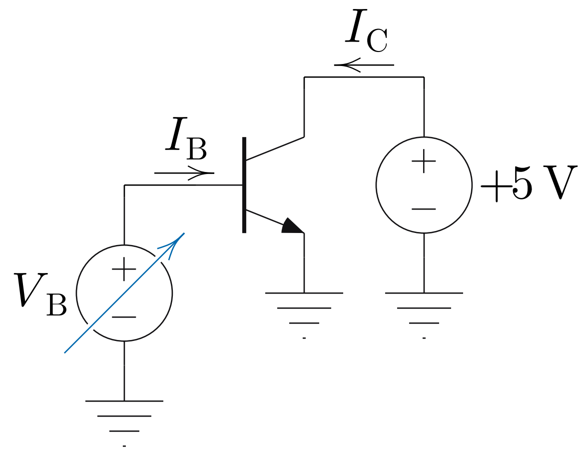
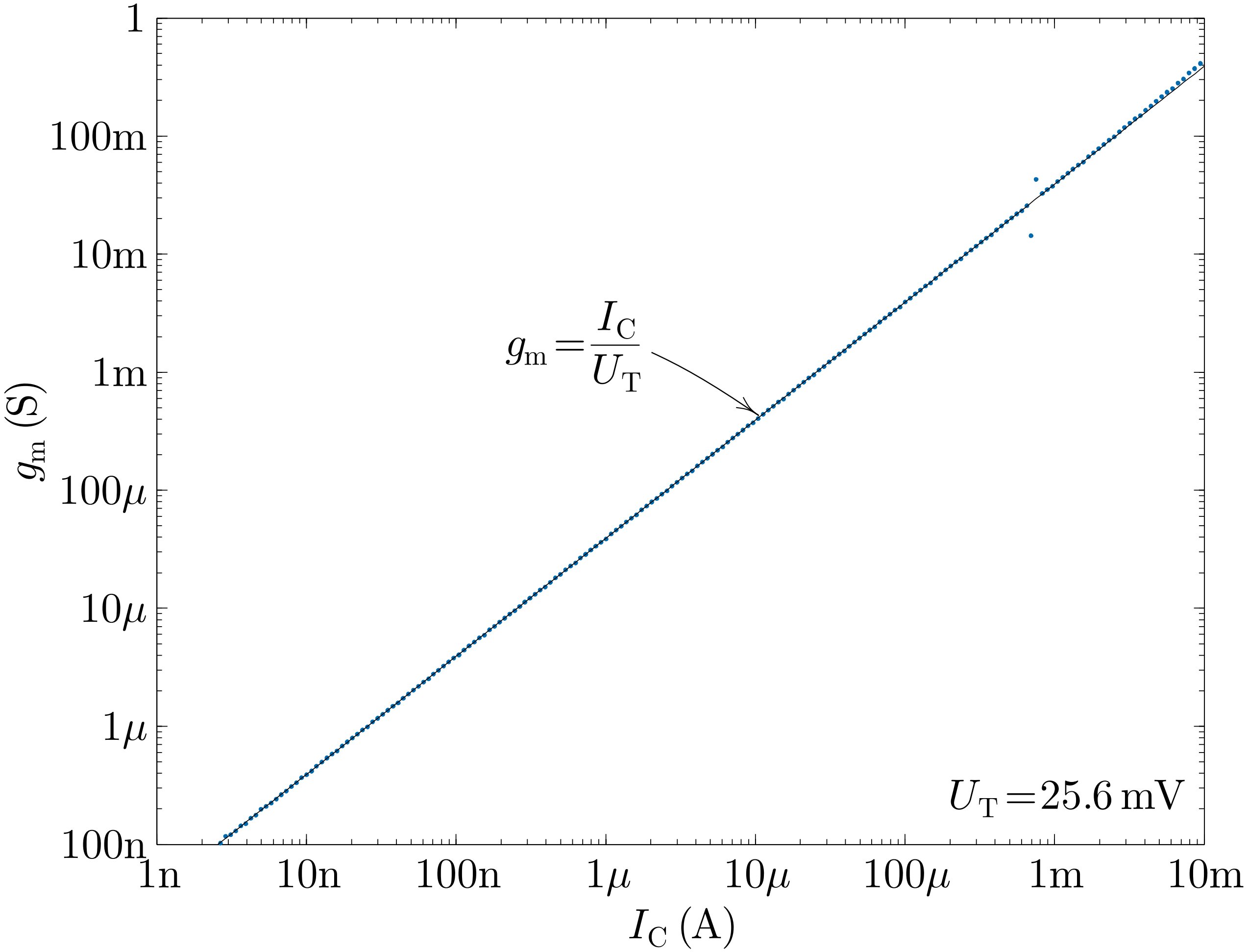
Gilbert also meant the word translinear to refer to circuit analysis and design principles that bridge the gap between the familiar territory of linear circuits and the uncharted domain of nonlinear circuits.

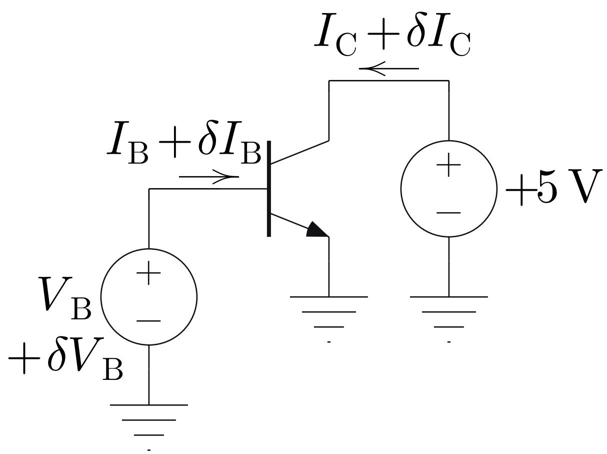
Gummel Plot of a Forward-Active Bipolar Transistor





Translinearity of the Forward-Active Bipolar Transistor





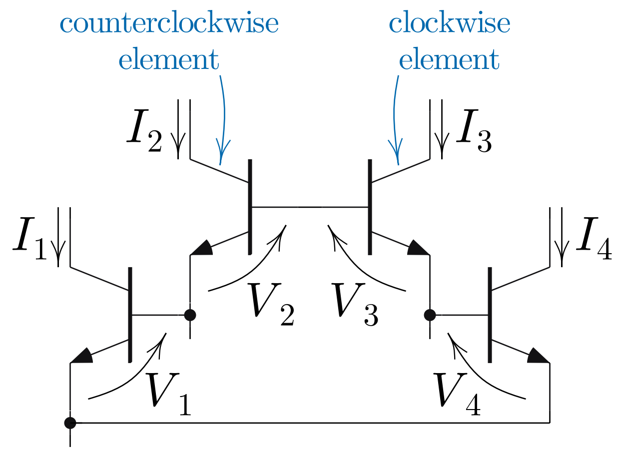
$$g_m = \frac{\partial I_C}{\partial V_B} = \frac{I_C}{U_T}$$

$$\delta I_C \approx g_m \delta V_B$$

The Translinear Principle

Consider a closed loop of base-emitter junctions of four closely matched *npn* bipolar transistors biased in the forward-active region and operating at the same temperature. Kirchhoff's voltage law (KVL) implies that

$$
\begin{aligned}
V_1 + V_2 &= V_3 + V_4 \\
U_\mathrm{T} \log \frac{I_1}{I_\mathrm{s}} + U_\mathrm{T} \log \frac{I_2}{I_\mathrm{s}} &= U_\mathrm{T} \log \frac{I_3}{I_\mathrm{s}} + U_\mathrm{T} \log \frac{I_4}{I_\mathrm{s}} \\
\log \frac{I_1 I_2}{I_\mathrm{s}^2} &= \log \frac{I_3 I_4}{I_\mathrm{s}^2} \\
\underbrace{I_1 I_2}_{\text{CCW}} &= \underbrace{I_3 I_4}_{\text{CW}}.
\end{aligned}
$$

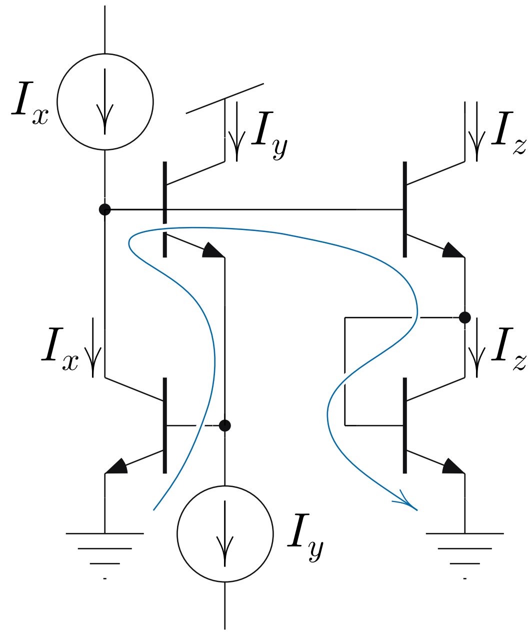


This result is a particular case of Gilbert's *translinear principle* (TLP): The product of the clockwise currents is equal to the product of the counterclockwise currents.

Static Translinear Circuits: Geometric Mean

We neglect both base currents (i.e., $\beta_\mathrm{F} = \infty$) and the Early effect, and we assume that all transistors operate in their forward-active regions. Then, we have that

$$\text{TLP} \implies I_x I_y = I_z^2 \implies I_z = \sqrt{I_x I_y}.$$



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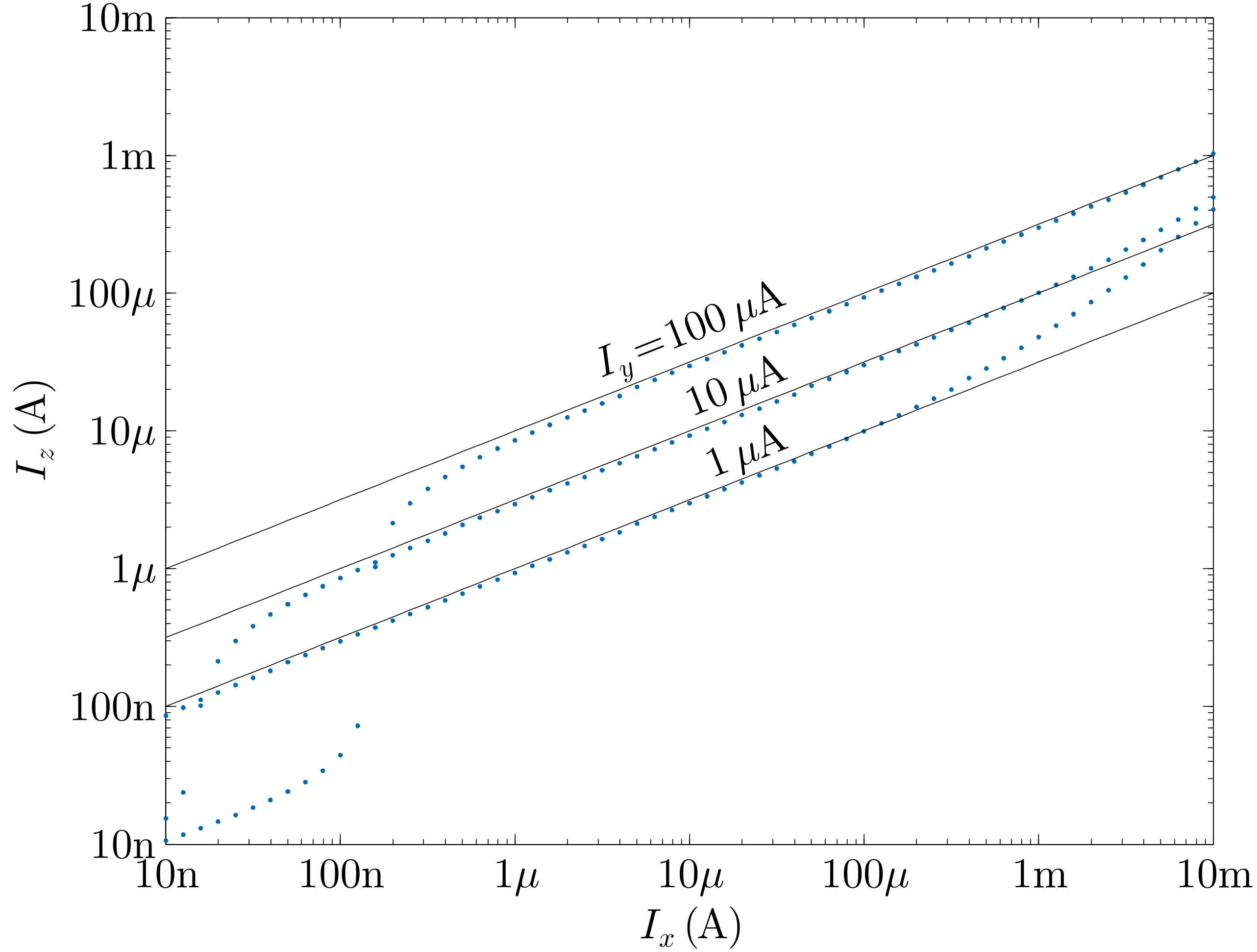

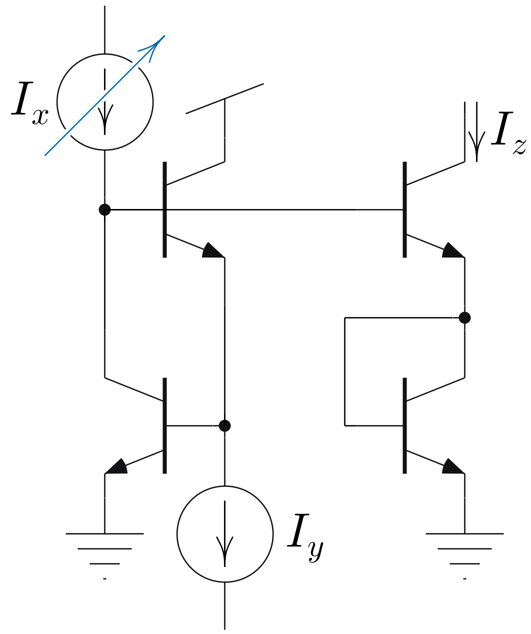

$$I_z = \sqrt{I_x I_y}$$

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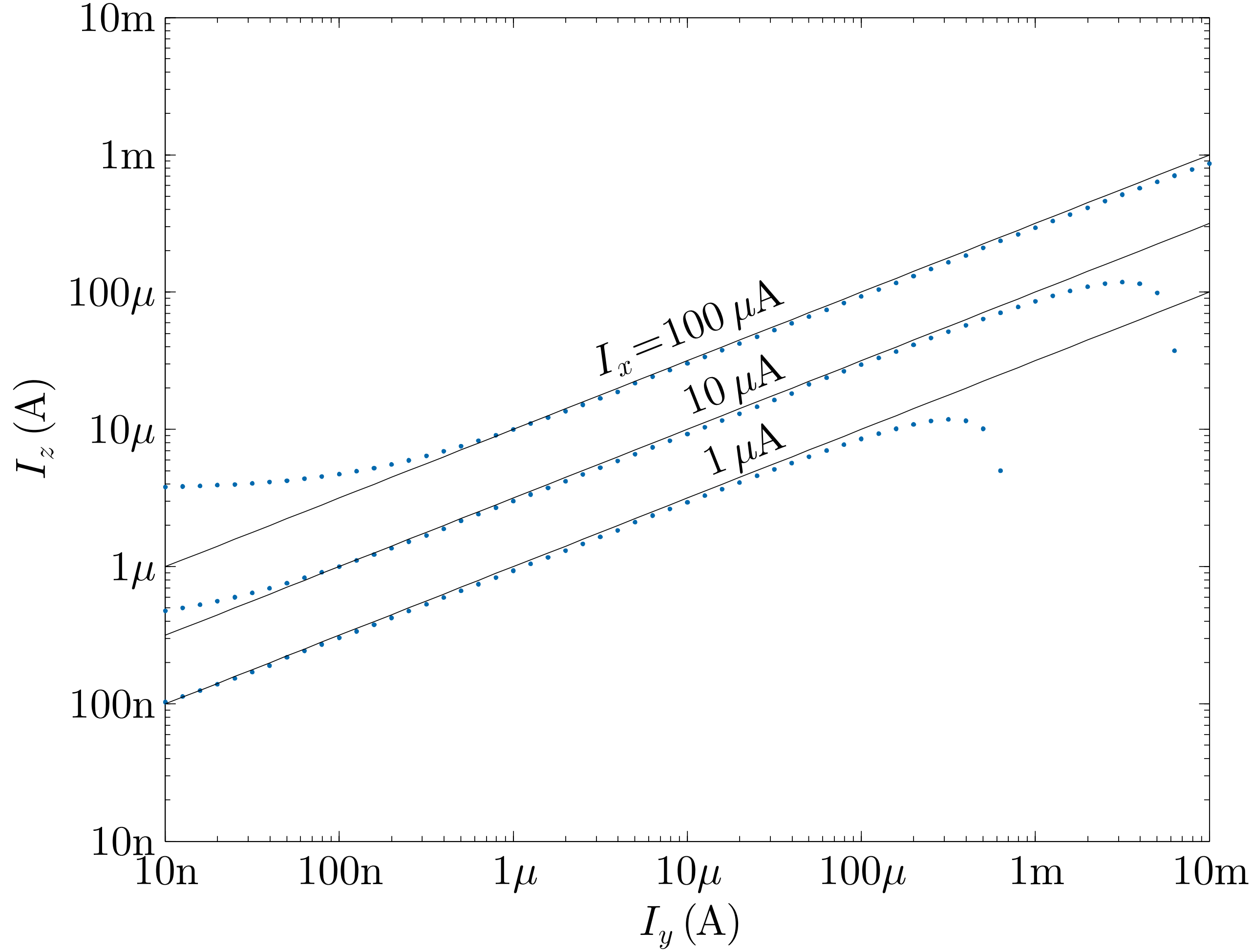

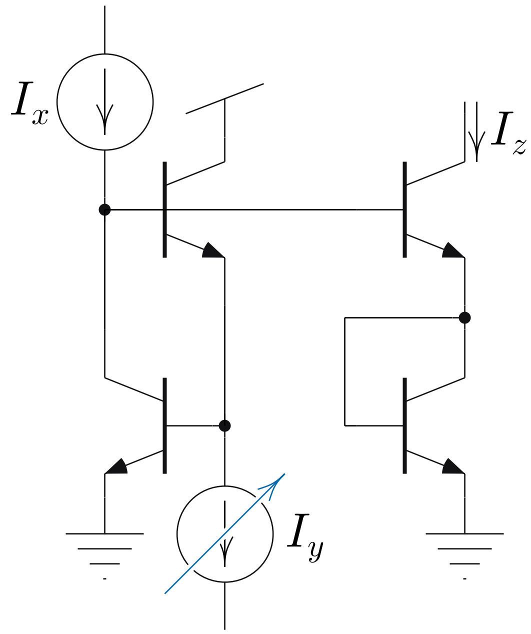

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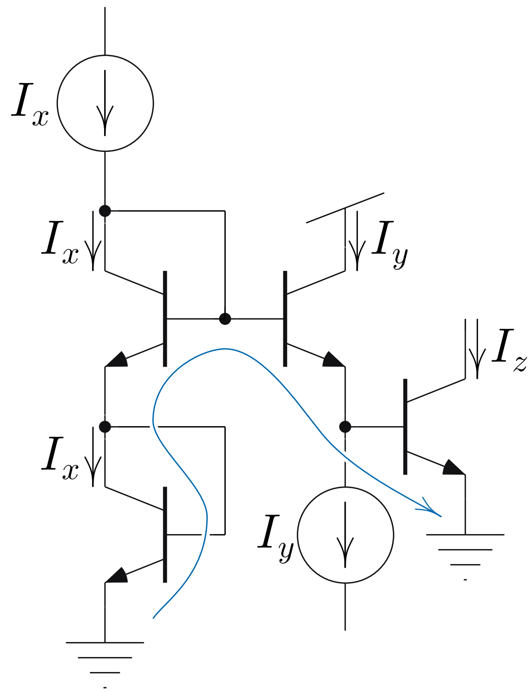
Static Translinear Circuits: Squaring/Reciprocal

Again, we neglect both base currents (i.e., $\beta_F = \infty$) and the Early effect, and we assume that all transistors operate in their forward-active regions. Then, we have that

$$\text{TLP} \implies I_x^2 = I_y I_z \implies I_z = \frac{I_x^2}{I_y}.$$



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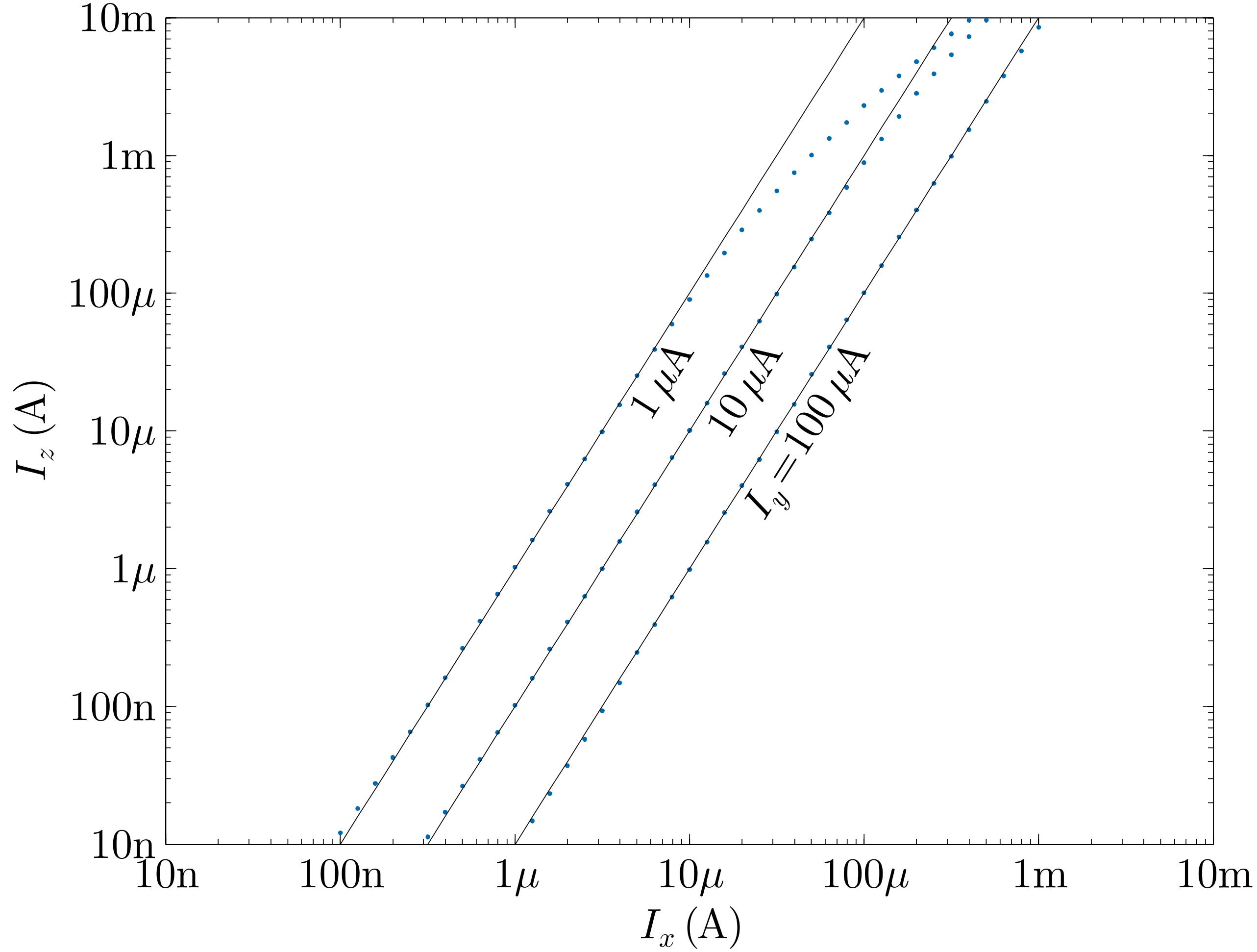

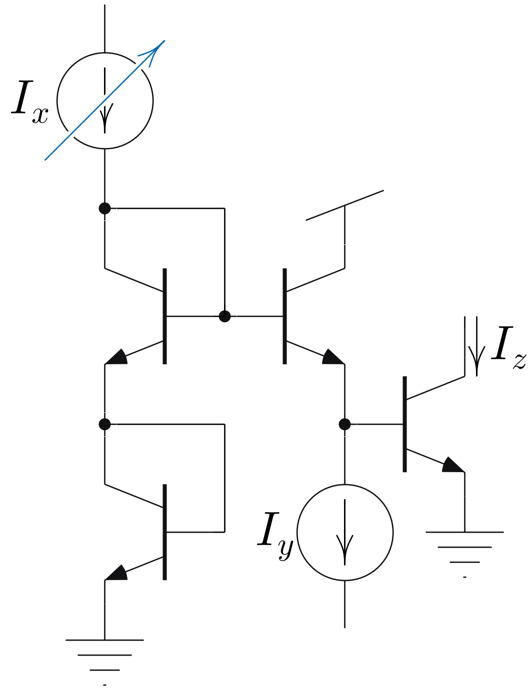

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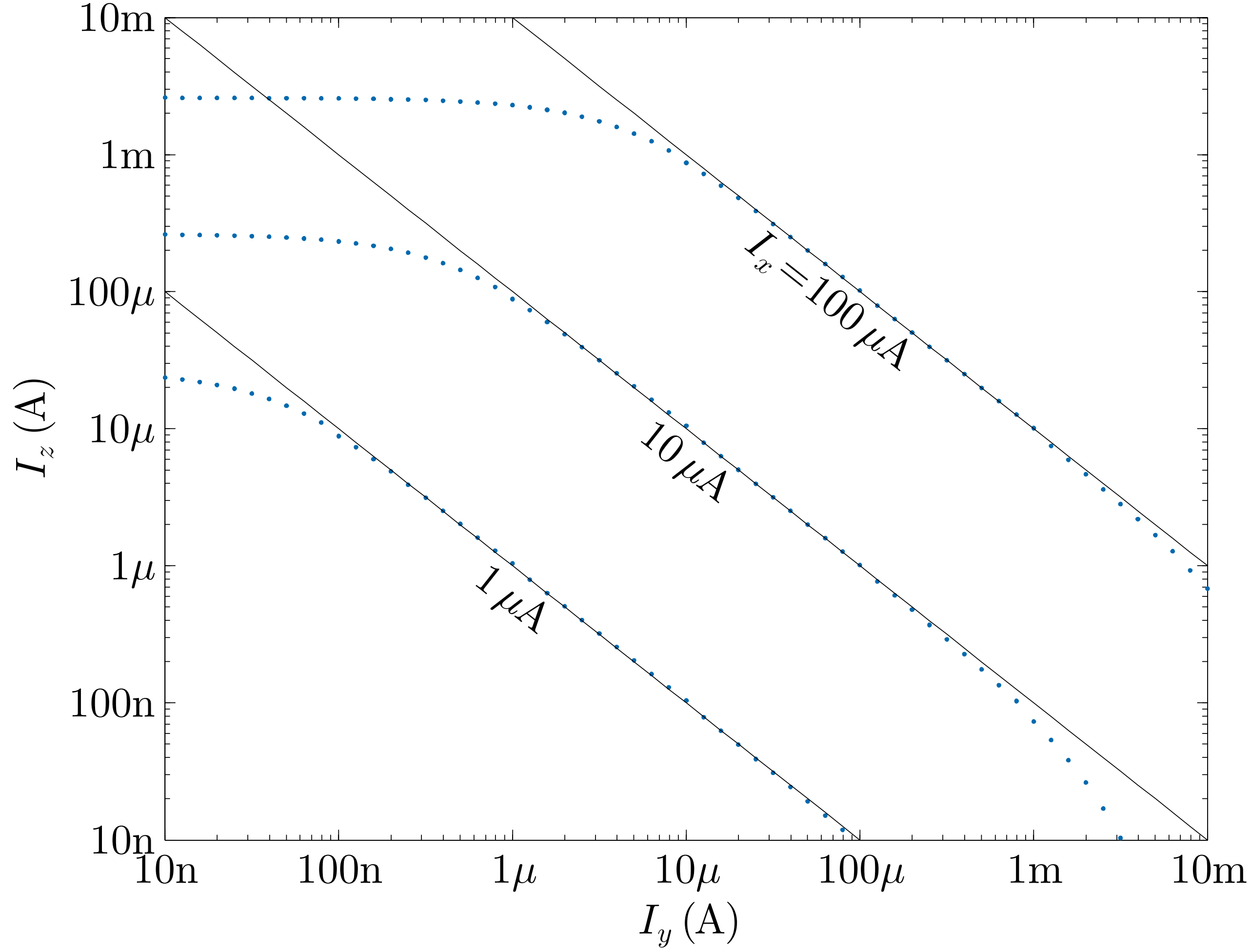

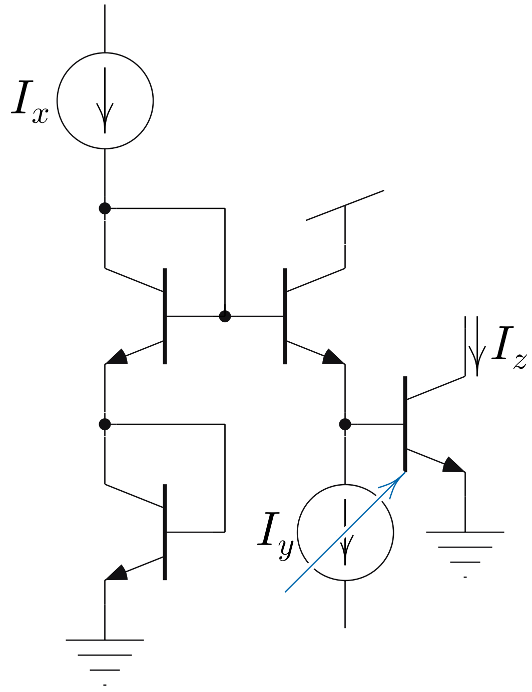

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Static Translinear Circuits: Pythagorator

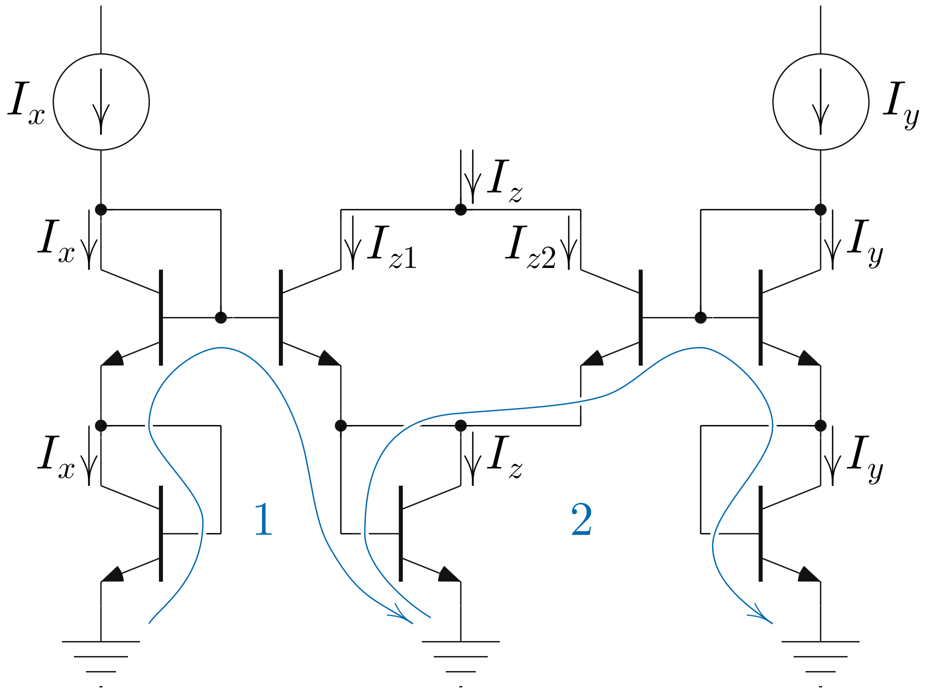
Again, we neglect both base currents and the Early effect, and we assume that all transistors operate in their forward-active regions. Then, we have that

$$\text{TLP 1} \implies I_x^2 = I_{z1} I_z \implies I_{z1} = \frac{I_x^2}{I_z}$$

$$\text{TLP 2} \implies I_y^2 = I_{z2} I_z \implies I_{z2} = \frac{I_y^2}{I_z}$$

$$\text{KCL} \implies I_z = I_{z1} + I_{z2} = \frac{I_x^2}{I_z} + \frac{I_y^2}{I_z}$$

$$\implies I_z^2 = I_x^2 + I_y^2 \implies I_z = \sqrt{I_x^2 + I_y^2}.$$



This circuit is called the pythagorator.

Dynamic Translinear Circuits: First-Order Low-Pass Filter

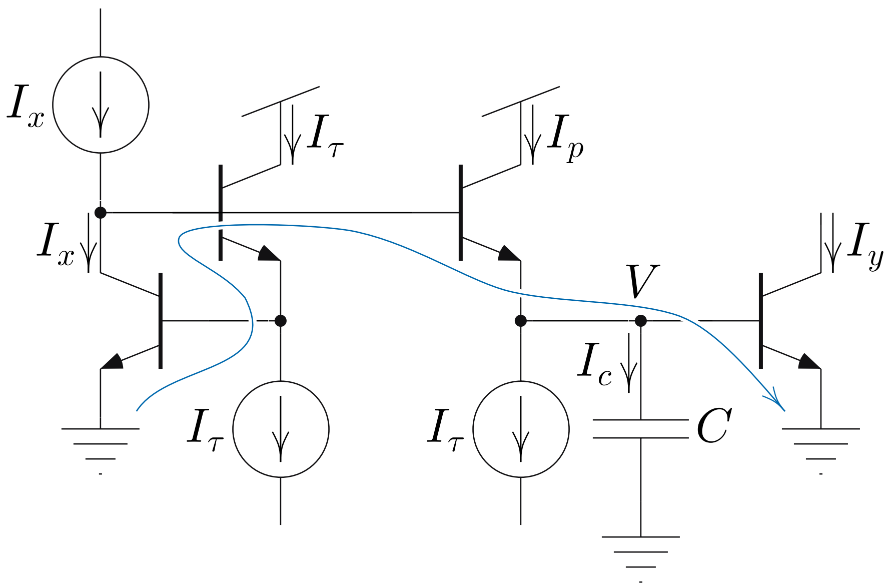
Next, consider the *dynamic* translinear circuit shown below, comprising a translinear loop and a capacitor. We shall again neglect base currents and the Early effect. We also assume that all transistors operate in the forward-active region. Then, we have that

$$\text{TLP} \implies I_\tau I_x = I_p I_y \implies I_p = \frac{I_\tau I_x}{I_y}$$

$$I_\mathrm{c} = C\frac{dV}{dt} = C\frac{d}{dt}\left(U_\mathrm{T}\log\frac{I_y}{I_\mathrm{s}}\right) = \frac{CU_\mathrm{T}}{I_y}\frac{dI_y}{dt}$$

$$\text{KCL} \implies I_\mathrm{c} + I_\tau = I_p \implies \frac{CU_\mathrm{T}}{I_y}\frac{dI_y}{dt} + I_\tau = \frac{I_\tau I_x}{I_y}$$

$$\implies \underbrace{\frac{CU_\mathrm{T}}{I_\tau}}_{\tau}\frac{dI_y}{dt} + I_y = I_x \implies \tau\frac{dI_y}{dt} + I_y = I_x.$$



This circuit is a first-order log-domain filter.

Dynamic Translinear Circuits: RMS-to-DC Converter

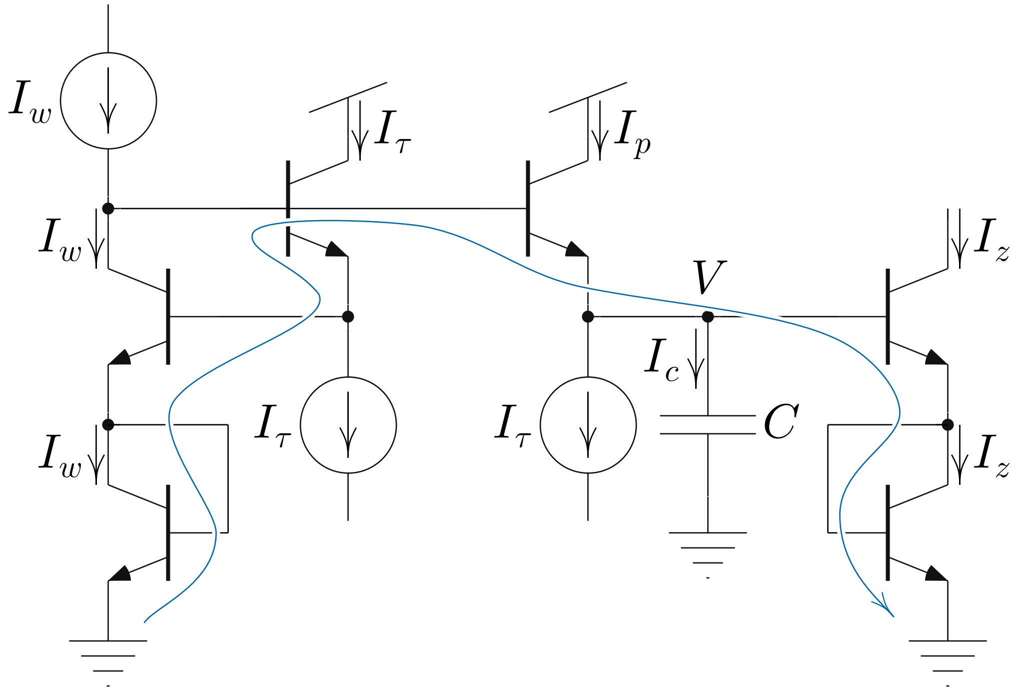
$$\text{TLP} \implies I_w^2 I_\tau = I_p I_z^2 \implies I_p = \frac{I_\tau I_w^2}{I_z^2}$$

$$I_\text{c} = C\frac{d}{dt}\left(2U_\text{T}\log\frac{I_z}{I_\text{s}}\right) = 2\,\frac{CU_\text{T}}{I_z}\frac{dI_z}{dt}$$

$$\text{KCL} \implies I_\text{c} + I_\tau = I_p$$

$$\implies \frac{2CU_\text{T}}{I_z}\frac{dI_z}{dt} + I_\tau = \frac{I_\tau I_w^2}{I_z^2}$$

$$\implies \underbrace{\frac{CU_\text{T}}{I_\tau}}_{\tau}\,2\,I_z\frac{dI_z}{dt} + I_z^2 = I_w^2$$



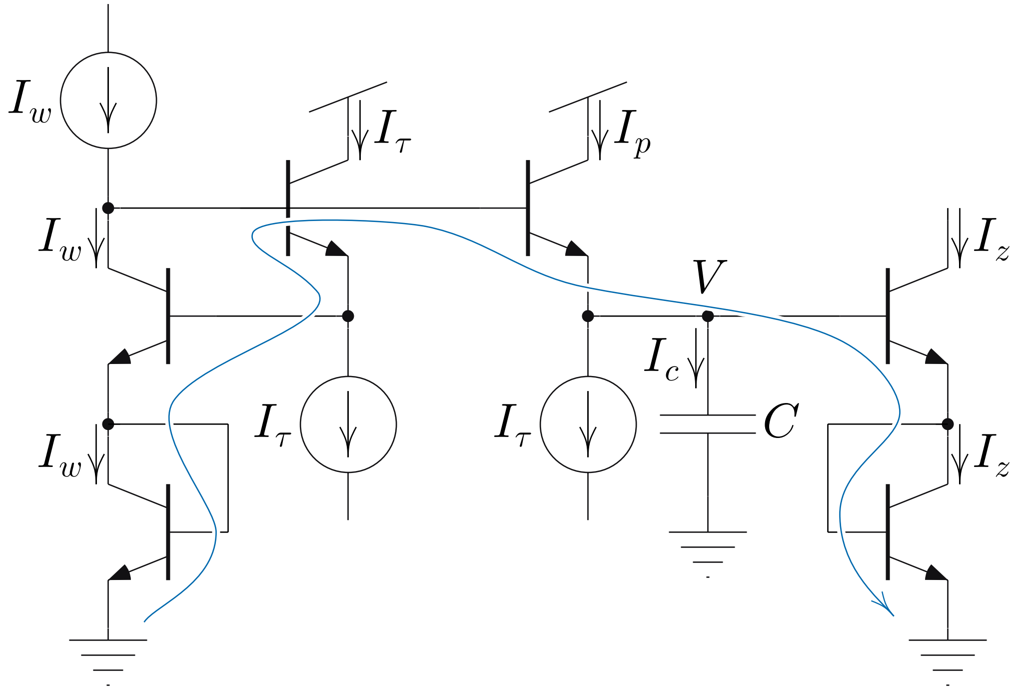
Dynamic Translinear Circuits: RMS-to-DC Converter

$$\Longrightarrow \tau\left(2\,I_z\frac{dI_z}{dt}\right)+I_z^2 = I_w^2$$

$$\Longrightarrow \tau\frac{d}{dt}\left(I_z^2\right)+I_z^2 = I_w^2$$

$$\Longrightarrow \tau\frac{d}{dt}\Bigg(\underbrace{\frac{I_z^2}{I_1}}_{I_y}\Bigg)+\underbrace{\frac{I_z^2}{I_1}}_{I_y} = \underbrace{\frac{I_w^2}{I_1}}_{I_x}$$

$$\underbrace{I_z=\sqrt{I_1 I_y}}_{\text{root}} \qquad \underbrace{\tau\frac{dI_y}{dt}+I_y=I_x}_{\text{mean}} \qquad \underbrace{I_x=\frac{I_w^2}{I_1}}_{\text{square}}$$



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- Translinear circuits are *tunable* electronically over a wide dynamic range of parameters (e.g., gains, corner frequencies, quality factors).

- Translinear circuits are *robust*. Carefully designed translinear circuits are temperature insensitive and do not depend on device or technology parameters.

Simple EKV Model of the Saturated nMOS Transistor

We model the saturation current of an nMOS transistor by

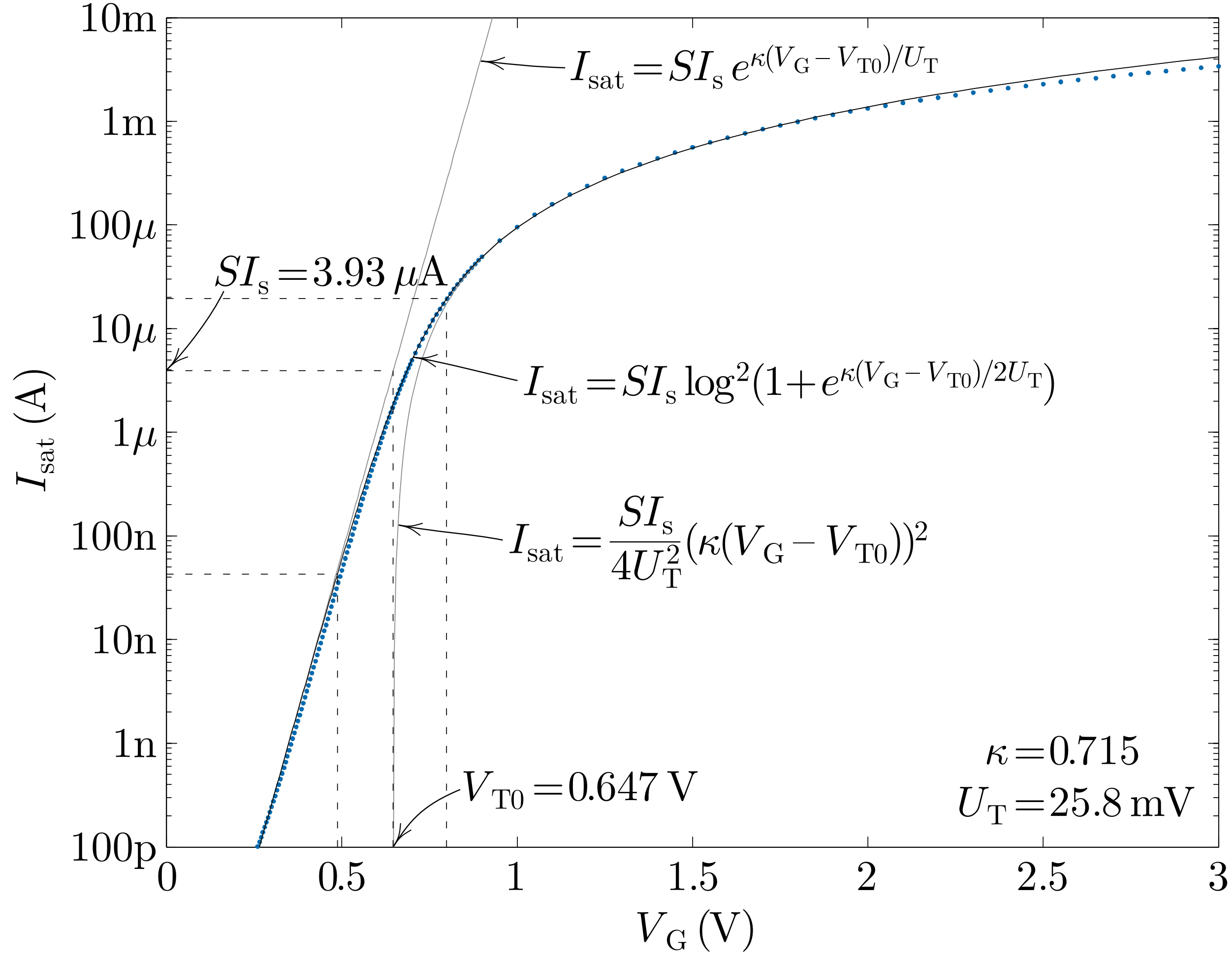
$$
\begin{aligned}
I_{\mathrm{sat}} &= SI_{\mathrm{s}} \log^2\left(1 + e^{(\kappa(V_{\mathrm{G}} - V_{\mathrm{T0}}) - V_{\mathrm{S}})/2U_{\mathrm{T}}}\right) \\
&\approx \begin{cases} SI_{\mathrm{s}} e^{(\kappa(V_{\mathrm{G}} - V_{\mathrm{T0}}) - V_{\mathrm{S}})/U_{\mathrm{T}}}, & \kappa\left(V_{\mathrm{G}} - V_{\mathrm{T0}}\right) - V_{\mathrm{S}} < 0 \\ \dfrac{SI_{\mathrm{s}}}{4U_{\mathrm{T}}^2}\left(\kappa\left(V_{\mathrm{G}} - V_{\mathrm{T0}}\right) - V_{\mathrm{S}}\right)^2, & \kappa\left(V_{\mathrm{G}} - V_{\mathrm{T0}}\right) - V_{\mathrm{S}} > 0, \end{cases}
\end{aligned}
$$

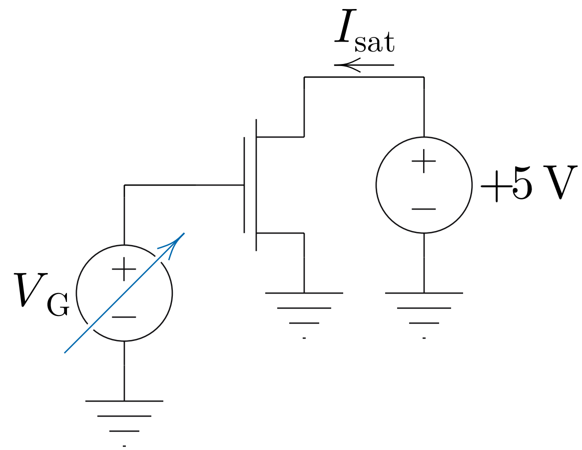
where

$$
U_{\mathrm{T}} = \frac{kT}{q}, \quad S = \frac{W}{L}, \quad I_{\mathrm{s}} = \frac{2\mu C_{\mathrm{ox}} U_{\mathrm{T}}^2}{\kappa}, \quad \text{and} \quad \kappa = \frac{C_{\mathrm{ox}}}{C_{\mathrm{ox}} + C_{\mathrm{dep}}}.
$$

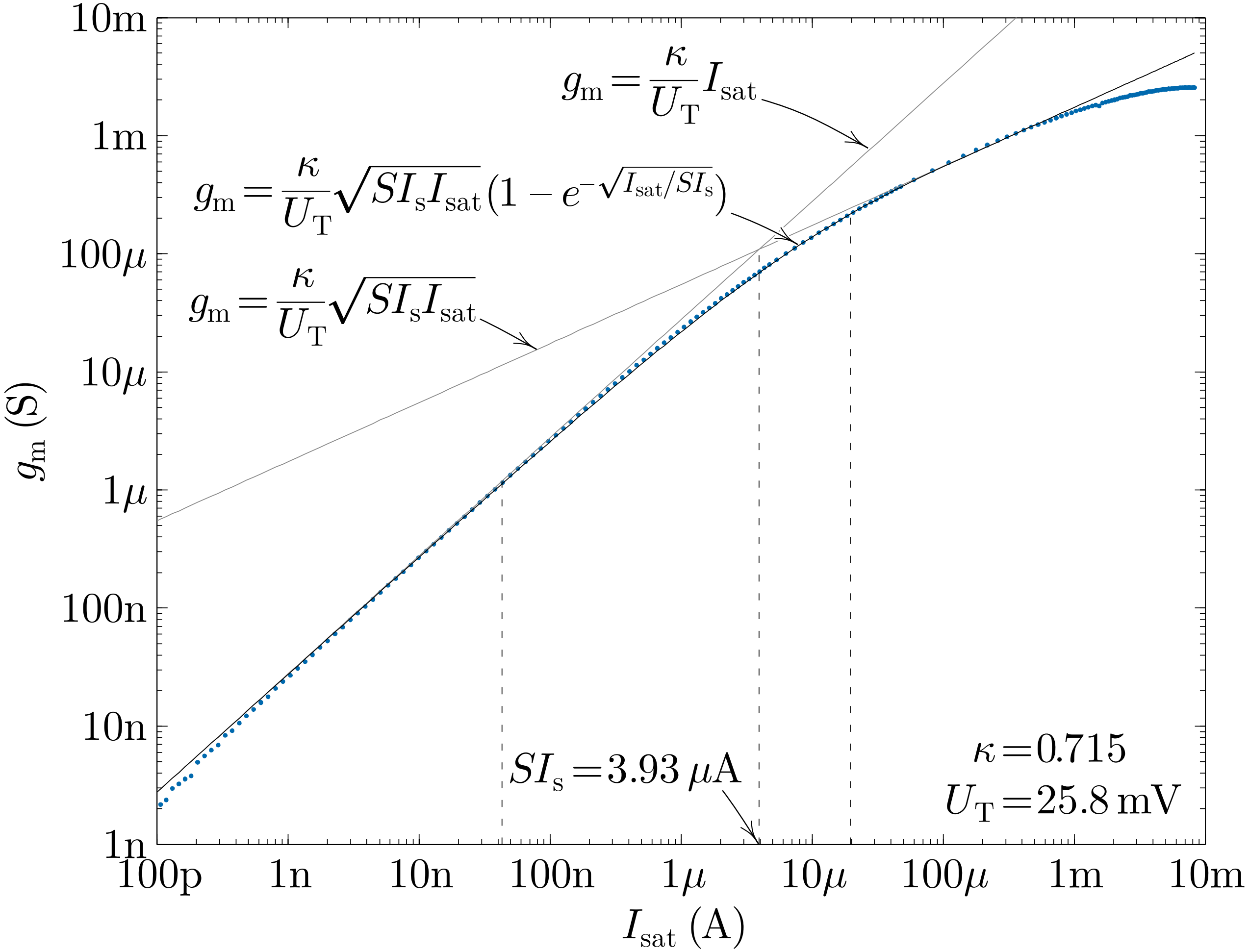
Weak inversion operation corresponds to $I_{\mathrm{sat}} \ll SI_{\mathrm{s}}$, moderate inversion operation corresponds to $I_{\mathrm{sat}} \approx SI_{\mathrm{s}}$, and strong inversion operation to $I_{\mathrm{sat}} \gg SI_{\mathrm{s}}$. Note that SI_{s} is approximately twice the saturation current at threshold.

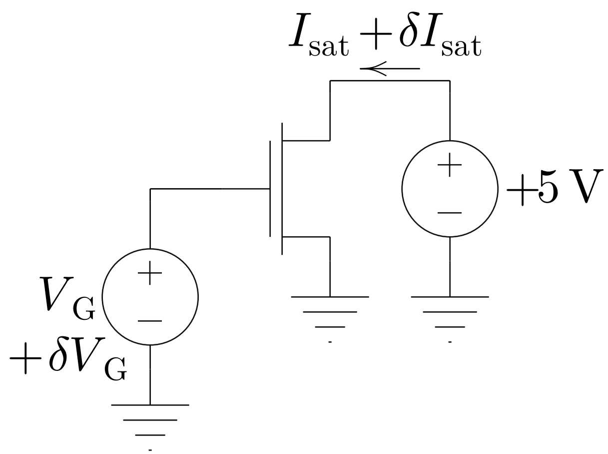
Saturation Current of an nMOS Transistor





Translinearity of the Saturated nMOS Transistor

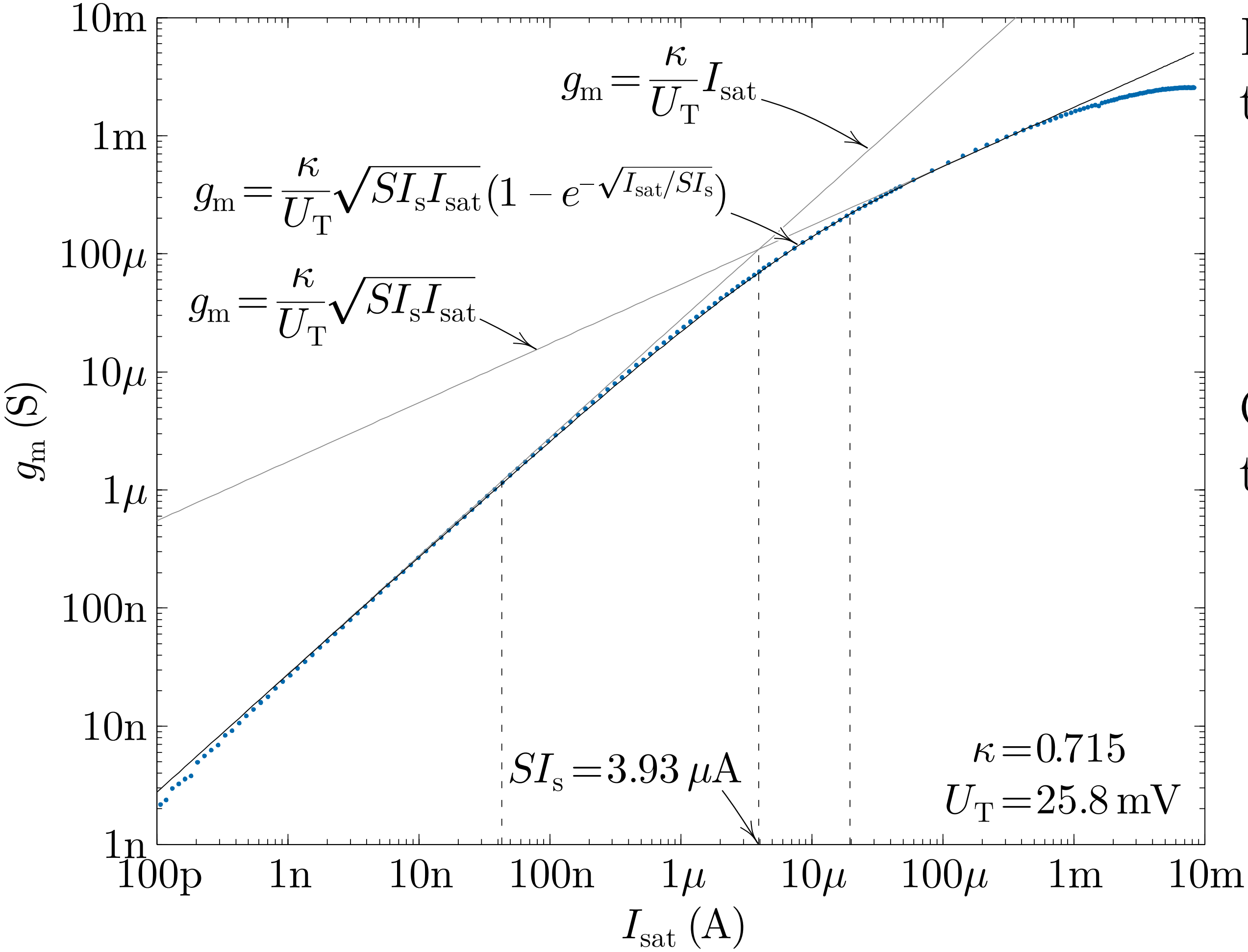




$$g_m = \frac{\partial I_{sat}}{\partial V_G} = \frac{\kappa I_{sat}}{U_T},\ I_{sat} \ll SI_s$$

$$\delta I_{sat} \approx g_m \delta V_G$$

Weak Inversion is Suited to Audio Signal Processing



For integrated continous-time filters, typically

$$f_c = \frac{g_m}{2\pi C}.$$

On-chip capacitors are typically on the order of 1 pF or of 10 pF.

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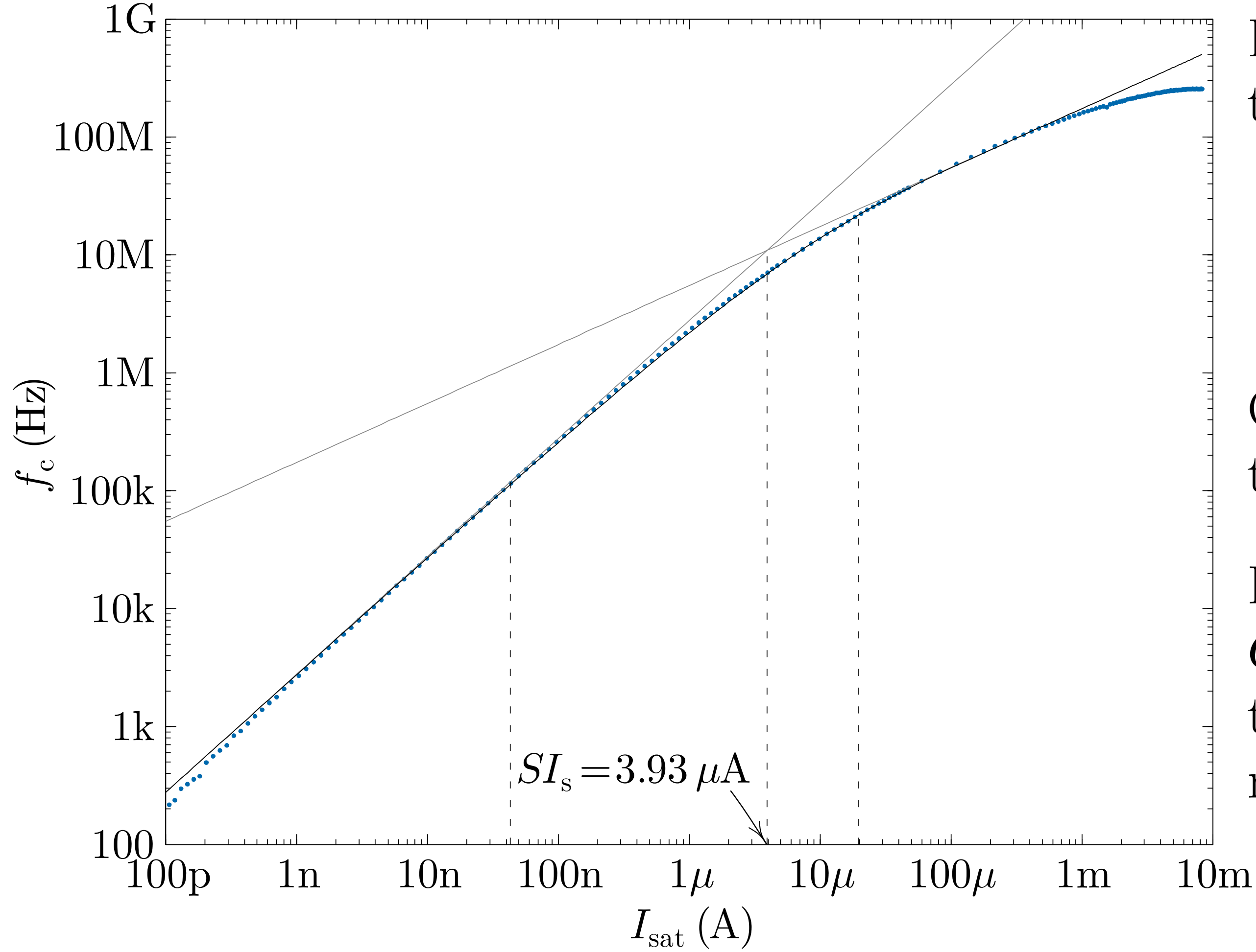

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$$f_c = \frac{g_m}{2\pi C}.$$

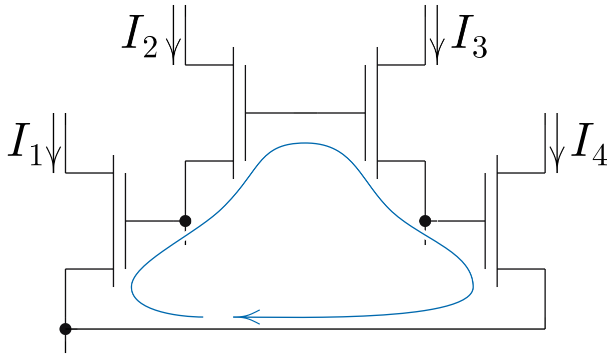
On-chip capacitors are typically on the order of 1 pF or of 10 pF.

If we choose a reasonable value for C, say $C = 5/\pi\,\text{pF} \approx 1.59\,\text{pF}$, then we find that weak inversion maps onto the audio band.

Weak-Inversion MOS Translinear Principle

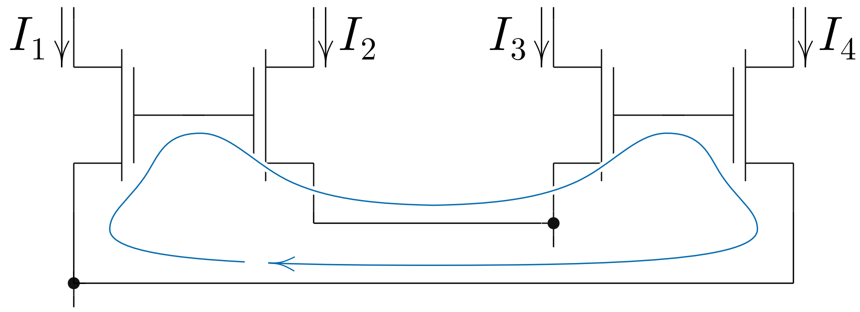
When designing weak-inversion MOS translinear circuits, if we restrict ourselves to using translinear loops that alternate between clockwise and counterclockwise elements, we obtain Gilbert's original TLP, with no dependence on the body effect (i.e., κ).

stacked loop



TLP: $I_1 I_2^{\kappa} = I_3^{\kappa} I_4$

alternating loop

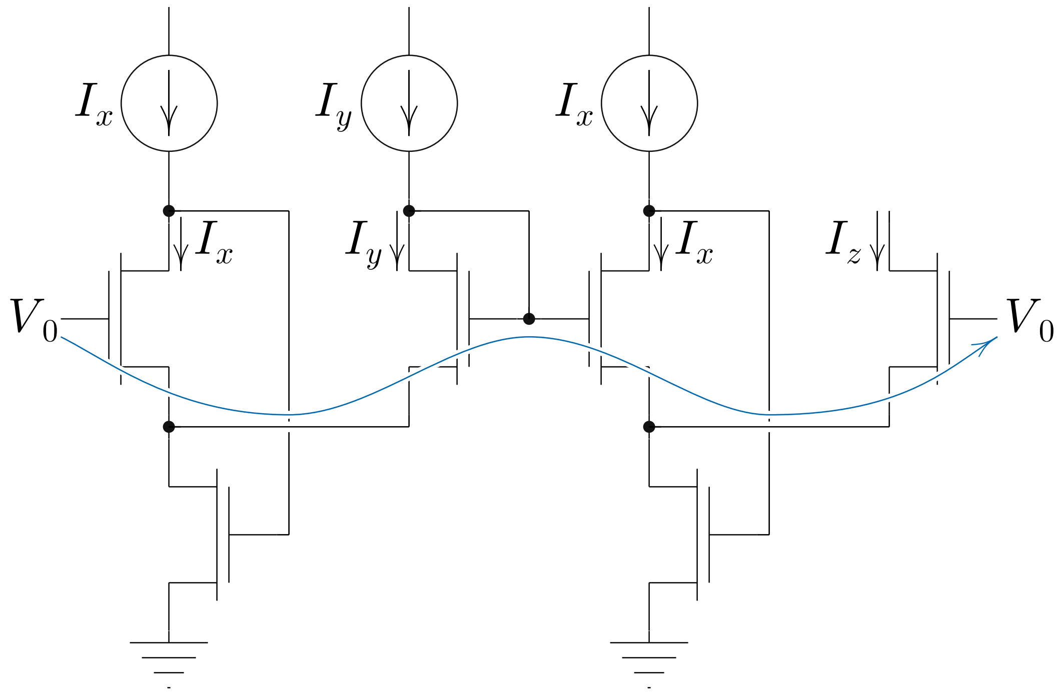


TLP: $I_1 I_3 = I_2 I_4$

This restriction does not limit the class of systems that we can implement. However, designs based on alternating loops generally consume more current but operate on a lower power supply voltage than do designs based on stacked loops.

Static Translinear Circuits: Squaring/Reciprocal

TLP $\Longrightarrow$ $I_x^2 = I_y I_z$ $\Longrightarrow$ $I_z = \dfrac{I_x^2}{I_y}$



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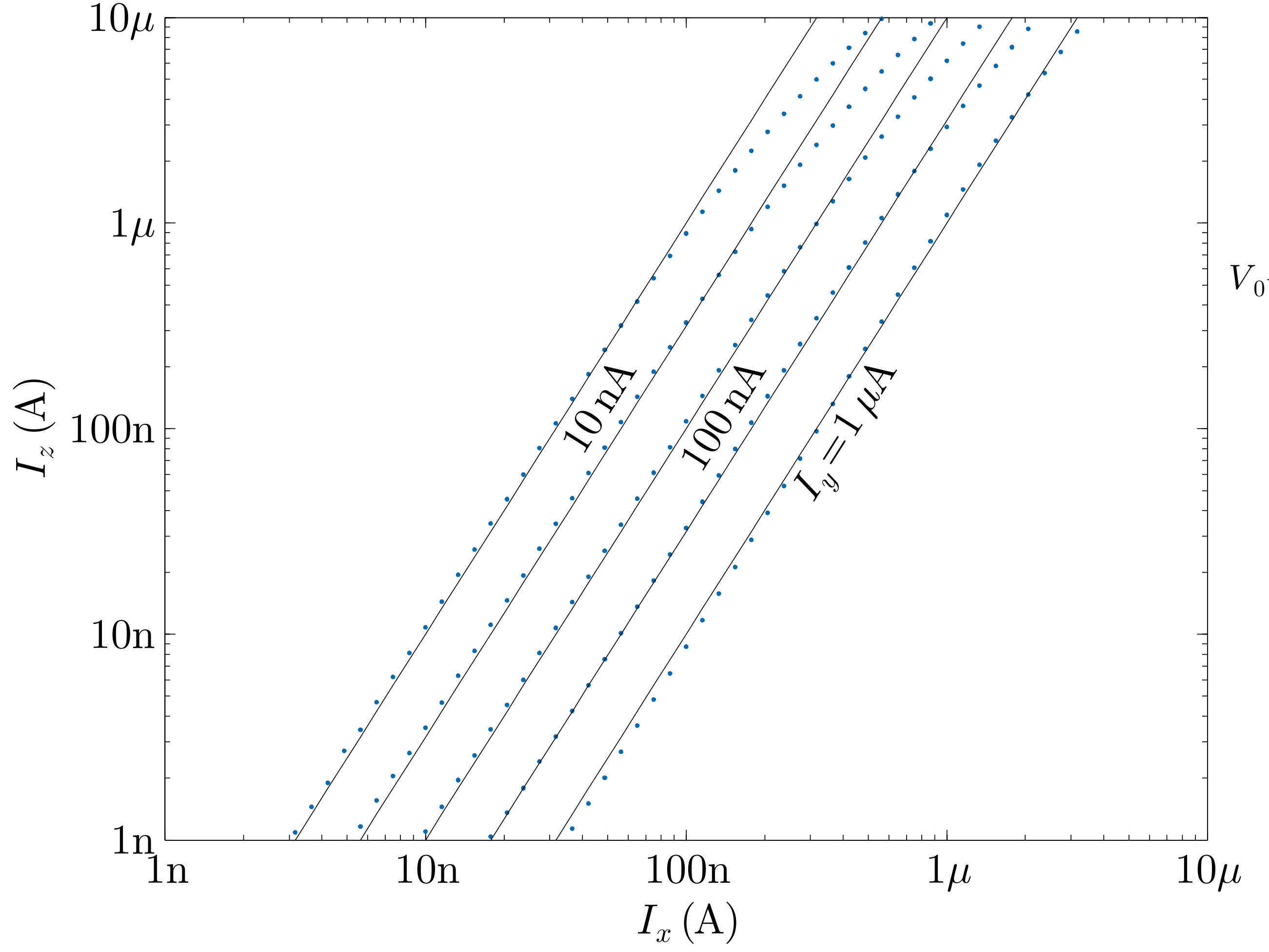

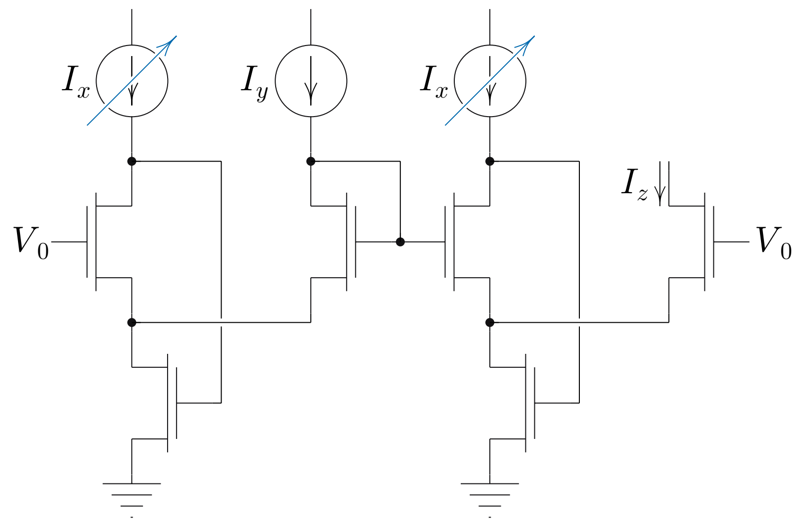

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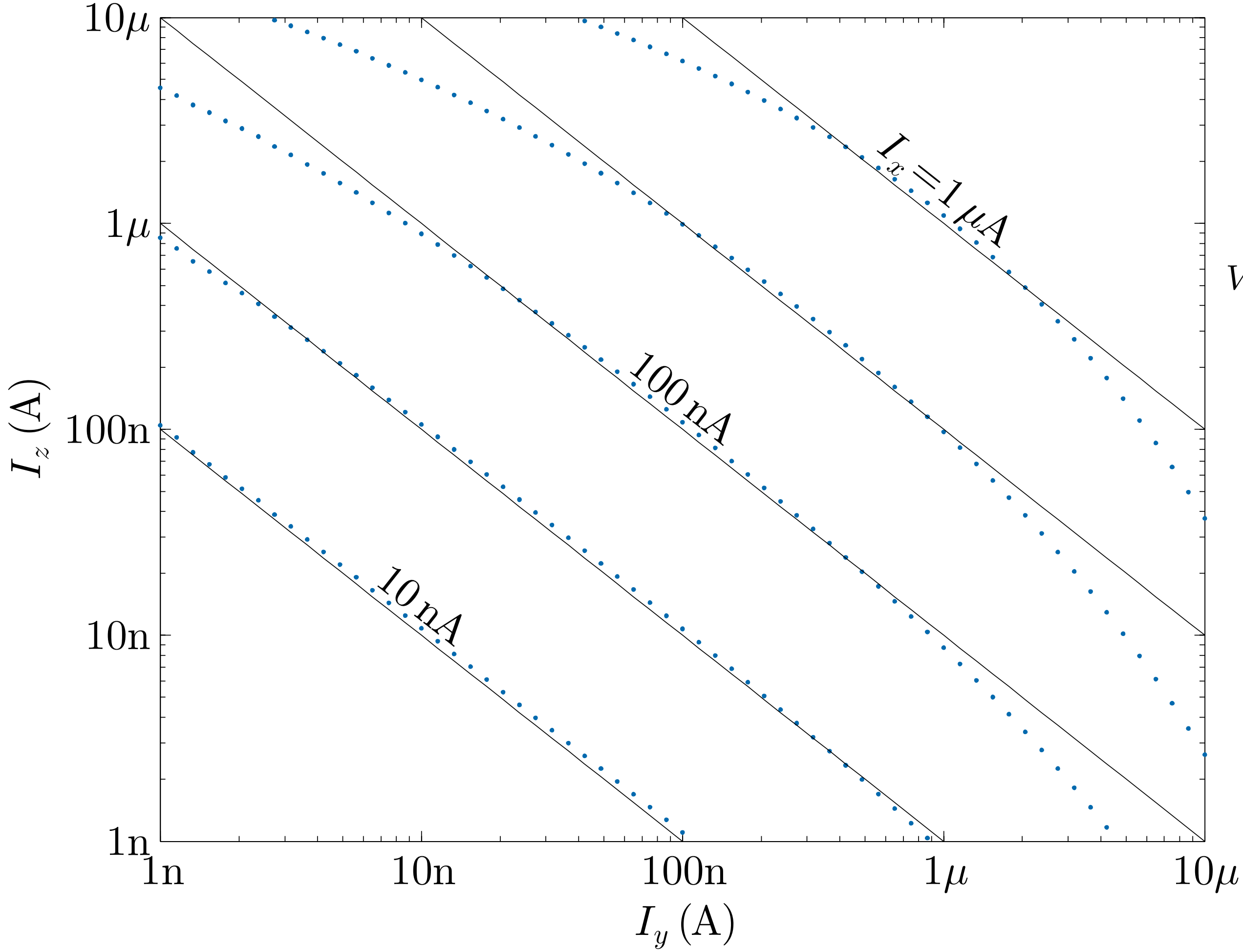

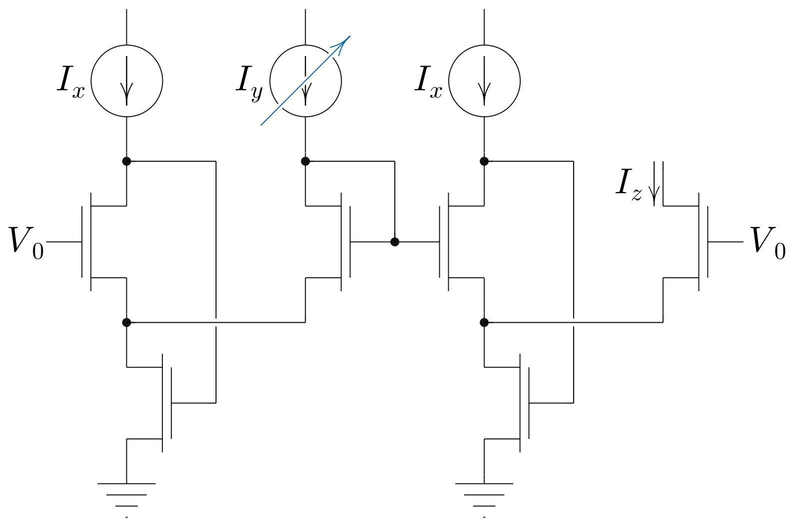

$$I_z = \frac{I_x^2}{I_y}$$

Static Translinear Circuit Synthesis: Pythagorator

Synthesize a two-dimensional vector-magnitude circuit implementing

$$r = \sqrt{x^2 + y^2}, \quad \text{where} \quad x > 0 \quad \text{and} \quad y > 0.$$

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We represent each signal as a ratio of a signal current to the unit current:

$$x \equiv \frac{I_x}{I_1}, \quad y \equiv \frac{I_y}{I_1}, \quad \text{and} \quad r \equiv \frac{I_r}{I_1}.$$

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We substitute these into the original equation and rearrange to obtain

$$\frac{I_r}{I_1} = \sqrt{\left(\frac{I_x}{I_1}\right)^2 + \left(\frac{I_y}{I_1}\right)^2}$$

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$$\frac{I_r}{I_1} = \sqrt{\left(\frac{I_x}{I_1}\right)^2 + \left(\frac{I_y}{I_1}\right)^2} \quad \Longrightarrow \quad I_r^2 = I_x^2 + I_y^2$$

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$$x \equiv \frac{I_x}{I_1}, \quad y \equiv \frac{I_y}{I_1}, \quad \text{and} \quad r \equiv \frac{I_r}{I_1}.$$

We substitute these into the original equation and rearrange to obtain

$$\frac{I_r}{I_1} = \sqrt{\left(\frac{I_x}{I_1}\right)^2 + \left(\frac{I_y}{I_1}\right)^2} \quad \Longrightarrow \quad I_r^2 = I_x^2 + I_y^2 \quad \Longrightarrow \quad I_r = \frac{I_x^2}{I_r} + \frac{I_y^2}{I_r}$$

Static Translinear Circuit Synthesis: Pythagorator

Synthesize a two-dimensional vector-magnitude circuit implementing

$$r = \sqrt{x^2 + y^2}, \quad \text{where} \quad x > 0 \quad \text{and} \quad y > 0.$$

We represent each signal as a ratio of a signal current to the unit current:

$$x \equiv \frac{I_x}{I_1}, \quad y \equiv \frac{I_y}{I_1}, \quad \text{and} \quad r \equiv \frac{I_r}{I_1}.$$

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Static Translinear Circuit Synthesis: Pythagorator

TLP: $I_{r1} I_r = I_x^2$

$I_{r2} I_r = I_y^2$

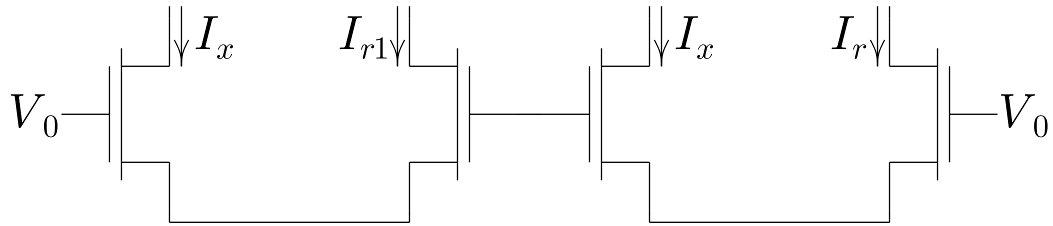
KCL: $I_r = I_{r1} + I_{r2}$

Static Translinear Circuit Synthesis: Pythagorator

TLP: $I_{r1} I_r = I_x^2$

$I_{r2} I_r = I_y^2$

KCL: $I_r = I_{r1} + I_{r2}$

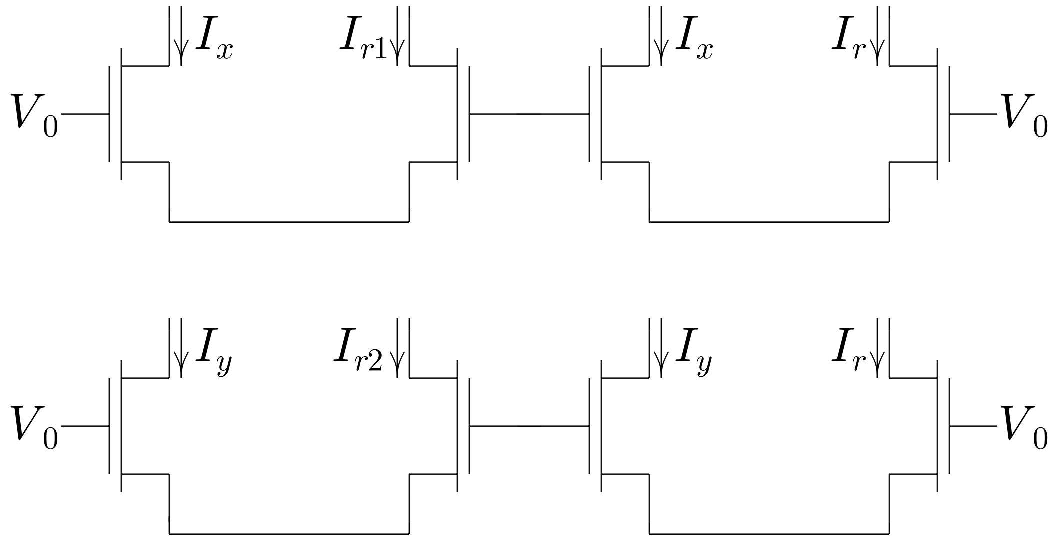


Static Translinear Circuit Synthesis: Pythagorator

TLP: $I_{r1} I_r = I_x^2$

$I_{r2} I_r = I_y^2$

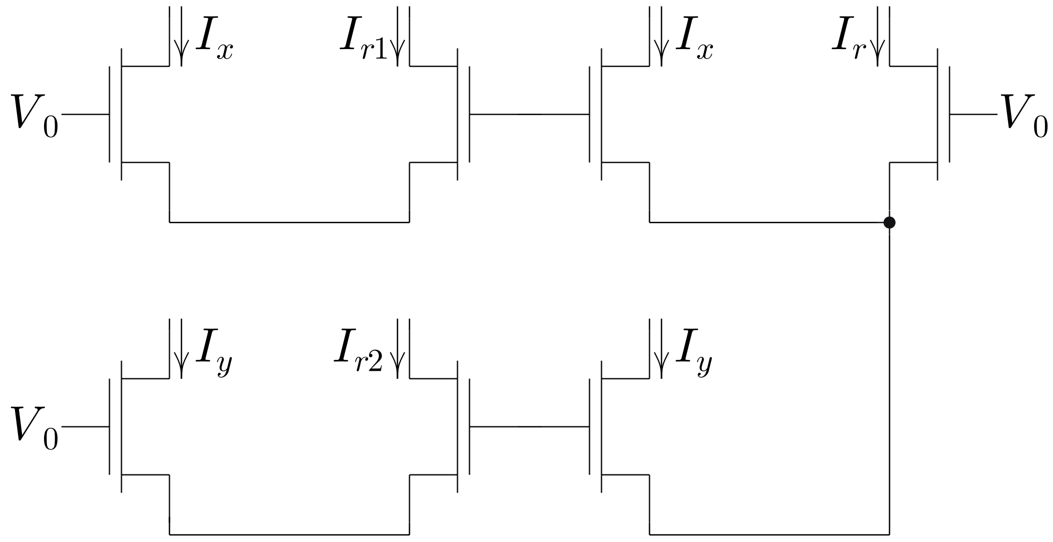
KCL: $I_r = I_{r1} + I_{r2}$



Static Translinear Circuit Synthesis: Pythagorator

TLP: $I_{r1} I_r = I_x^2$
$I_{r2} I_r = I_y^2$

KCL: $I_r = I_{r1} + I_{r2}$



Static Translinear Circuit Synthesis: Pythagorator

TLP: $I_{r1} I_r = I_x^2$

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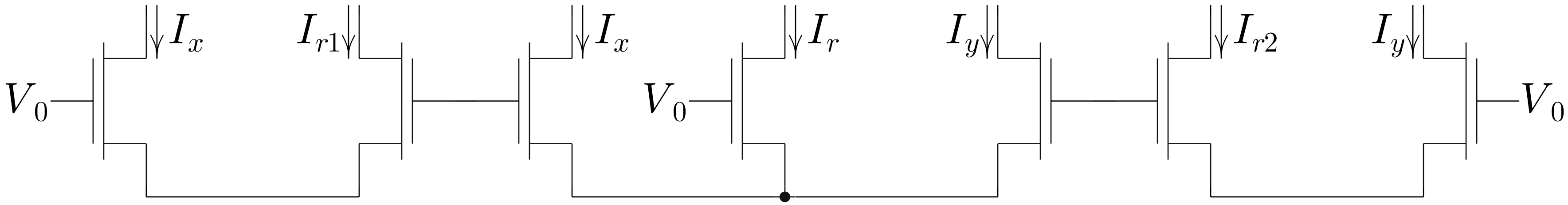
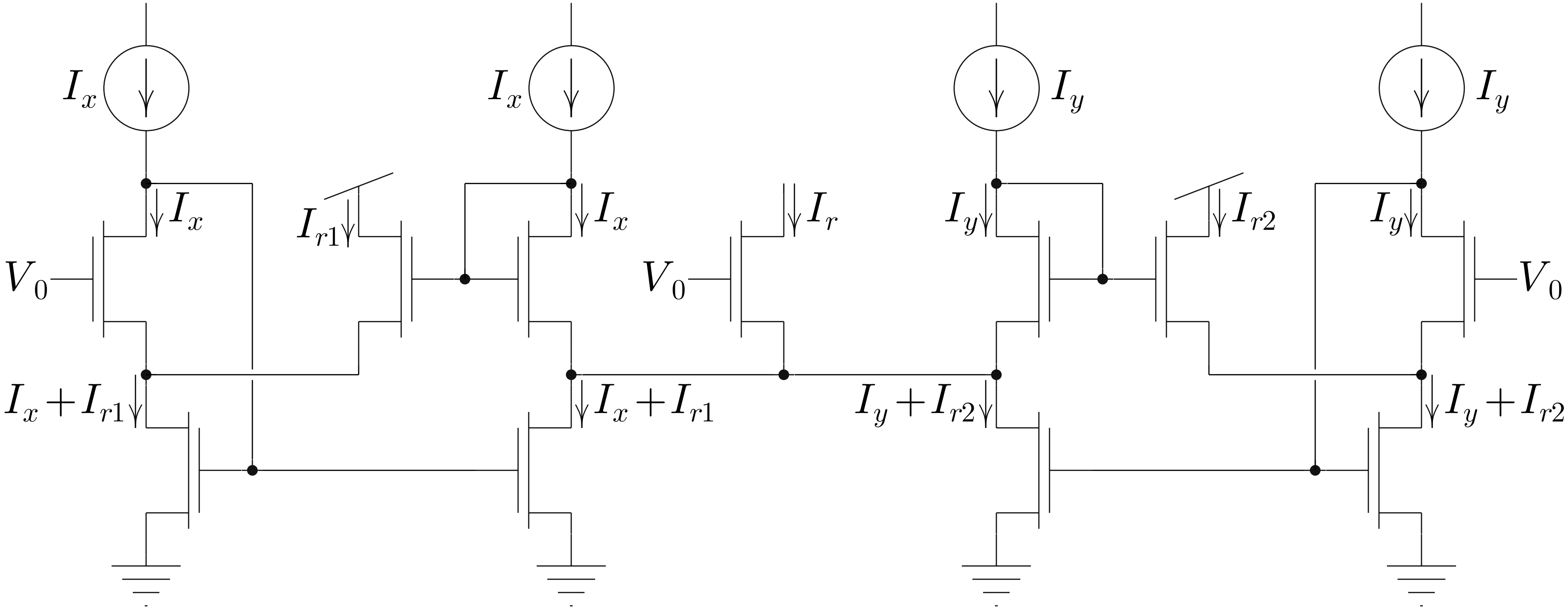


Static Translinear Circuit Synthesis: Pythagorator

TLP: $I_{r1} I_r = I_x^2$

$I_{r2} I_r = I_y^2$

KCL: $I_r = I_{r1} + I_{r2}$



Static Translinear Circuit Synthesis: Vector Normalizer

Synthesize a two-dimensional vector-normalization circuit implementing

$$u = \frac{x}{\sqrt{x^2 + y^2}} \quad \text{and} \quad v = \frac{y}{\sqrt{x^2 + y^2}}, \quad \text{where} \quad x > 0 \quad \text{and} \quad y > 0.$$

Static Translinear Circuit Synthesis: Vector Normalizer

Synthesize a two-dimensional vector-normalization circuit implementing

$$u = \frac{x}{\sqrt{x^2 + y^2}} \quad \text{and} \quad v = \frac{y}{\sqrt{x^2 + y^2}}, \quad \text{where} \quad x > 0 \quad \text{and} \quad y > 0.$$

Each equation shares $r \equiv \sqrt{x^2 + y^2}$, which we can use to decompose the system as

$$u = \frac{x}{r}, \quad v = \frac{y}{r}, \quad \text{and} \quad r = \sqrt{x^2 + y^2}, \quad \text{where} \quad x > 0 \quad \text{and} \quad y > 0.$$

Static Translinear Circuit Synthesis: Vector Normalizer

Synthesize a two-dimensional vector-normalization circuit implementing

$$u = \frac{x}{\sqrt{x^2 + y^2}} \quad \text{and} \quad v = \frac{y}{\sqrt{x^2 + y^2}}, \quad \text{where} \quad x > 0 \quad \text{and} \quad y > 0.$$

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We represent each signal as a ratio of a signal current to the unit current:

$$x \equiv \frac{I_x}{I_1}, \quad y \equiv \frac{I_y}{I_1}, \quad u \equiv \frac{I_u}{I_1}, \quad v \equiv \frac{I_v}{I_1}, \quad \text{and} \quad r \equiv \frac{I_r}{I_1}.$$

Static Translinear Circuit Synthesis: Vector Normalizer

We substitute these into the original equations and rearrange to obtain

$$\frac{I_u}{I_1} = \frac{I_x/I_1}{I_r/I_1} \quad \text{and} \quad \frac{I_v}{I_1} = \frac{I_y/I_1}{I_r/I_1}$$

Static Translinear Circuit Synthesis: Vector Normalizer

We substitute these into the original equations and rearrange to obtain

$$\frac{I_u}{I_1} = \frac{I_x/I_1}{I_r/I_1} \quad \text{and} \quad \frac{I_v}{I_1} = \frac{I_y/I_1}{I_r/I_1} \quad \Longrightarrow \quad I_u I_r = I_x I_1 \quad \text{and} \quad I_v I_r = I_y I_1$$

Static Translinear Circuit Synthesis: Vector Normalizer

We substitute these into the original equations and rearrange to obtain

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and

$$\frac{I_r}{I_1} = \sqrt{\left(\frac{I_x}{I_1}\right)^2 + \left(\frac{I_y}{I_1}\right)^2}$$

Static Translinear Circuit Synthesis: Vector Normalizer

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and

$$\frac{I_r}{I_1} = \sqrt{\left(\frac{I_x}{I_1}\right)^2 + \left(\frac{I_y}{I_1}\right)^2} \quad \Longrightarrow \quad I_r^2 = I_x^2 + I_y^2$$

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and

$$\frac{I_r}{I_1} = \sqrt{\left(\frac{I_x}{I_1}\right)^2 + \left(\frac{I_y}{I_1}\right)^2} \quad \Longrightarrow \quad I_r^2 = I_x^2 + I_y^2 \quad \Longrightarrow \quad I_r = \frac{I_x^2}{I_r} + \frac{I_y^2}{I_r}$$

Static Translinear Circuit Synthesis: Vector Normalizer

We substitute these into the original equations and rearrange to obtain

$$\frac{I_u}{I_1} = \frac{I_x/I_1}{I_r/I_1} \quad \text{and} \quad \frac{I_v}{I_1} = \frac{I_y/I_1}{I_r/I_1} \quad \Longrightarrow \quad I_u I_r = I_x I_1 \quad \text{and} \quad I_v I_r = I_y I_1$$

and

$$\frac{I_r}{I_1} = \sqrt{\left(\frac{I_x}{I_1}\right)^2 + \left(\frac{I_y}{I_1}\right)^2} \quad \Longrightarrow \quad I_r^2 = I_x^2 + I_y^2 \quad \Longrightarrow \quad I_r = \underbrace{\frac{I_x^2}{I_r}}_{I_{r1}} + \underbrace{\frac{I_y^2}{I_r}}_{I_{r2}}$$

Static Translinear Circuit Synthesis: Vector Normalizer

TLP:

$$I_{r1}I_r = I_x^2$$

$$I_{r2}I_r = I_y^2$$

$$I_uI_r = I_xI_1$$

$$I_vI_r = I_yI_1$$

KCL: $I_r = I_{r1}+I_{r2}$

Static Translinear Circuit Synthesis: Vector Normalizer

TLP: $I_{r1}I_r = I_x^2$

$I_{r2}I_r = I_y^2$

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$I_vI_r = I_yI_1$

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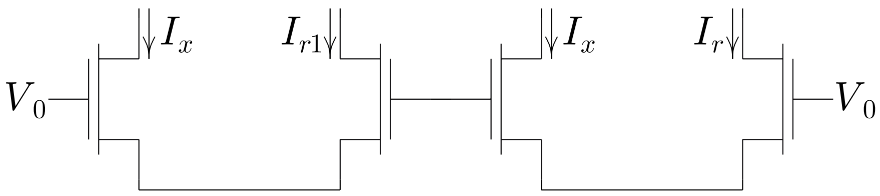

Static Translinear Circuit Synthesis: Vector Normalizer

TLP: $I_{r1} I_r = I_x^2$

$I_{r2} I_r = I_y^2$

$I_u I_r = I_x I_1$

$I_v I_r = I_y I_1$

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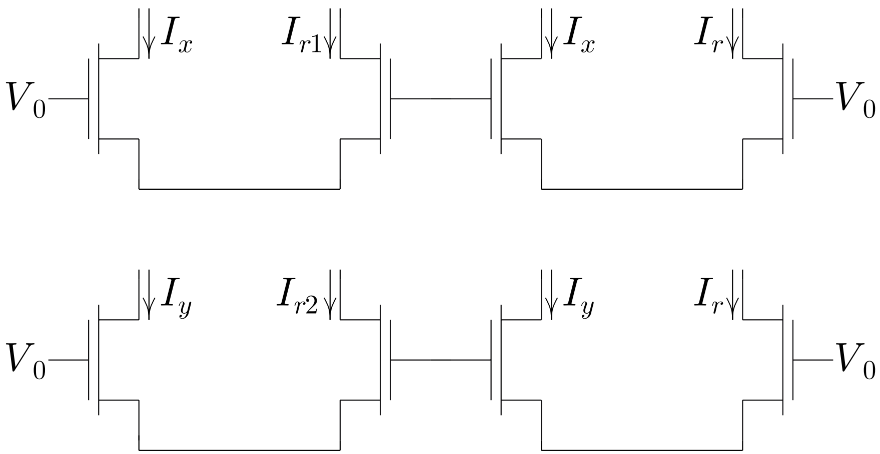
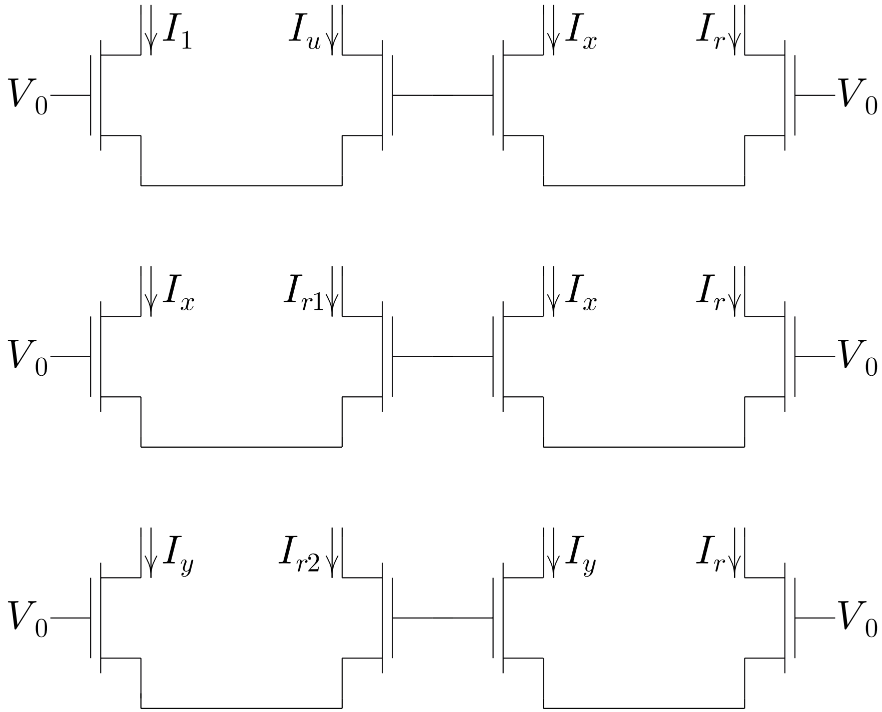


Static Translinear Circuit Synthesis: Vector Normalizer



TLP: $I_{r1}I_r = I_x^2$
$I_{r2}I_r = I_y^2$
$I_uI_r = I_xI_1$
$I_vI_r = I_yI_1$

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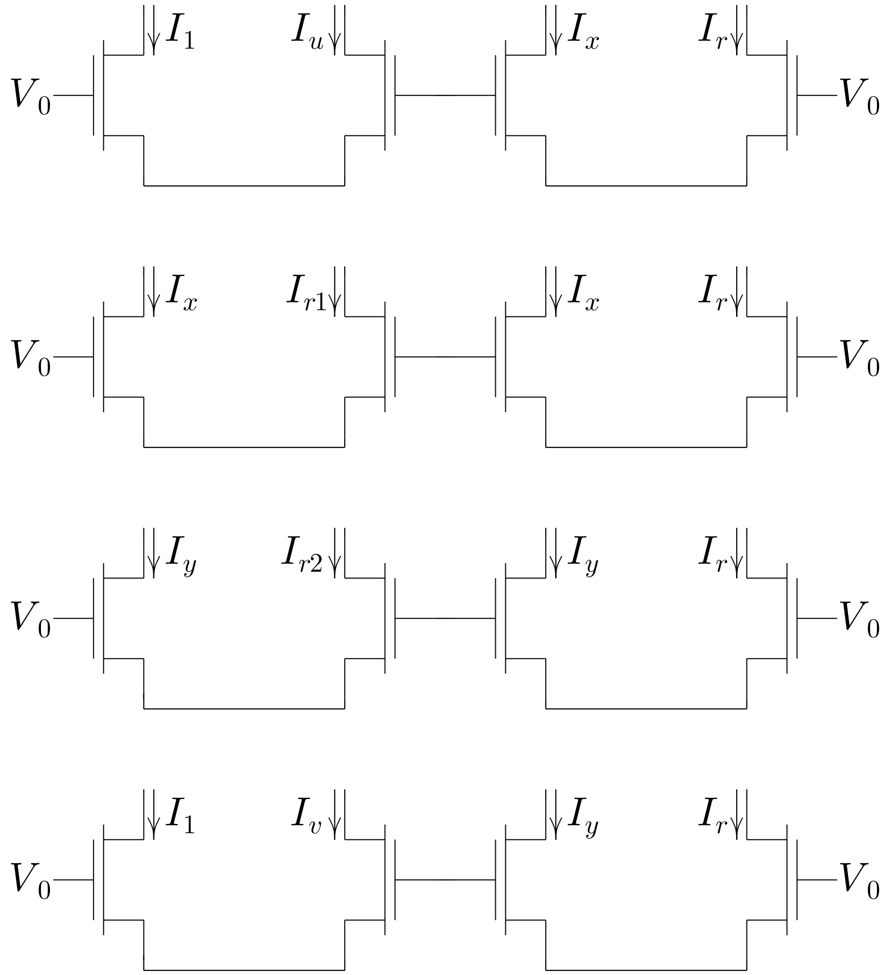

Static Translinear Circuit Synthesis: Vector Normalizer

TLP: $I_{r1}I_r = I_x^2$
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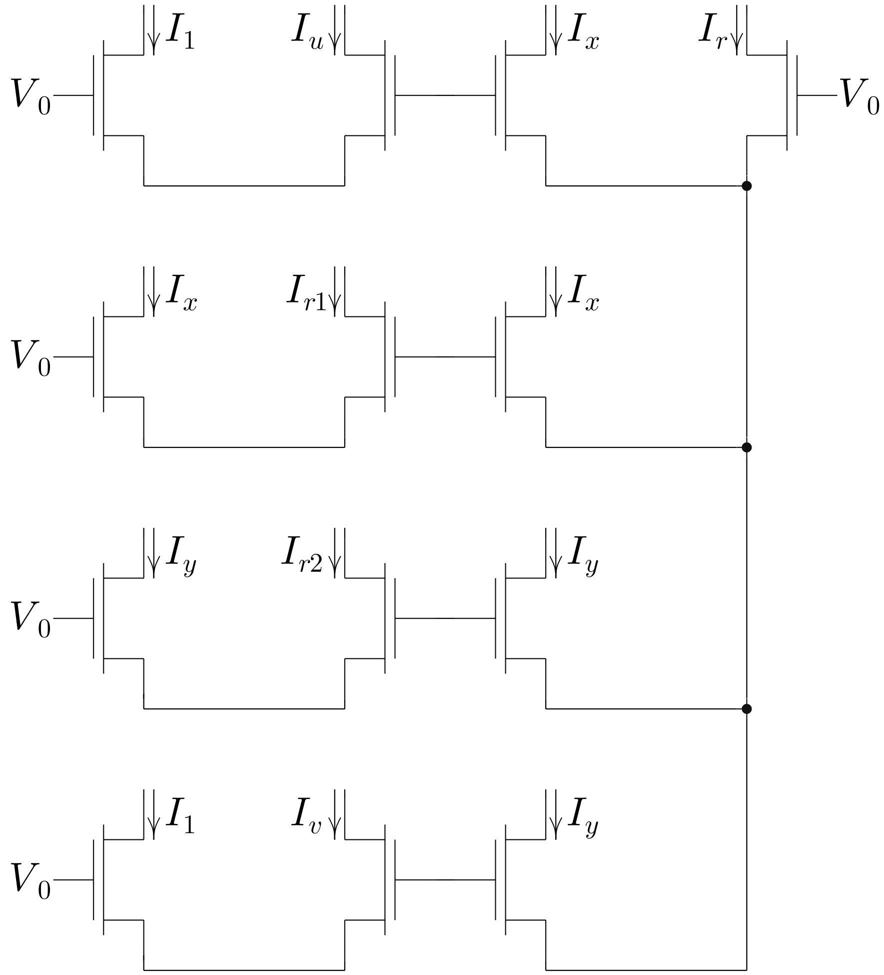

Static Translinear Circuit Synthesis: Vector Normalizer

TLP: $I_{r1}I_r = I_x^2$

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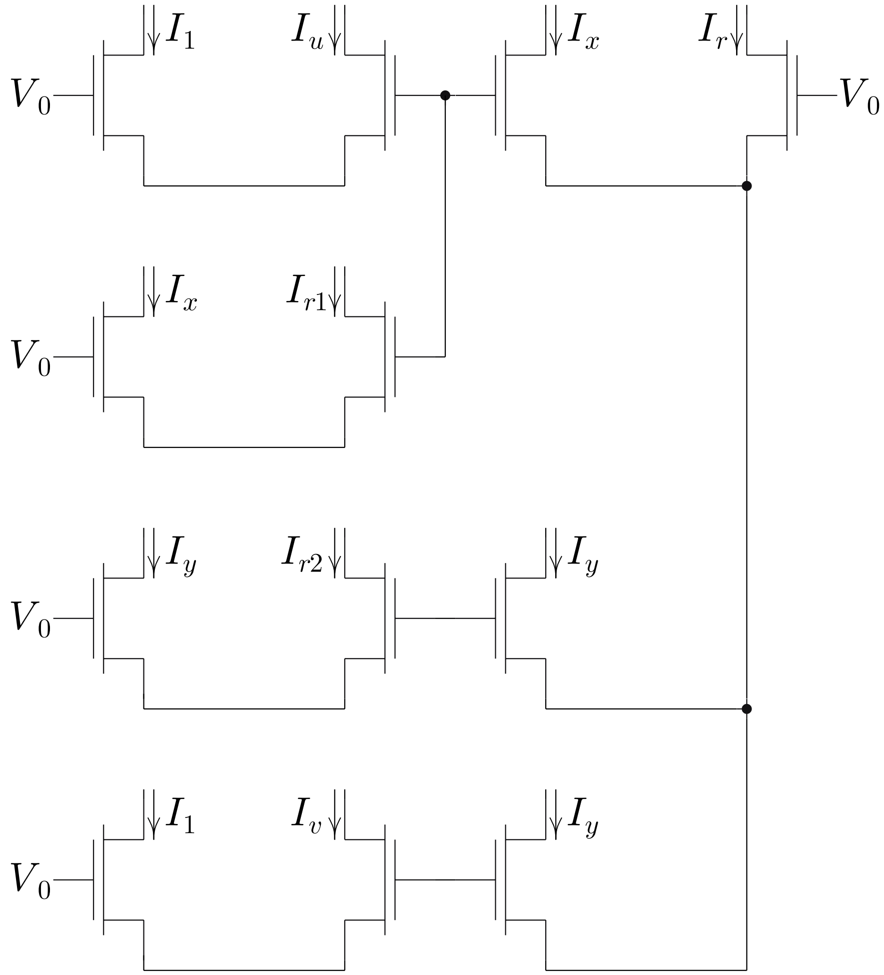

Static Translinear Circuit Synthesis: Vector Normalizer

TLP: $I_{r1} I_r = I_x^2$
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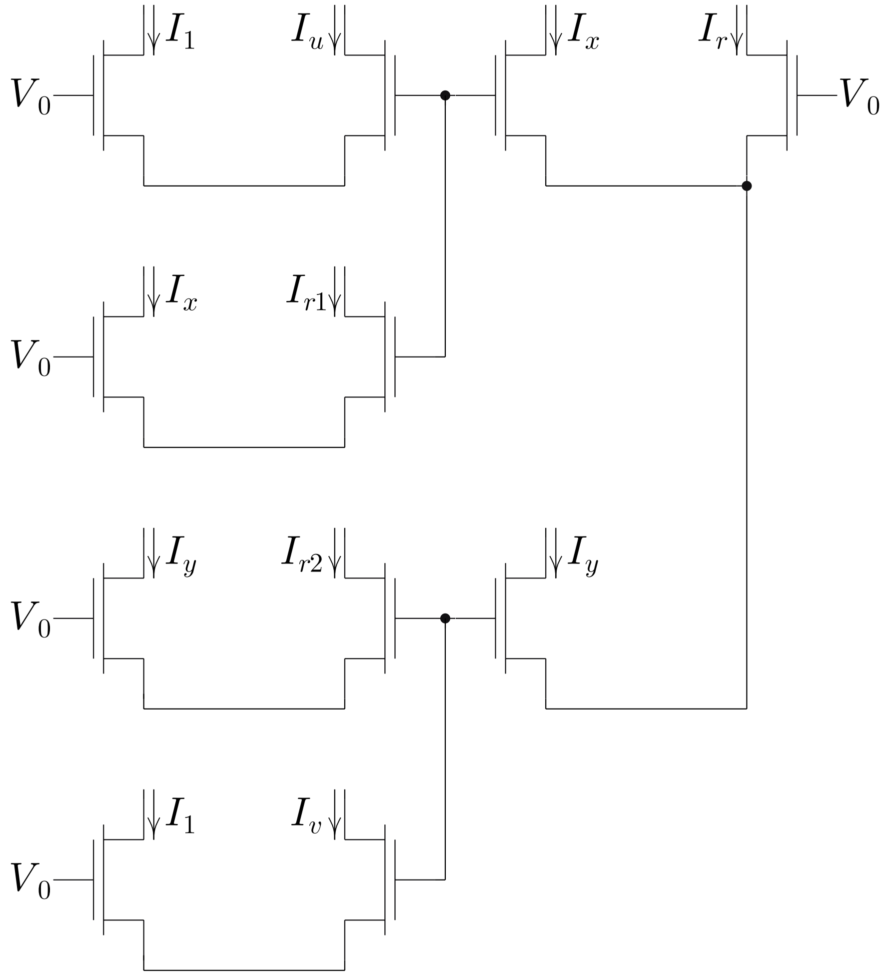
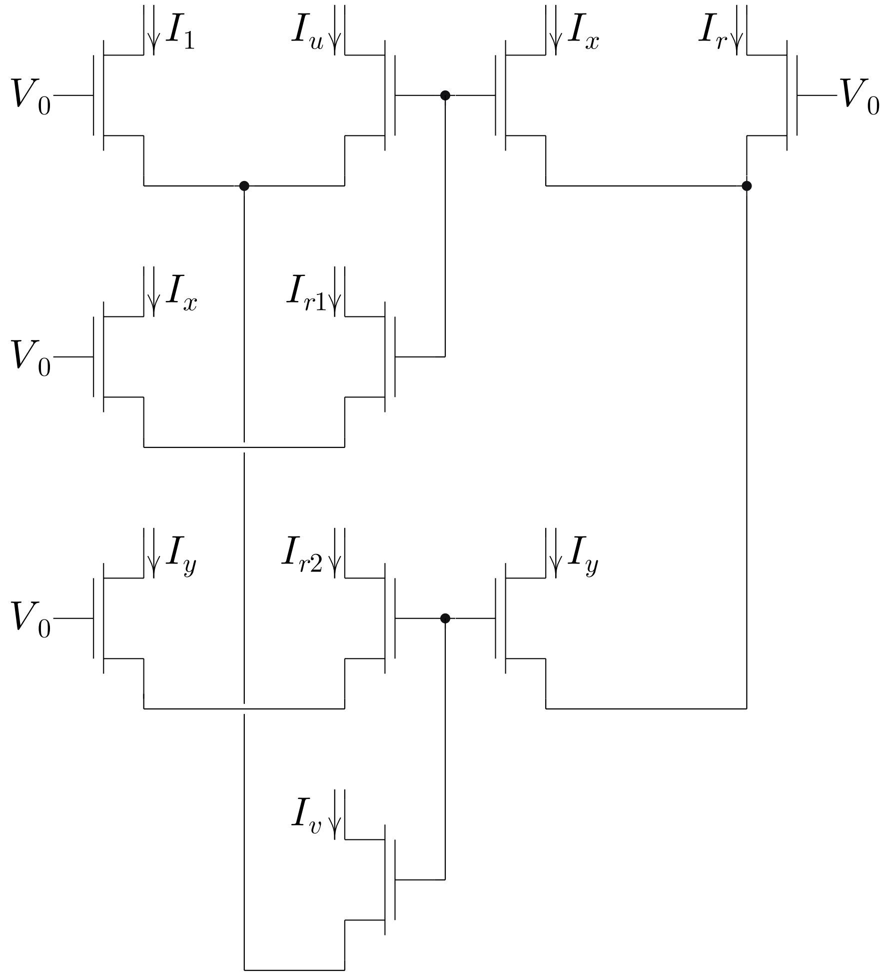


Static Translinear Circuit Synthesis: Vector Normalizer



TLP: $I_{r1}I_r = I_x^2$
$I_{r2}I_r = I_y^2$
$I_uI_r = I_xI_1$
$I_vI_r = I_yI_1$

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Static Translinear Circuit Synthesis: Vector Normalizer

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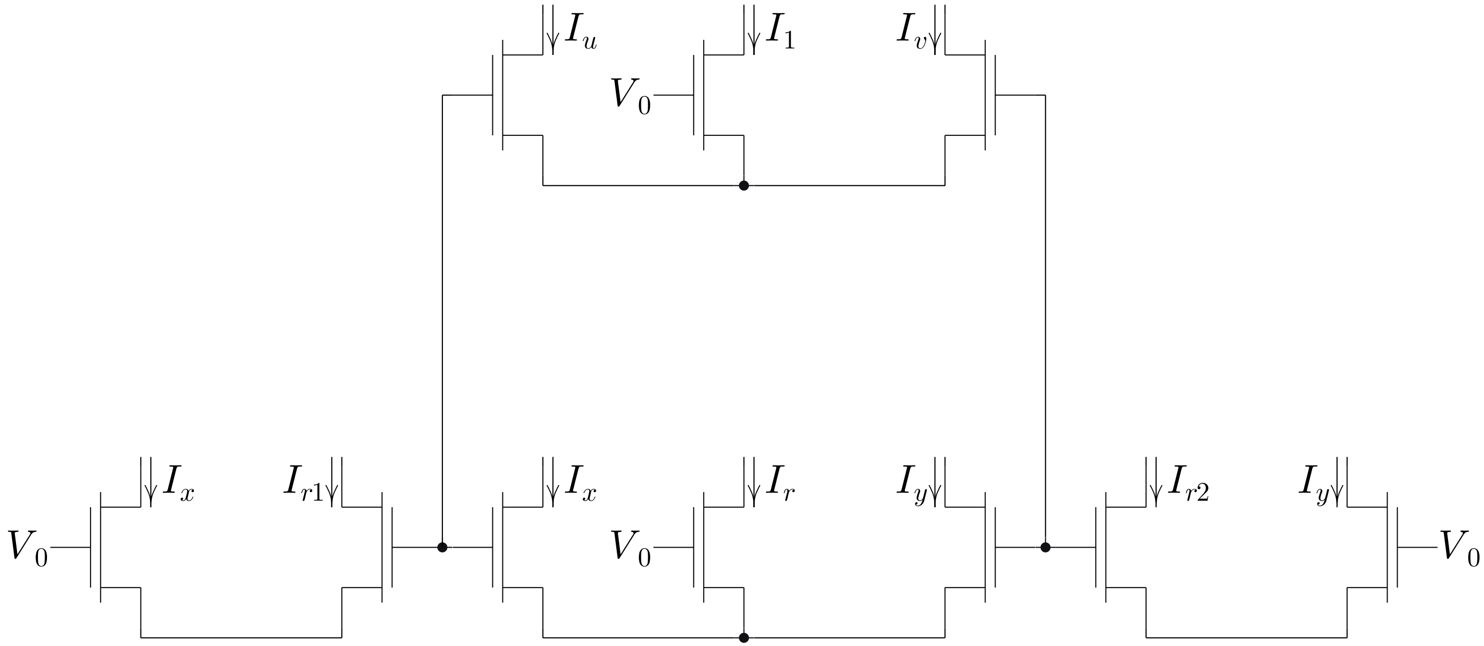

Static Translinear Circuit Synthesis: Vector Normalizer

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$I_{r2} I_r = I_y^2$
$I_u I_r = I_x I_1$
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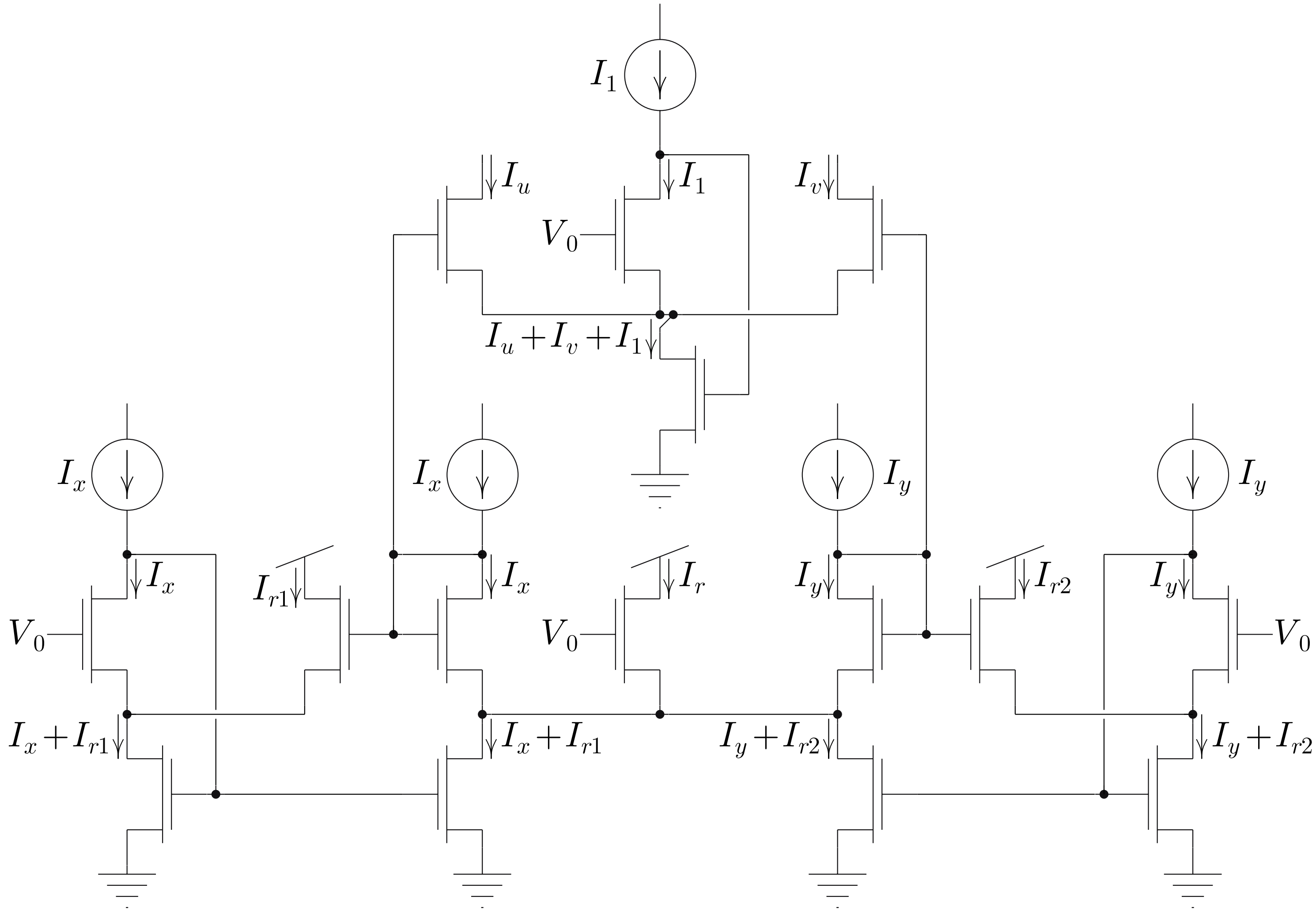
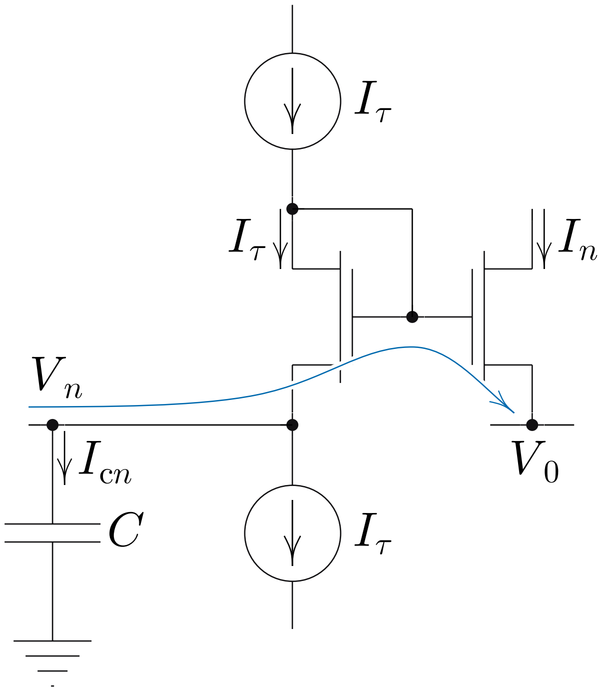


Dynamic Translinear Circuit Synthesis: Output Structures

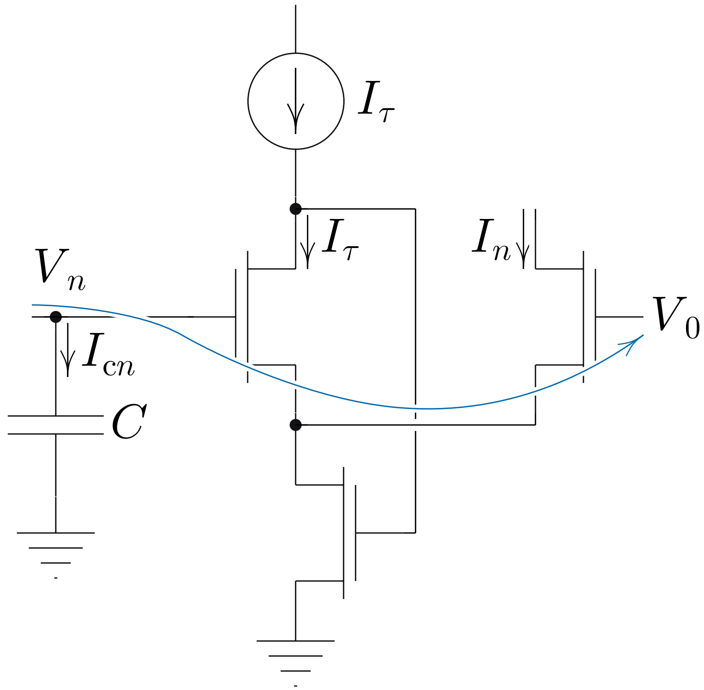
noninverting



$$I_n = I_\tau e^{(V_n - V_0)/U_\mathrm{T}}$$

$$\frac{\partial I_n}{\partial V_n} = \frac{I_n}{U_\mathrm{T}}$$

inverting



$$I_n = I_\tau e^{\kappa(V_0 - V_n)/U_\mathrm{T}}$$

$$\frac{\partial I_n}{\partial V_n} = -\frac{\kappa I_n}{U_\mathrm{T}}$$

Dynamic Translinear Circuit Synthesis: First-Order LPF

Synthesize a first-order low-pass filter described by

$$\tau \frac{dy}{dt} + y = x, \quad \text{where} \quad x > 0.$$

Dynamic Translinear Circuit Synthesis: First-Order LPF

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We represent each signal as a ratio of a signal current to the unit current:

$$x \equiv \frac{I_x}{I_1} \quad \text{and} \quad y \equiv \frac{I_y}{I_1}.$$

Substituting these into the ODE, we obtain

$$\tau \frac{d}{dt}\left(\frac{I_y}{I_1}\right) + \frac{I_y}{I_1} = \frac{I_x}{I_1}$$

Dynamic Translinear Circuit Synthesis: First-Order LPF

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$$\tau \frac{dy}{dt} + y = x, \quad \text{where} \quad x > 0.$$

We represent each signal as a ratio of a signal current to the unit current:

$$x \equiv \frac{I_x}{I_1} \quad \text{and} \quad y \equiv \frac{I_y}{I_1}.$$

Substituting these into the ODE, we obtain

$$\tau \frac{d}{dt}\left(\frac{I_y}{I_1}\right) + \frac{I_y}{I_1} = \frac{I_x}{I_1} \quad \Longrightarrow \quad \tau \frac{dI_y}{dt} + I_y = I_x.$$

Dynamic Translinear Circuit Synthesis: First-Order LPF

To implement the time derivative, we introduce a log-compressed voltage state variable, V_y. Using the chain rule, we can express the preceding equation as

$$\tau \frac{\partial I_y}{\partial V_y} \cdot \frac{dV_y}{dt} + I_y = I_x$$

Dynamic Translinear Circuit Synthesis: First-Order LPF

To implement the time derivative, we introduce a log-compressed voltage state variable, V_y. Using the chain rule, we can express the preceding equation as

$$\tau \frac{\partial I_y}{\partial V_y} \cdot \frac{dV_y}{dt} + I_y = I_x \quad \Longrightarrow \quad \tau \left(-\frac{\kappa}{U_\mathrm{T}} I_y \right) \frac{dV_y}{dt} + I_y = I_x$$

Dynamic Translinear Circuit Synthesis: First-Order LPF

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$$\Longrightarrow -\frac{\kappa\tau}{U_\mathrm{T}} \cdot \frac{dV_y}{dt} + 1 = \frac{I_x}{I_y}$$

Dynamic Translinear Circuit Synthesis: First-Order LPF

To implement the time derivative, we introduce a log-compressed voltage state variable, V_y. Using the chain rule, we can express the preceding equation as

$$\tau \frac{\partial I_y}{\partial V_y} \cdot \frac{dV_y}{dt} + I_y = I_x \quad \Longrightarrow \quad \tau \left(-\frac{\kappa}{U_\mathrm{T}} I_y \right) \frac{dV_y}{dt} + I_y = I_x$$

$$\Longrightarrow \; -\frac{\kappa\tau}{U_\mathrm{T}} \cdot \frac{dV_y}{dt} + 1 = \frac{I_x}{I_y} \quad \Longrightarrow \quad -\frac{\kappa\tau}{CU_\mathrm{T}} \cdot C \frac{dV_y}{dt} + 1 = \frac{I_x}{I_y}$$

Dynamic Translinear Circuit Synthesis: First-Order LPF

To implement the time derivative, we introduce a log-compressed voltage state variable, V_y. Using the chain rule, we can express the preceding equation as

$$\tau\frac{\partial I_y}{\partial V_y}\cdot\frac{dV_y}{dt}+I_y=I_x \quad\Longrightarrow\quad \tau\left(-\frac{\kappa}{U_{\mathrm{T}}}I_y\right)\frac{dV_y}{dt}+I_y=I_x$$

$$\Longrightarrow\ -\frac{\kappa\tau}{U_{\mathrm{T}}}\cdot\frac{dV_y}{dt}+1=\frac{I_x}{I_y} \quad\Longrightarrow\quad -\underbrace{\frac{\kappa\tau}{CU_{\mathrm{T}}}}_{1/I_\tau}\cdot\underbrace{C\frac{dV_y}{dt}}_{I_c}+1=\frac{I_x}{I_y}$$

Dynamic Translinear Circuit Synthesis: First-Order LPF

To implement the time derivative, we introduce a log-compressed voltage state variable, V_y. Using the chain rule, we can express the preceding equation as

$$\tau \frac{\partial I_y}{\partial V_y} \cdot \frac{dV_y}{dt} + I_y = I_x \quad \Longrightarrow \quad \tau \left(-\frac{\kappa}{U_\mathrm{T}} I_y \right) \frac{dV_y}{dt} + I_y = I_x$$

$$\Longrightarrow -\frac{\kappa\tau}{U_\mathrm{T}} \cdot \frac{dV_y}{dt} + 1 = \frac{I_x}{I_y} \quad \Longrightarrow \quad -\underbrace{\frac{\kappa\tau}{CU_\mathrm{T}}}_{1/I_\tau} \cdot \underbrace{C\frac{dV_y}{dt}}_{I_c} + 1 = \frac{I_x}{I_y}$$

$$\Longrightarrow -\frac{I_c}{I_\tau} + 1 = \frac{I_x}{I_y}$$

Dynamic Translinear Circuit Synthesis: First-Order LPF

To implement the time derivative, we introduce a log-compressed voltage state variable, V_y. Using the chain rule, we can express the preceding equation as

$$\tau\frac{\partial I_y}{\partial V_y}\cdot\frac{dV_y}{dt}+I_y=I_x \quad\Longrightarrow\quad \tau\left(-\frac{\kappa}{U_{\mathrm{T}}}I_y\right)\frac{dV_y}{dt}+I_y=I_x$$

$$\Longrightarrow -\frac{\kappa\tau}{U_{\mathrm{T}}}\cdot\frac{dV_y}{dt}+1=\frac{I_x}{I_y} \quad\Longrightarrow\quad -\underbrace{\frac{\kappa\tau}{CU_{\mathrm{T}}}}_{1/I_\tau}\cdot\underbrace{C\frac{dV_y}{dt}}_{I_c}+1=\frac{I_x}{I_y}$$

$$\Longrightarrow -\frac{I_c}{I_\tau}+1=\frac{I_x}{I_y} \quad\Longrightarrow\quad I_\tau-I_c=\frac{I_\tau I_x}{I_y}.$$

Dynamic Translinear Circuit Synthesis: First-Order LPF

To implement the time derivative, we introduce a log-compressed voltage state variable, V_y. Using the chain rule, we can express the preceding equation as

$$\tau \frac{\partial I_y}{\partial V_y} \cdot \frac{dV_y}{dt} + I_y = I_x \quad \Longrightarrow \quad \tau \left(-\frac{\kappa}{U_\mathrm{T}} I_y \right) \frac{dV_y}{dt} + I_y = I_x$$

$$\Longrightarrow -\frac{\kappa\tau}{U_\mathrm{T}} \cdot \frac{dV_y}{dt} + 1 = \frac{I_x}{I_y} \quad \Longrightarrow \quad -\underbrace{\frac{\kappa\tau}{CU_\mathrm{T}}}_{1/I_\tau} \cdot \underbrace{C\frac{dV_y}{dt}}_{I_c} + 1 = \frac{I_x}{I_y}$$

$$\Longrightarrow -\frac{I_c}{I_\tau} + 1 = \frac{I_x}{I_y} \quad \Longrightarrow \quad I_\tau - I_c = \frac{I_\tau I_x}{\underbrace{I_y}_{I_p}}.$$

Dynamic Translinear Circuit Synthesis: First-Order LPF

TLP: $I_p I_y = I_x I_\tau$ KCL: $I_c + I_p = I_\tau$

Dynamic Translinear Circuit Synthesis: First-Order LPF

TLP: $I_p I_y = I_x I_\tau$ KCL: $I_c + I_p = I_\tau$

Dynamic Translinear Circuit Synthesis: First-Order LPF

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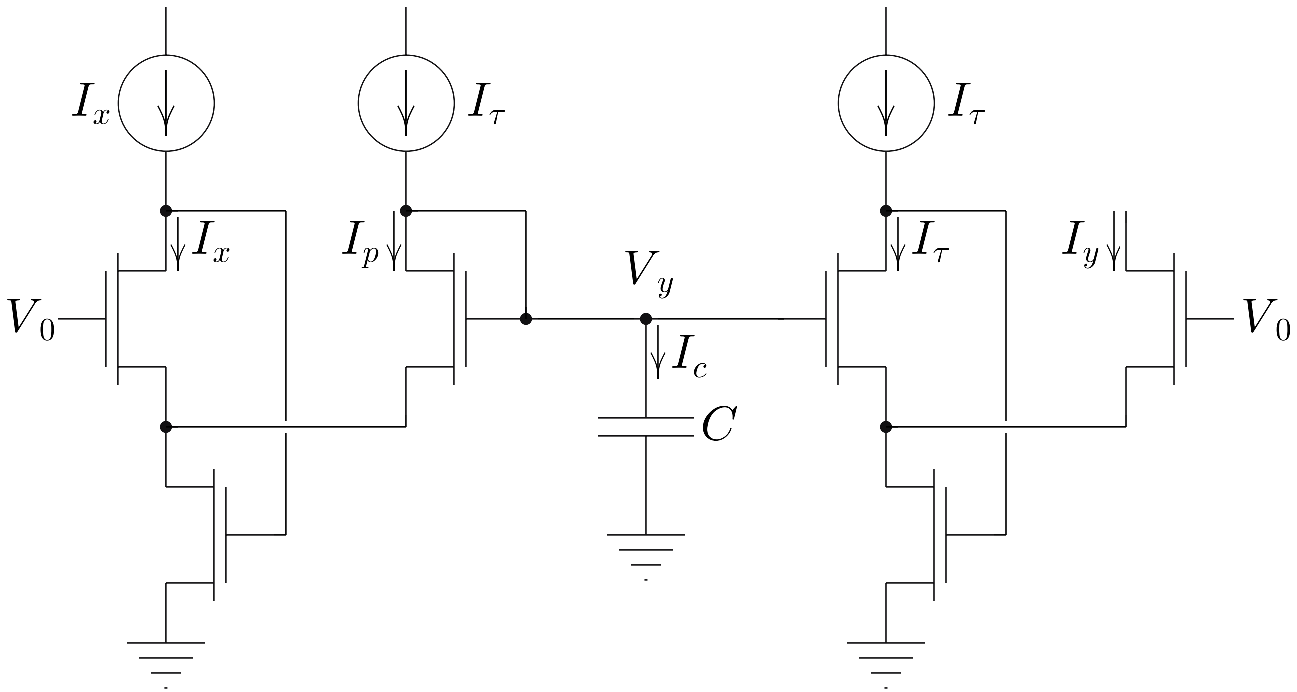

Dynamic Translinear Circuit Synthesis: RMS-DC Converter

Synthesize an RMS-to-DC converter described by

$$x = w^2, \quad \tau \frac{dy}{dt} + y = x, \quad \text{and} \quad z = \sqrt{y}.$$

Dynamic Translinear Circuit Synthesis: RMS-DC Converter

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We can eliminate x and y from the system description by substituting

$$x = w^2, \quad y = z^2, \quad \text{and} \quad \frac{dy}{dt} = 2z\frac{dz}{dt}$$

into the linear ODE describing the low-pass filter, obtaining a first-order nonlinear ODE describing the system given by

$$2\tau z\frac{dz}{dt} + z^2 = w^2.$$

Dynamic Translinear Circuit Synthesis: RMS-DC Converter

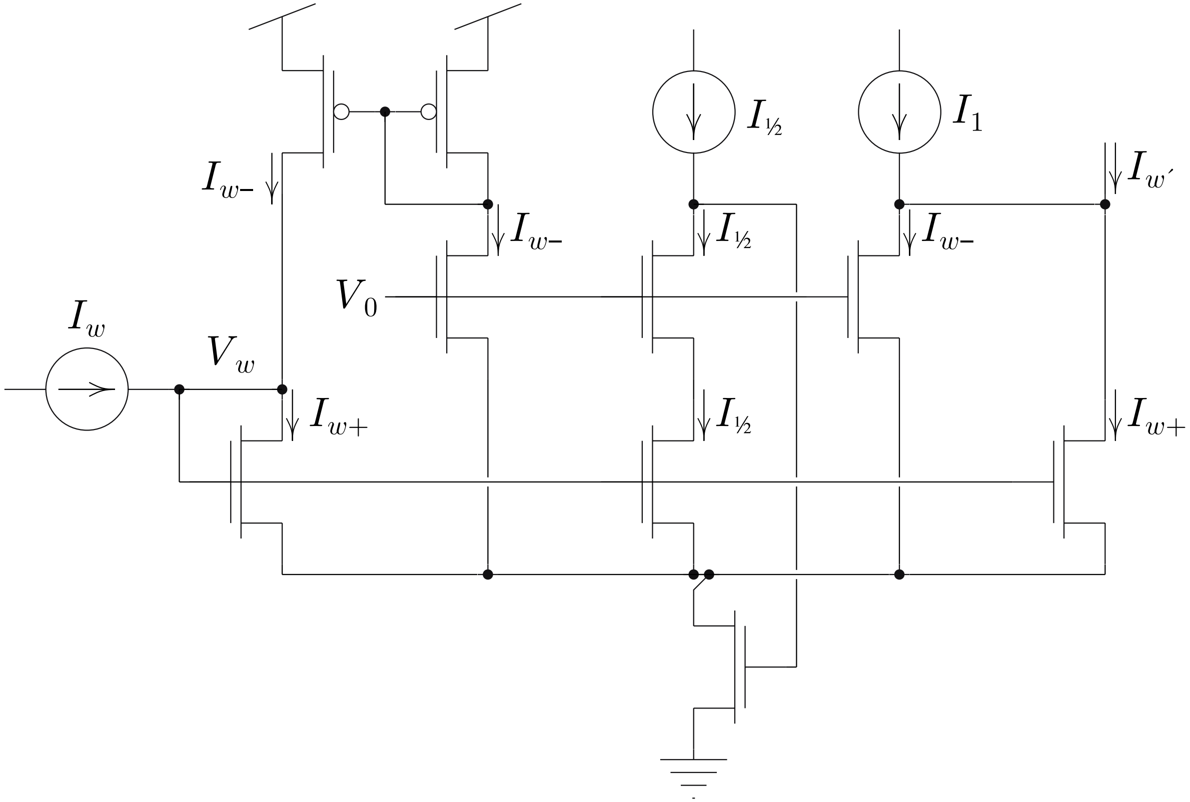
$$w_+ \equiv \frac{I_{w+}}{I_1} = \tfrac{1}{2}\left(1 + e^{\kappa(V_w - V_0)/U_\mathrm{T}}\right)$$

$$w_- \equiv \frac{I_{w-}}{I_1} = \tfrac{1}{2}\left(1 + e^{-\kappa(V_w - V_0)/U_\mathrm{T}}\right)$$

$$w \equiv \frac{I_w}{I_1} = w_+ - w_- = \sinh\frac{\kappa\left(V_w - V_0\right)}{U_\mathrm{T}}$$

$$w' \equiv \frac{I_{w'}}{I_1} = w_+ + w_- - 1 = \cosh\frac{\kappa\left(V_w - V_0\right)}{U_\mathrm{T}}$$

$$\boxed{w^2 = \left(w'\right)^2 - 1}$$



Dynamic Translinear Circuit Synthesis: RMS-DC Converter

The input signal, w, can be positive or negative. To remedy this situation, we adopt a sinh representation for w and and define an associated signal, w', as just described. Substituting $w^2 = (w')^2 - 1$ into the nonlinear ODE, we obtain

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We represent each signal as a ratio of a signal current to the unit current:

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Dynamic Translinear Circuit Synthesis: RMS-DC Converter

To implement the time derivative, we introduce a log-compressed voltage state variable, V_z. Using the chain rule, we can express the preceding equation as

$$2\tau I_z \frac{\partial I_z}{\partial V_z} \cdot \frac{dV_z}{dt} + I_z^2 = I_{w'}^2 - I_1^2$$

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Dynamic Translinear Circuit Synthesis: RMS-DC Converter

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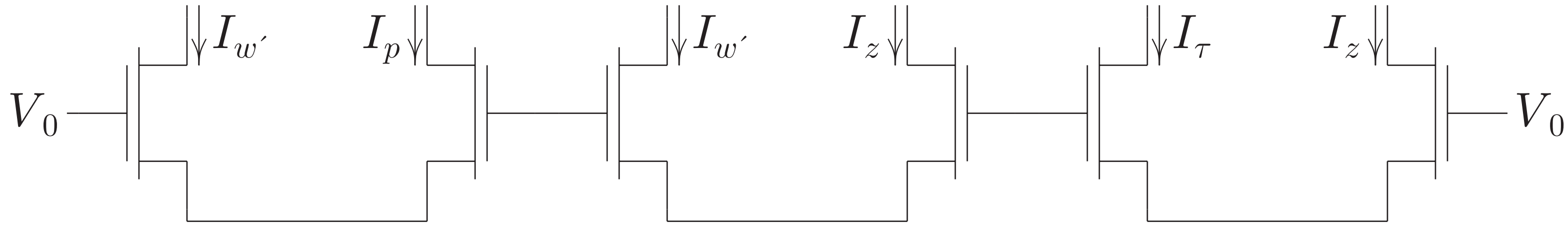

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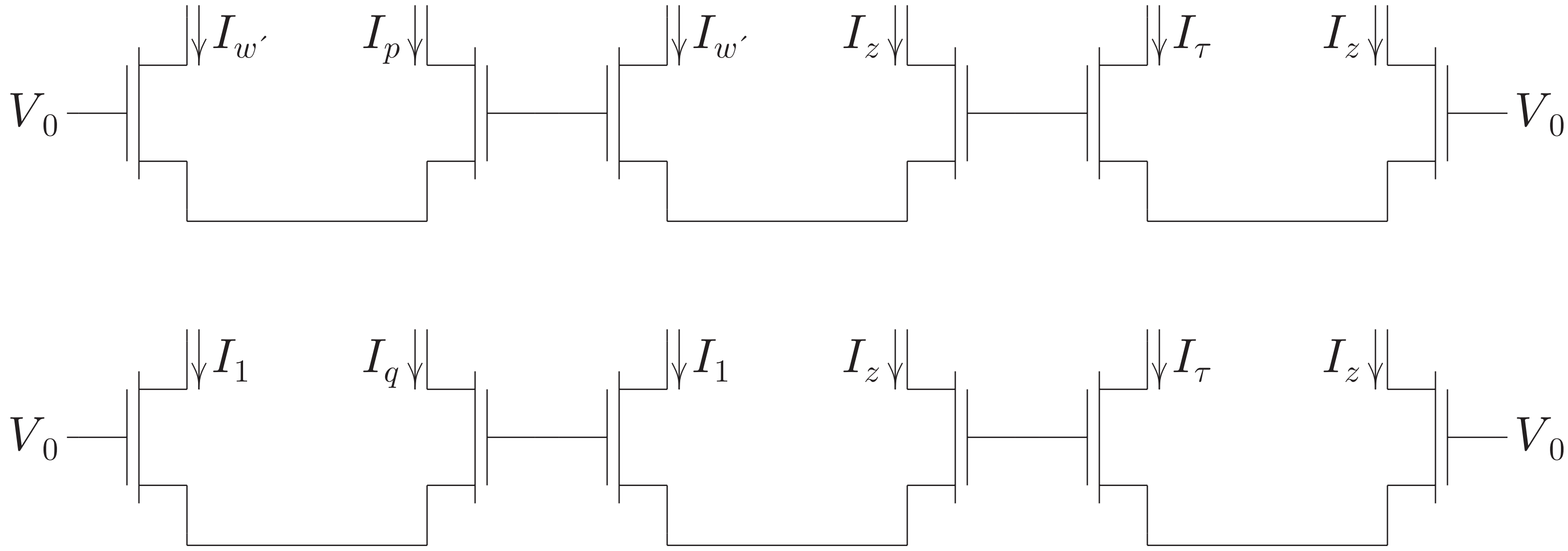

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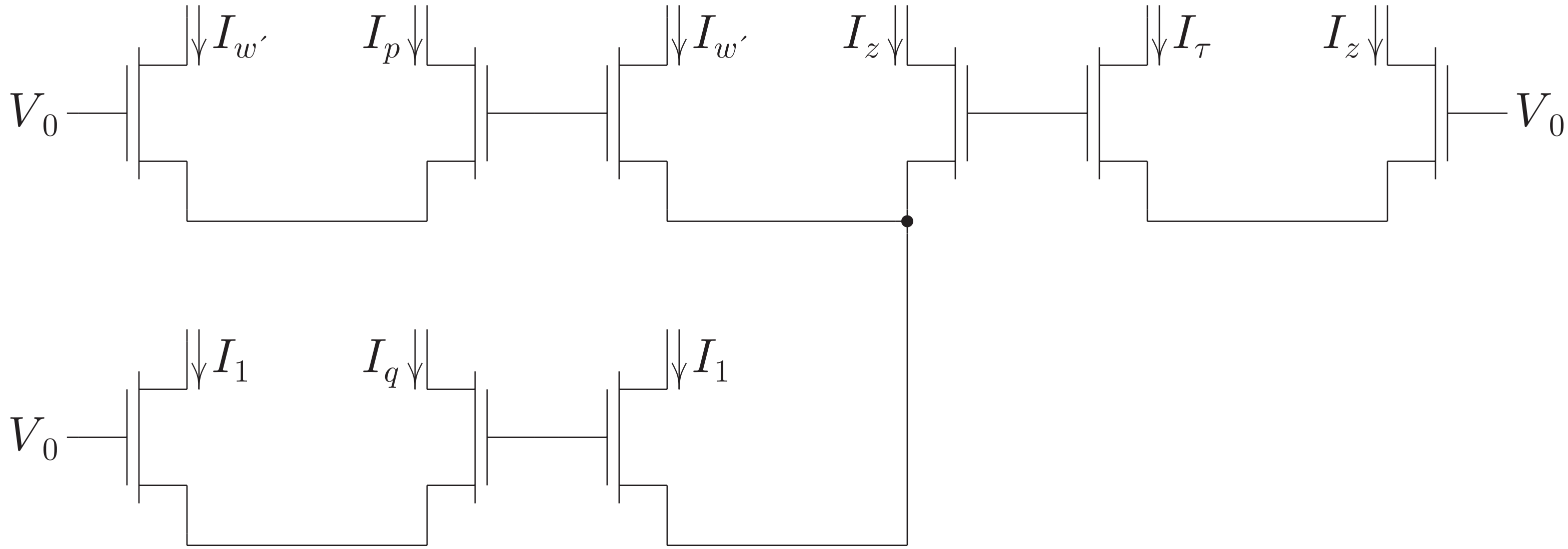

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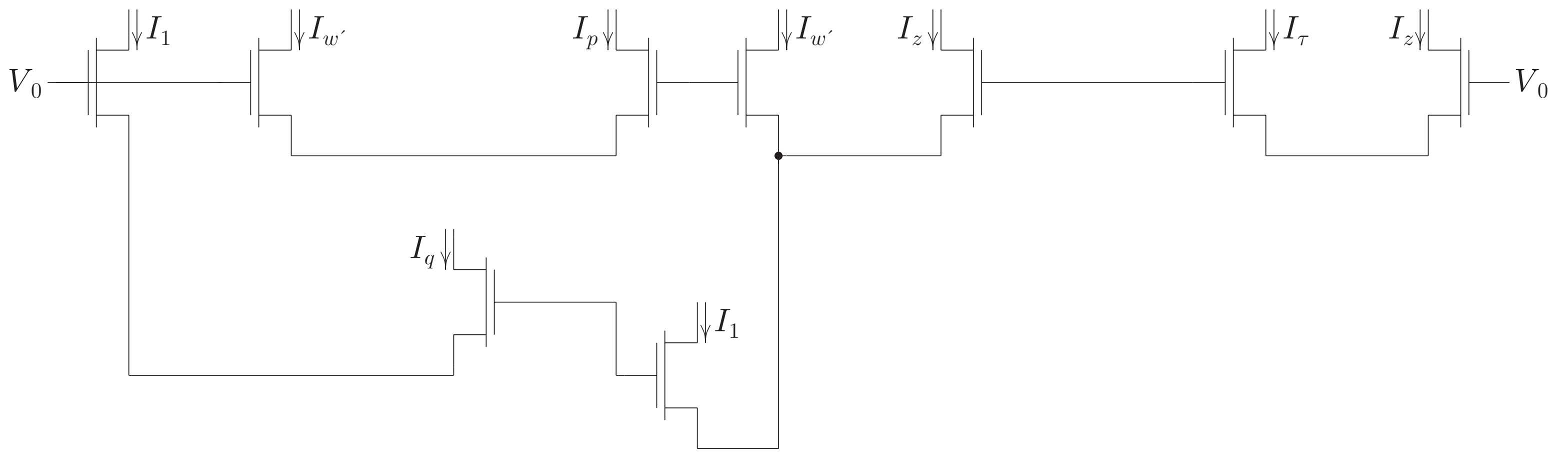

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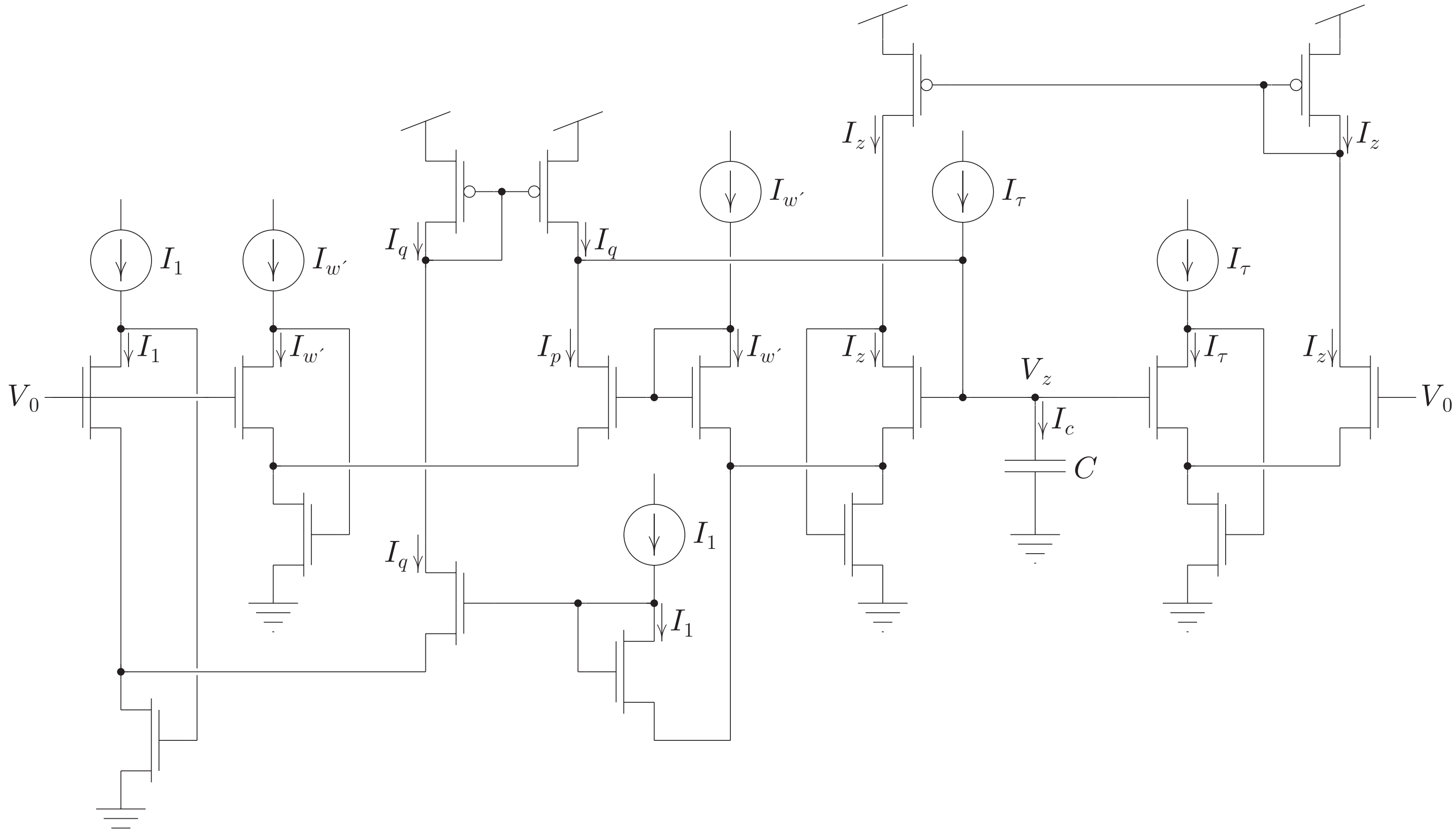

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