Static and Dynamic Translinear Circuits

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May 20, 2010

Translinear Circuits: What's in a Name?

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The word translinear refers to the exponential I/V characteristic of the bipolar transistor—its transconductance is linear in its collector current:

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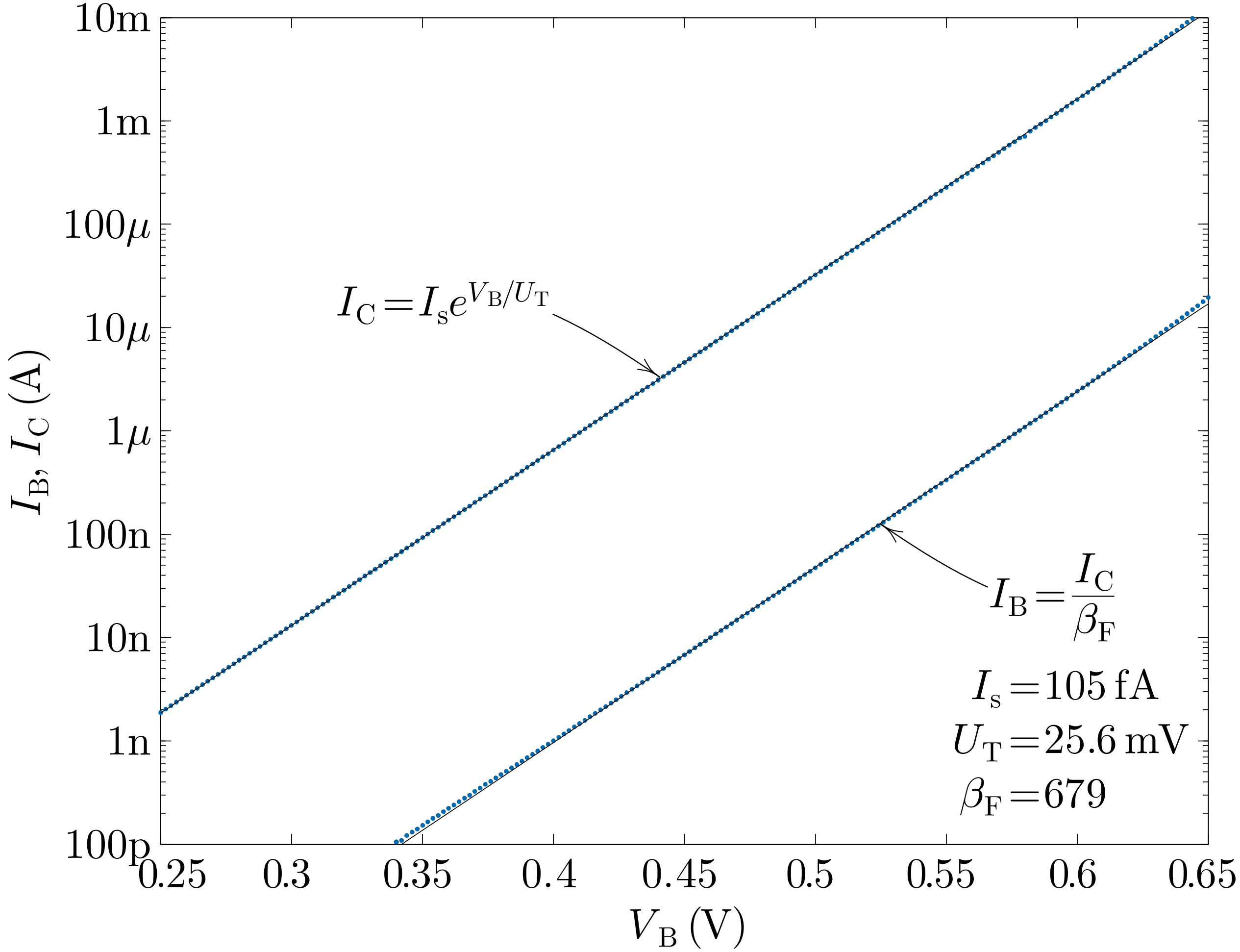
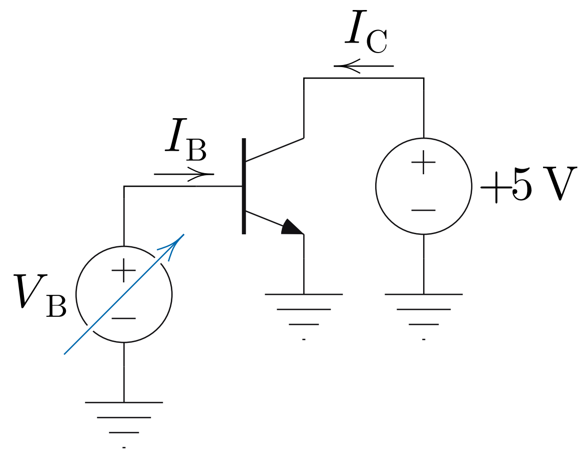
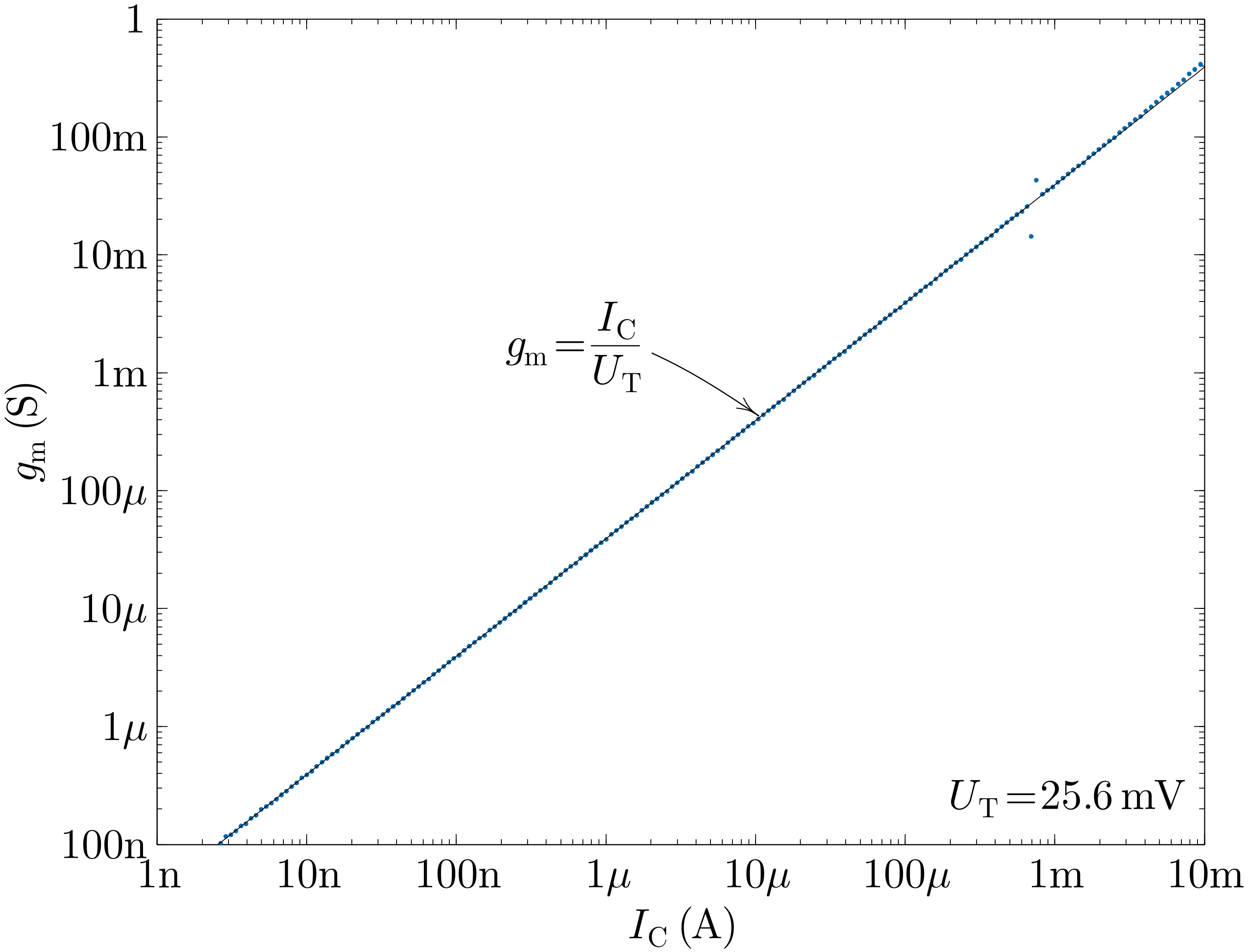
Gilbert also meant the word translinear to refer to circuit analysis and design principles that bridge the gap between the familiar territory of linear circuits and the uncharted domain of nonlinear circuits.

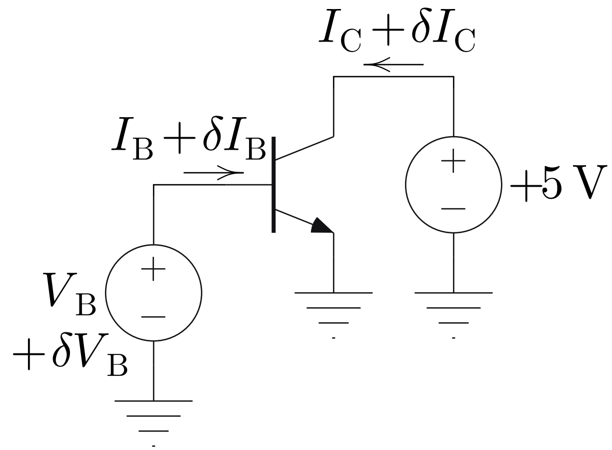
Gummel Plot of a Forward-Active Bipolar Transistor





Translinearity of the Forward-Active Bipolar Transistor





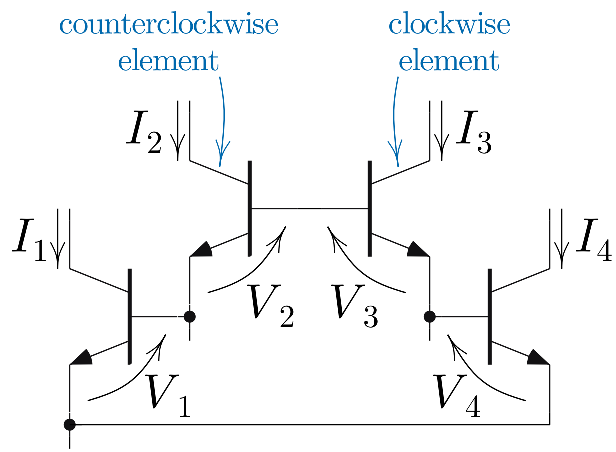
$$g_m = \frac{\partial I_C}{\partial V_B} = \frac{I_C}{U_T}$$

$$\delta I_C \approx g_m \delta V_B$$

The Translinear Principle

Consider a closed loop of base-emitter junctions of four closely matched *npn* bipolar transistors biased in the forward-active region and operating at the same temperature. Kirchhoff's voltage law (KVL) implies that

$$\begin{aligned} V_1 + V_2 &= V_3 + V_4 \\ U_\mathrm{T}\log\frac{I_1}{I_\mathrm{s}} + U_\mathrm{T}\log\frac{I_2}{I_\mathrm{s}} &= U_\mathrm{T}\log\frac{I_3}{I_\mathrm{s}} + U_\mathrm{T}\log\frac{I_4}{I_\mathrm{s}} \\ \log\frac{I_1 I_2}{I_\mathrm{s}^2} &= \log\frac{I_3 I_4}{I_\mathrm{s}^2} \\ \underbrace{I_1 I_2}_{\text{CCW}} &= \underbrace{I_3 I_4}_{\text{CW}}. \end{aligned}$$



This result is a particular case of Gilbert's *translinear principle* (TLP): The product of the clockwise currents is equal to the product of the counterclockwise currents.

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- Translinear circuits are *robust*. Carefully designed translinear circuits are temperature insensitive and do not depend on device or technology parameters.

Simple EKV Model of the Saturated nMOS Transistor

We model the saturation current of an nMOS transistor by

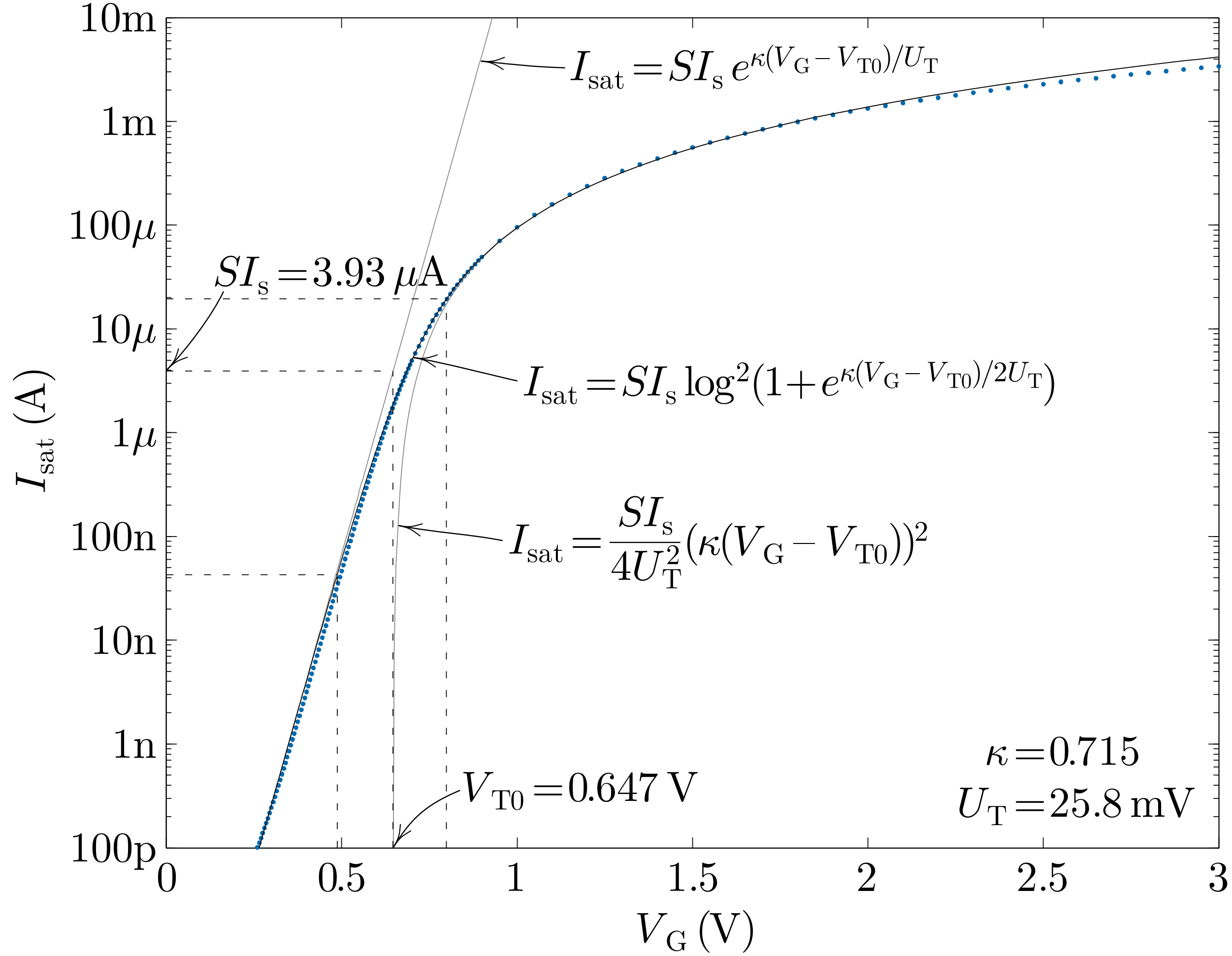
$$
\begin{aligned}
I_{\text{sat}} &= SI_{\text{s}} \log^2\left(1 + e^{(\kappa(V_{\text{G}} - V_{\text{T0}}) - V_{\text{S}})/2U_{\text{T}}}\right) \\
&\approx \begin{cases} SI_{\text{s}} e^{(\kappa(V_{\text{G}} - V_{\text{T0}}) - V_{\text{S}})/U_{\text{T}}}, & \kappa\left(V_{\text{G}} - V_{\text{T0}}\right) - V_{\text{S}} < 0 \\ \dfrac{SI_{\text{s}}}{4U_{\text{T}}^2}\left(\kappa\left(V_{\text{G}} - V_{\text{T0}}\right) - V_{\text{S}}\right)^2, & \kappa\left(V_{\text{G}} - V_{\text{T0}}\right) - V_{\text{S}} > 0, \end{cases}
\end{aligned}
$$

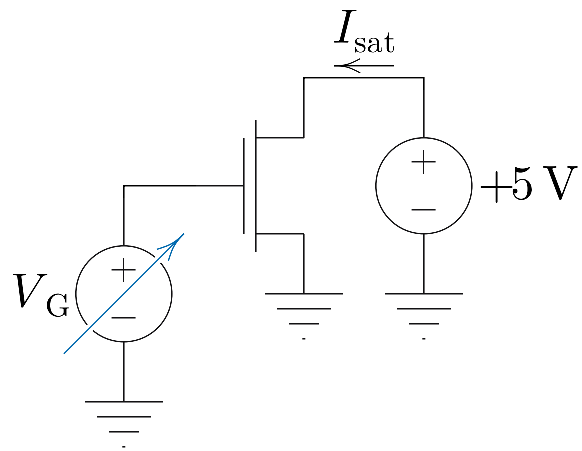
where

$$
U_{\text{T}} = \frac{kT}{q}, \quad S = \frac{W}{L}, \quad I_{\text{s}} = \frac{2\mu C_{\text{ox}} U_{\text{T}}^2}{\kappa}, \quad \text{and} \quad \kappa = \frac{C_{\text{ox}}}{C_{\text{ox}} + C_{\text{dep}}}.
$$

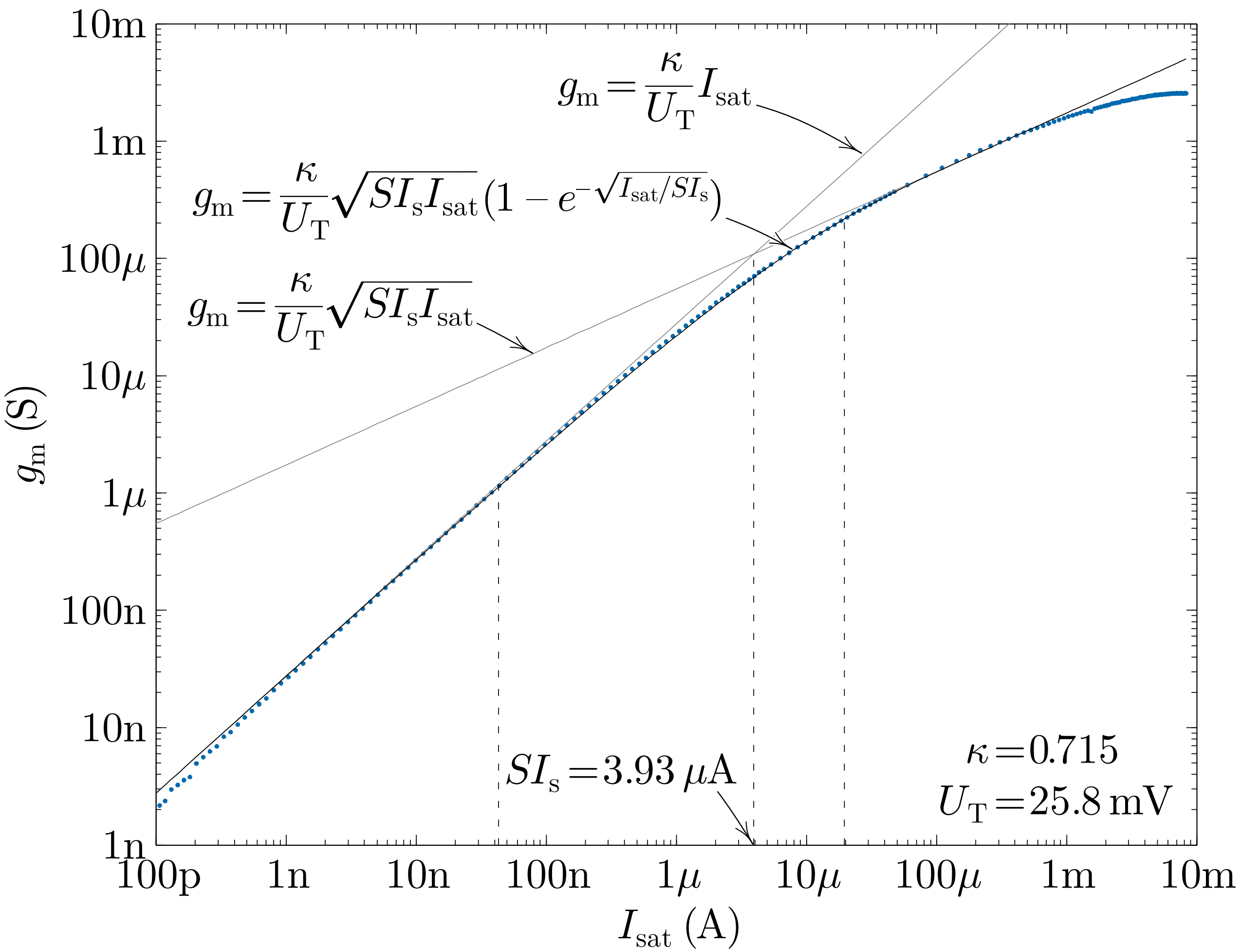
Weak inversion operation corresponds to $I_{\text{sat}} \ll SI_{\text{s}}$, moderate inversion operation corresponds to $I_{\text{sat}} \approx SI_{\text{s}}$, and strong inversion operation to $I_{\text{sat}} \gg SI_{\text{s}}$. Note that SI_{s} is approximately twice the saturation current at threshold.

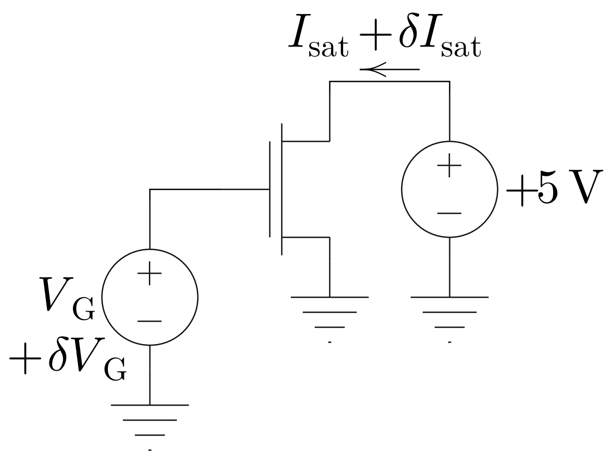
Saturation Current of an nMOS Transistor





Translinearity of the Saturated nMOS Transistor





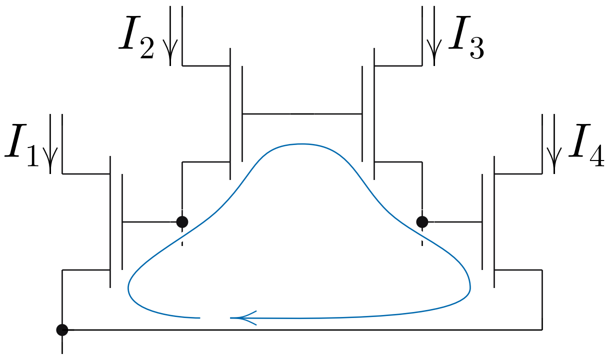
$$g_m = \frac{\partial I_{sat}}{\partial V_G} = \frac{\kappa I_{sat}}{U_T},\ I_{sat} \ll SI_s$$

$$\delta I_{sat} \approx g_m \delta V_G$$

Weak-Inversion MOS Translinear Principle

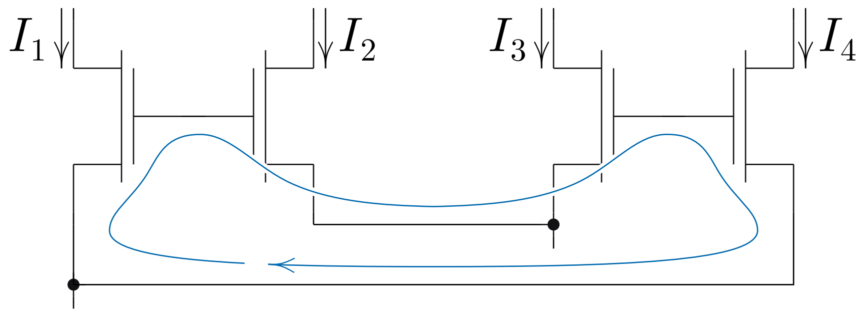
When designing weak-inversion MOS translinear circuits, if we restrict ourselves to using translinear loops that alternate between clockwise and counterclockwise elements, we obtain Gilbert's original TLP, with no dependence on the body effect (i.e., κ).

stacked loop



TLP: $I_1 I_2^\kappa = I_3^\kappa I_4$

alternating loop

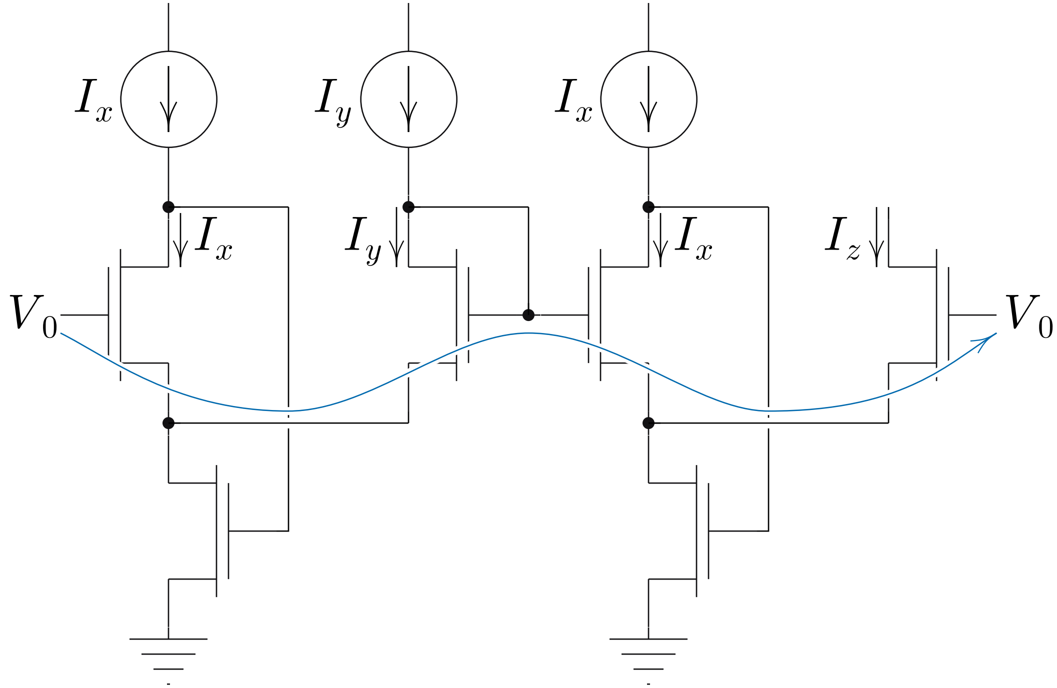


TLP: $I_1 I_3 = I_2 I_4$

This restriction does not limit the class of systems that we can implement. However, designs based on alternating loops generally consume more current but operate on a lower power supply voltage than do designs based on stacked loops.

Static Translinear Circuits: Squaring/Reciprocal

TLP $\Longrightarrow$ $I_x^2 = I_y I_z$ $\Longrightarrow$ $I_z = \dfrac{I_x^2}{I_y}$



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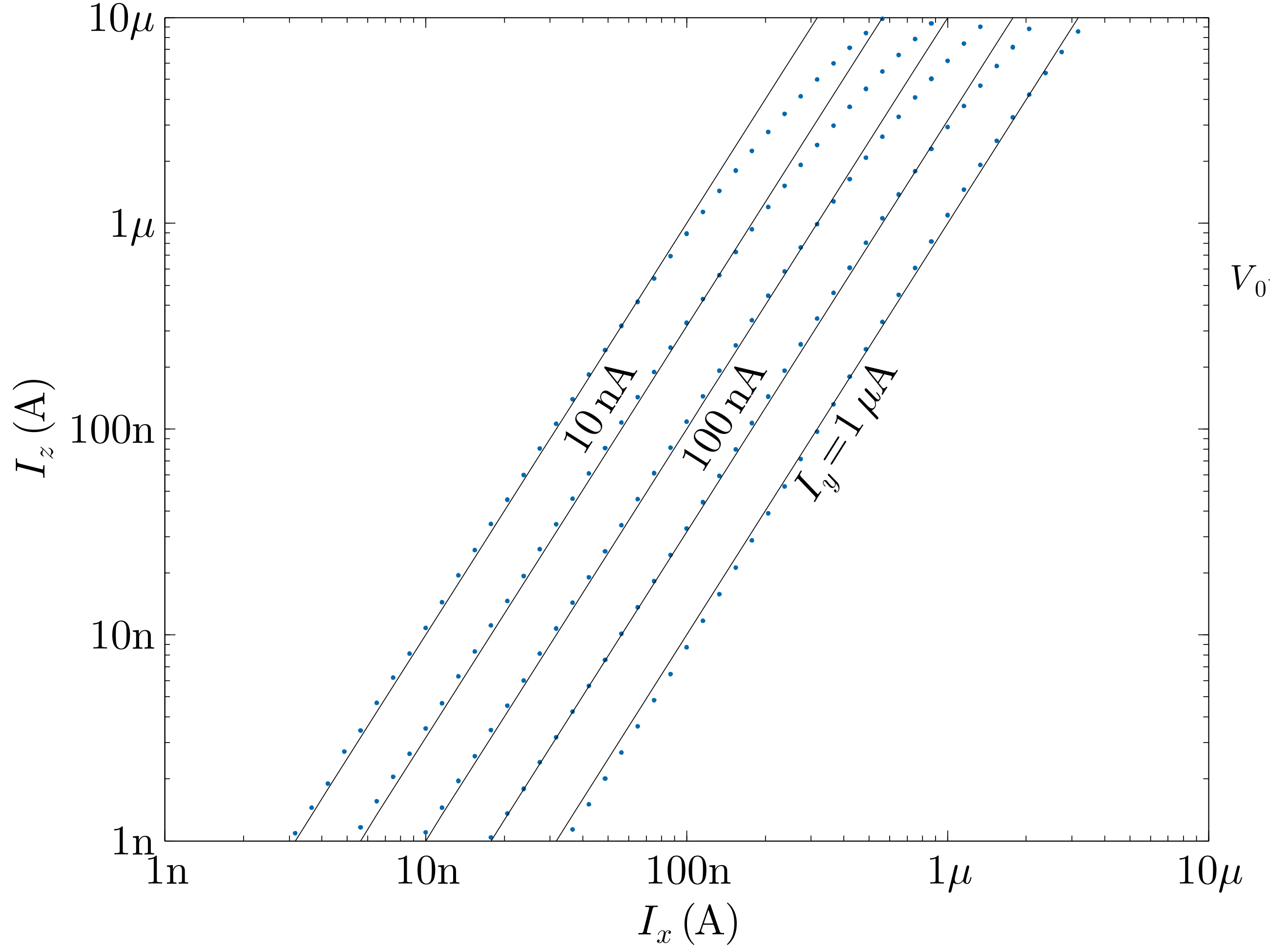

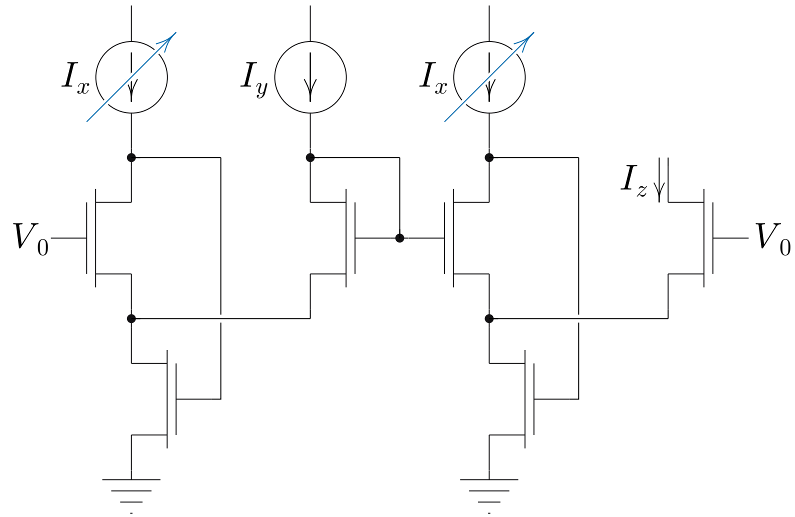

$$I_z = \frac{I_x^2}{I_y}$$

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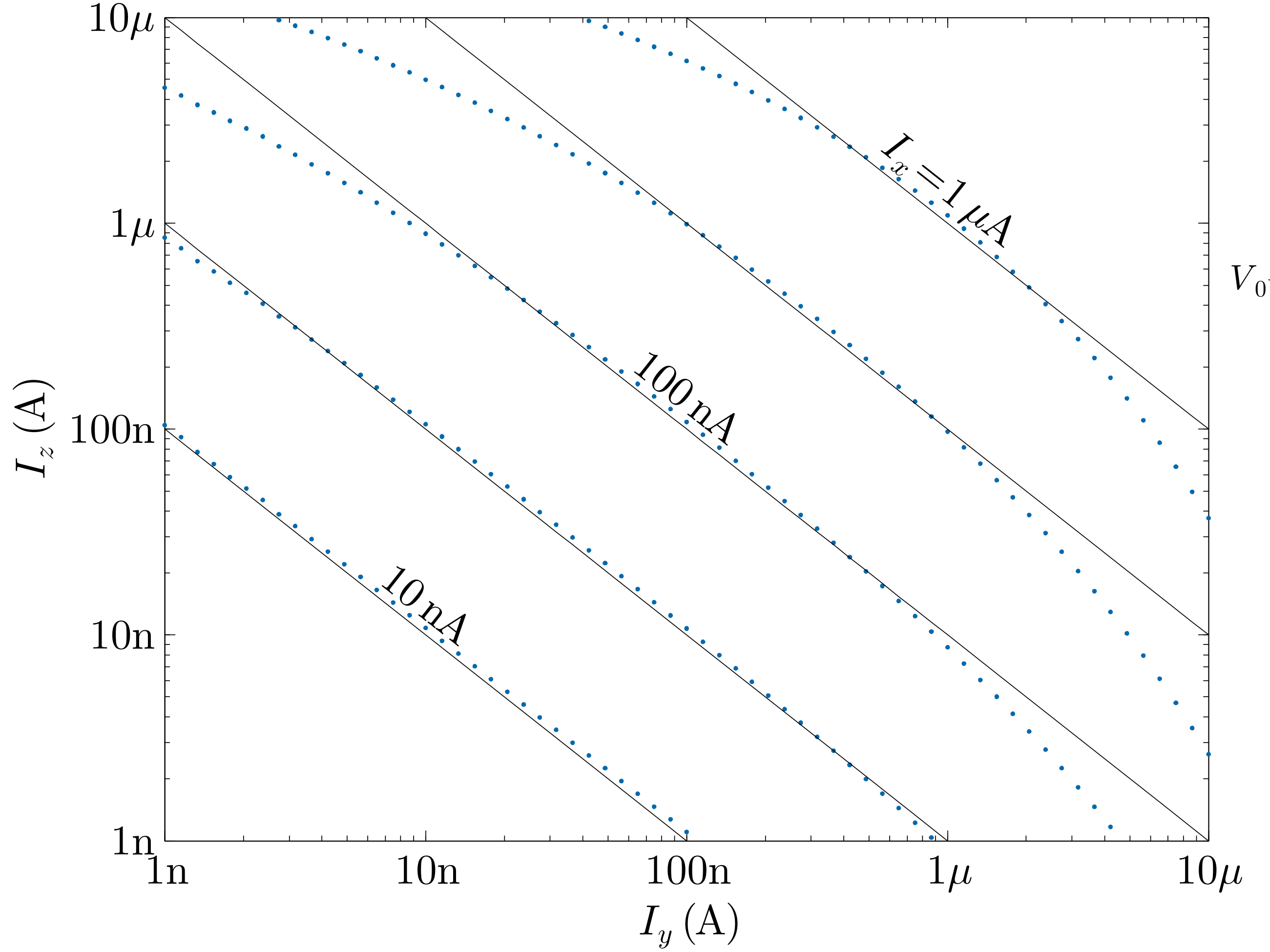

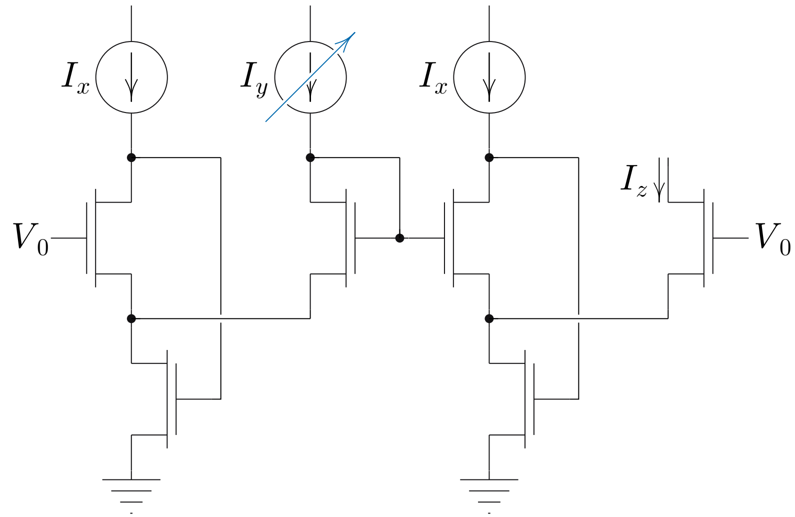

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Static Translinear Circuit Synthesis: Pythagorator

Synthesize a two-dimensional vector-magnitude circuit implementing

$$r = \sqrt{x^2 + y^2}, \quad \text{where} \quad x > 0 \quad \text{and} \quad y > 0.$$

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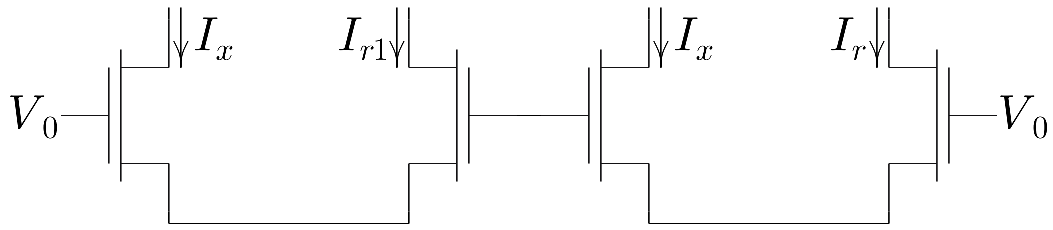

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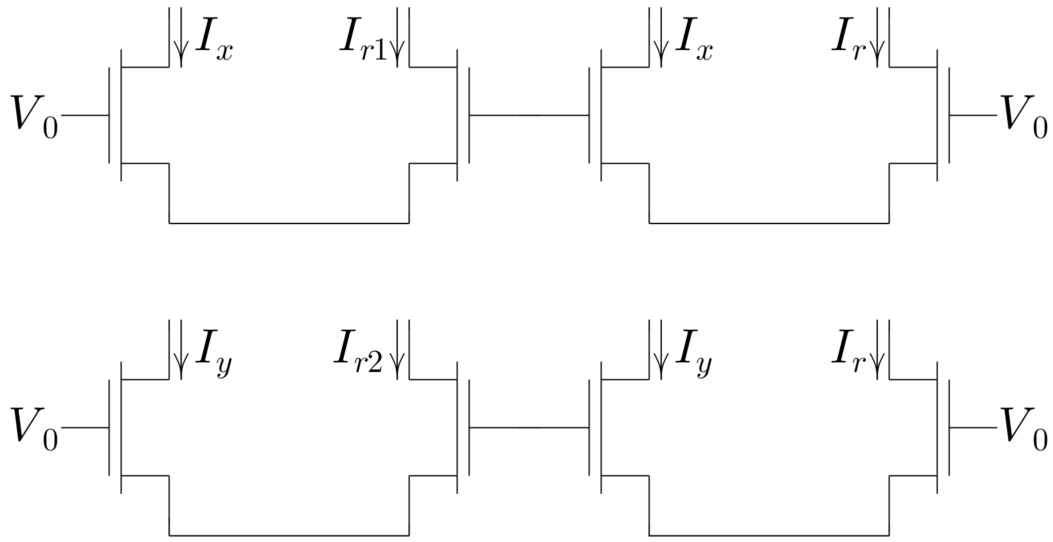

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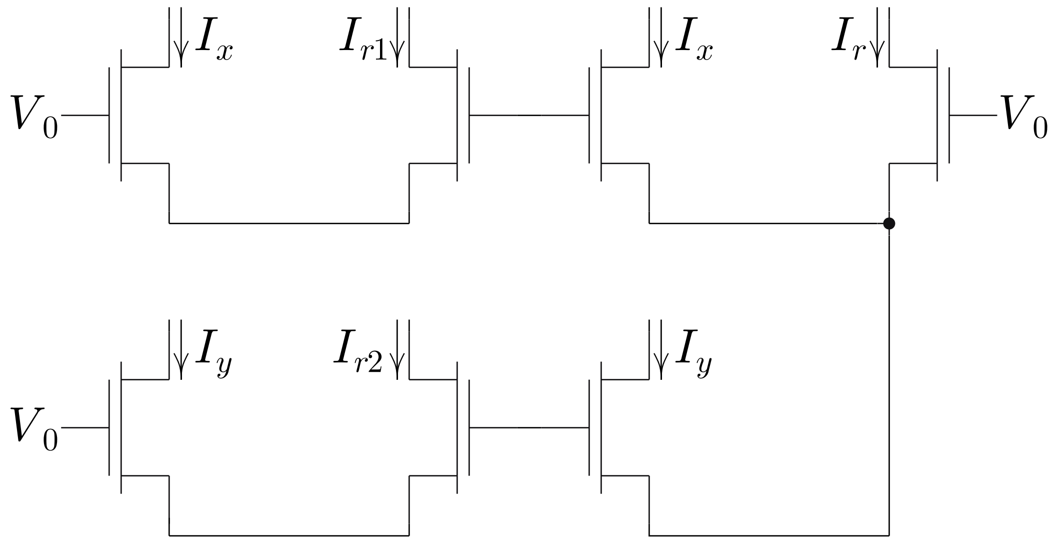

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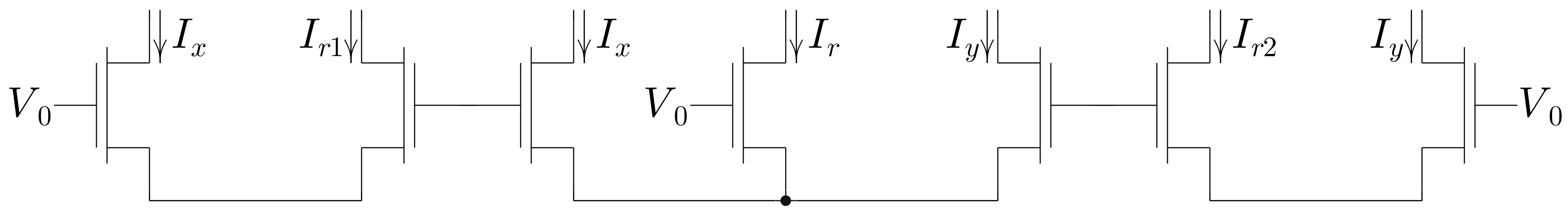

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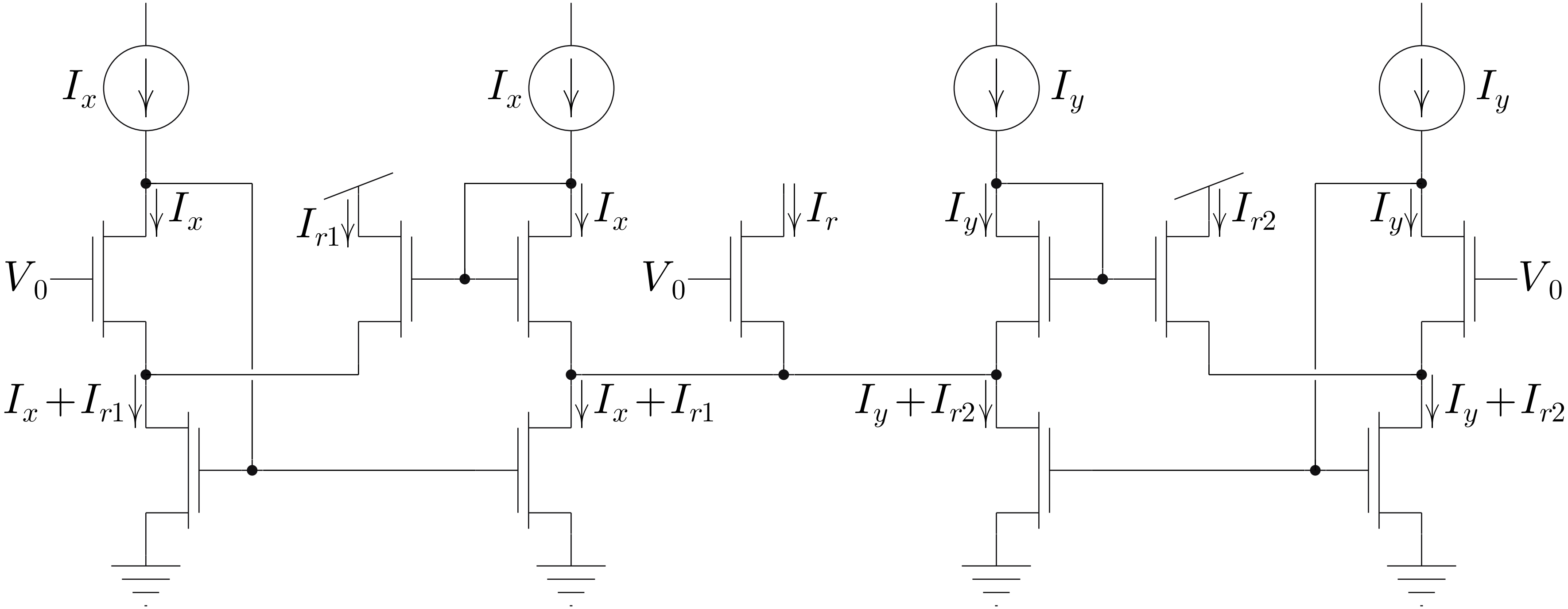

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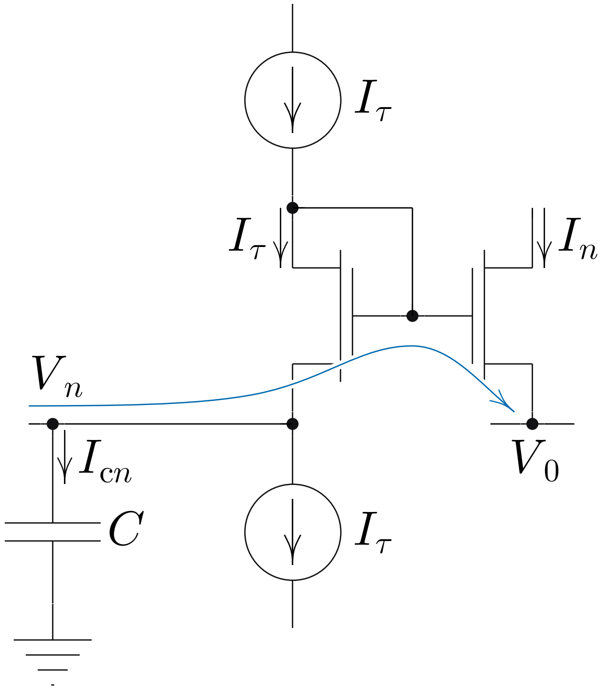
Dynamic Translinear Circuit Synthesis: Output Structures

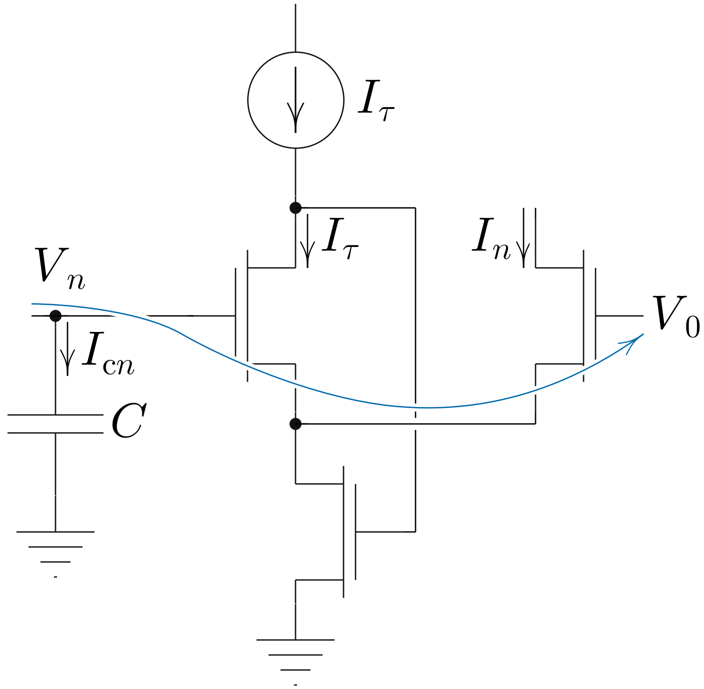
noninverting



$$I_n = I_\tau e^{(V_n - V_0)/U_\mathrm{T}}$$

$$\frac{\partial I_n}{\partial V_n} = \frac{I_n}{U_\mathrm{T}}$$

inverting



$$I_n = I_\tau e^{\kappa(V_0 - V_n)/U_\mathrm{T}}$$

$$\frac{\partial I_n}{\partial V_n} = -\frac{\kappa I_n}{U_\mathrm{T}}$$

Dynamic Translinear Circuit Synthesis: First-Order LPF

Synthesize a first-order low-pass filter described by

$$\tau \frac{dy}{dt} + y = x, \quad \text{where} \quad x > 0.$$

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Dynamic Translinear Circuit Synthesis: First-Order LPF

To implement the time derivative, we introduce a log-compressed voltage state variable, V_y. Using the chain rule, we can express the preceding equation as

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$$\Longrightarrow -\frac{\kappa\tau}{U_\mathrm{T}}\cdot\frac{dV_y}{dt}+1=\frac{I_x}{I_y}$$

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$$\Longrightarrow -\frac{I_c}{I_\tau} + 1 = \frac{I_x}{I_y}$$

Dynamic Translinear Circuit Synthesis: First-Order LPF

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$$\Longrightarrow \; -\frac{I_c}{I_\tau} + 1 = \frac{I_x}{I_y} \quad \Longrightarrow \quad I_\tau - I_c = \frac{I_\tau I_x}{I_y}.$$

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Dynamic Translinear Circuit Synthesis: First-Order LPF

TLP: $I_p I_y = I_x I_\tau$ KCL: $I_c + I_p = I_\tau$

Dynamic Translinear Circuit Synthesis: First-Order LPF

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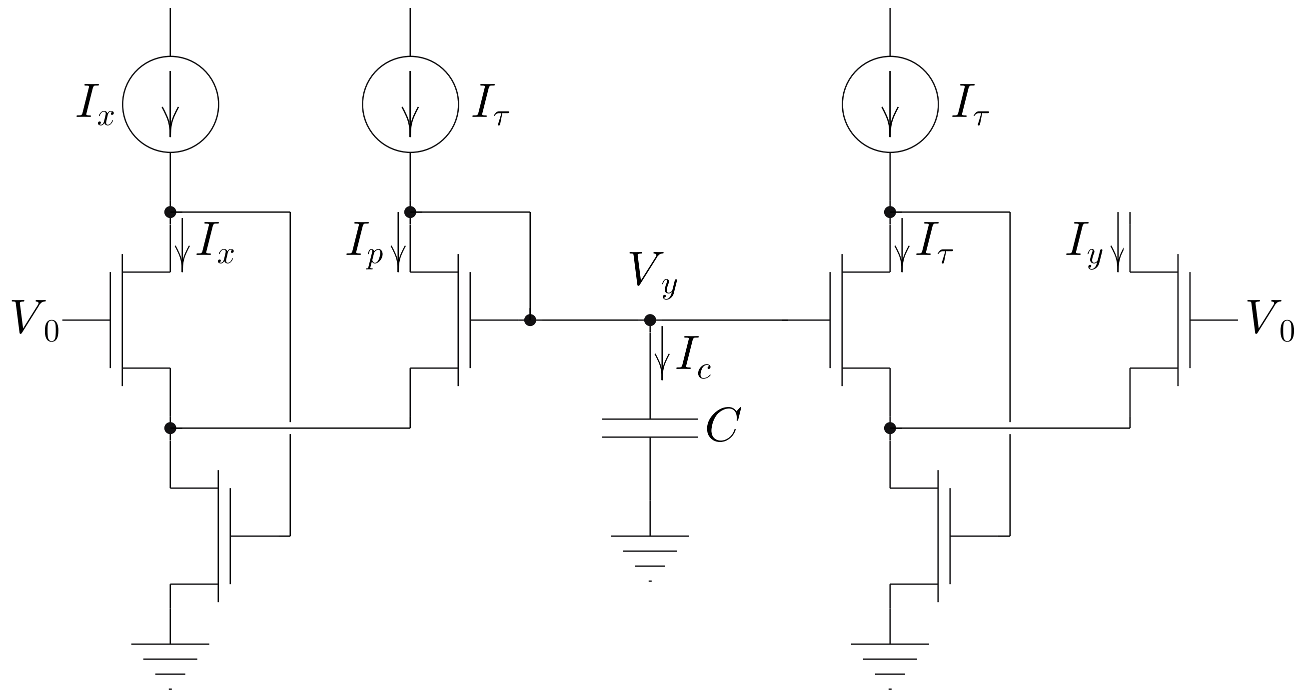


Dynamic Translinear Circuit Synthesis: RMS-DC Converter

Synthesize an RMS-to-DC converter described by

$$x = w^2, \quad \tau\frac{dy}{dt} + y = x, \quad \text{and} \quad z = \sqrt{y}.$$

Dynamic Translinear Circuit Synthesis: RMS-DC Converter

Synthesize an RMS-to-DC converter described by

$$x = w^2, \quad \tau\frac{dy}{dt} + y = x, \quad \text{and} \quad z = \sqrt{y}.$$

We can eliminate x and y from the system description by substituting

$$x = w^2, \quad y = z^2, \quad \text{and} \quad \frac{dy}{dt} = 2z\frac{dz}{dt}$$

into the linear ODE describing the low-pass filter, obtaining a first-order nonlinear ODE describing the system given by

$$2\tau z\frac{dz}{dt} + z^2 = w^2.$$

Dynamic Translinear Circuit Synthesis: RMS-DC Converter

$$w_+ \equiv \frac{I_{w+}}{I_1} = \tfrac{1}{2}\left(1 + e^{\kappa(V_w - V_0)/U_\mathrm{T}}\right)$$

$$w_- \equiv \frac{I_{w-}}{I_1} = \tfrac{1}{2}\left(1 + e^{-\kappa(V_w - V_0)/U_\mathrm{T}}\right)$$

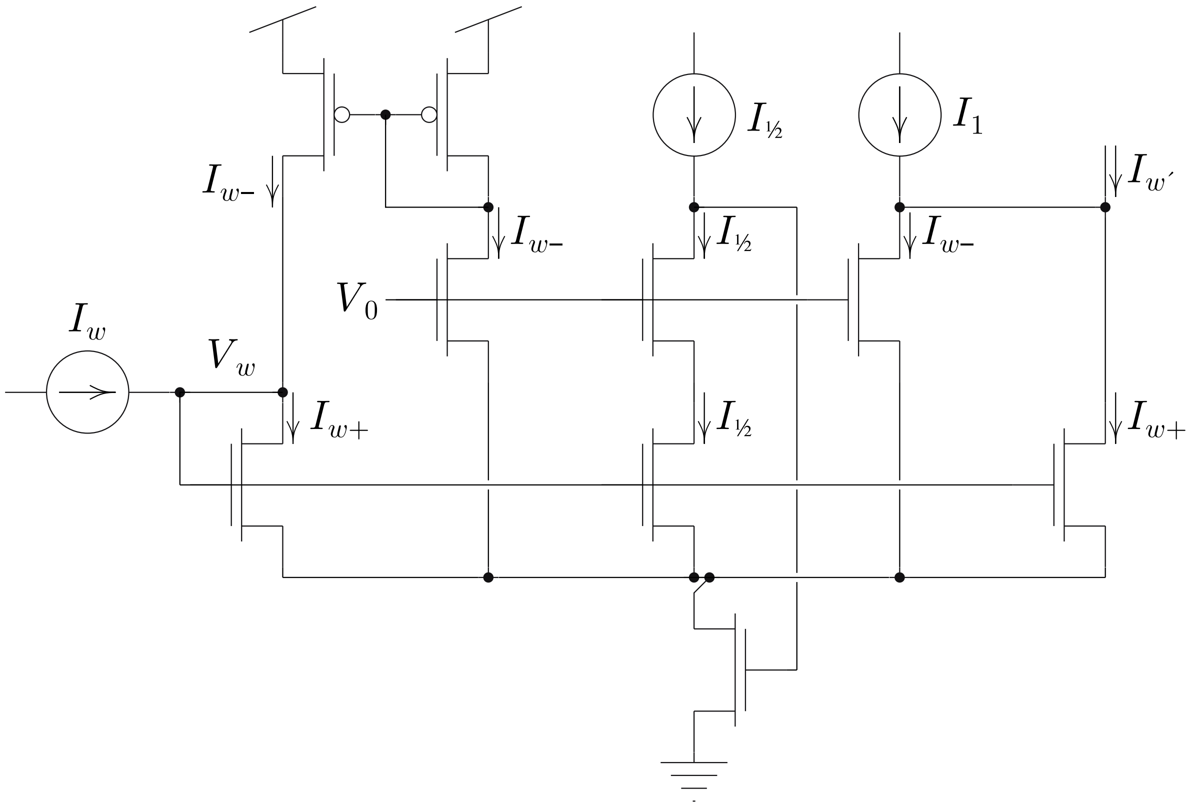
$$w \equiv \frac{I_w}{I_1} = w_+ - w_-$$

$$= \sinh \frac{\kappa\left(V_w - V_0\right)}{U_\mathrm{T}}$$

$$w' \equiv \frac{I_{w'}}{I_1} = w_+ + w_- - 1$$

$$= \cosh \frac{\kappa\left(V_w - V_0\right)}{U_\mathrm{T}}$$

$$\boxed{w^2 = \left(w'\right)^2 - 1}$$



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Dynamic Translinear Circuit Synthesis: RMS-DC Converter

The input signal, w, can be positive or negative. To remedy this situation, we adopt a sinh representation for w and and define an associated signal, w', as just described. Substituting $w^2 = (w')^2 - 1$ into the nonlinear ODE, we obtain

$$2\tau z \frac{dz}{dt} + z^2 = (w')^2 - 1.$$

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We represent each signal as a ratio of a signal current to the unit current:

$$w' \equiv \frac{I_{w'}}{I_1} \quad \text{and} \quad z \equiv \frac{I_z}{I_1}.$$

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$$2\tau \frac{I_z}{I_1} \cdot \frac{d}{dt}\left(\frac{I_z}{I_1}\right) + \left(\frac{I_z}{I_1}\right)^2 = \left(\frac{I_{w'}}{I_1}\right)^2 - 1$$

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Dynamic Translinear Circuit Synthesis: RMS-DC Converter

To implement the time derivative, we introduce a log-compressed voltage state variable, V_z. Using the chain rule, we can express the preceding equation as

$$2\tau I_z \frac{\partial I_z}{\partial V_z} \cdot \frac{dV_z}{dt} + I_z^2 = I_{w'}^2 - I_1^2$$

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Dynamic Translinear Circuit Synthesis: RMS-DC Converter

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$I_q I_z^2 = I_\tau I_1^2$

KCL: $I_c + I_p = I_\tau + I_q$

Dynamic Translinear Circuit Synthesis: RMS-DC Converter

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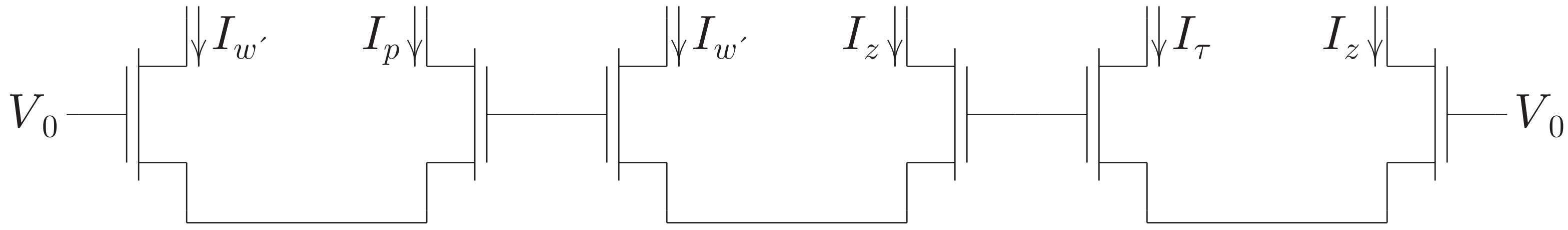

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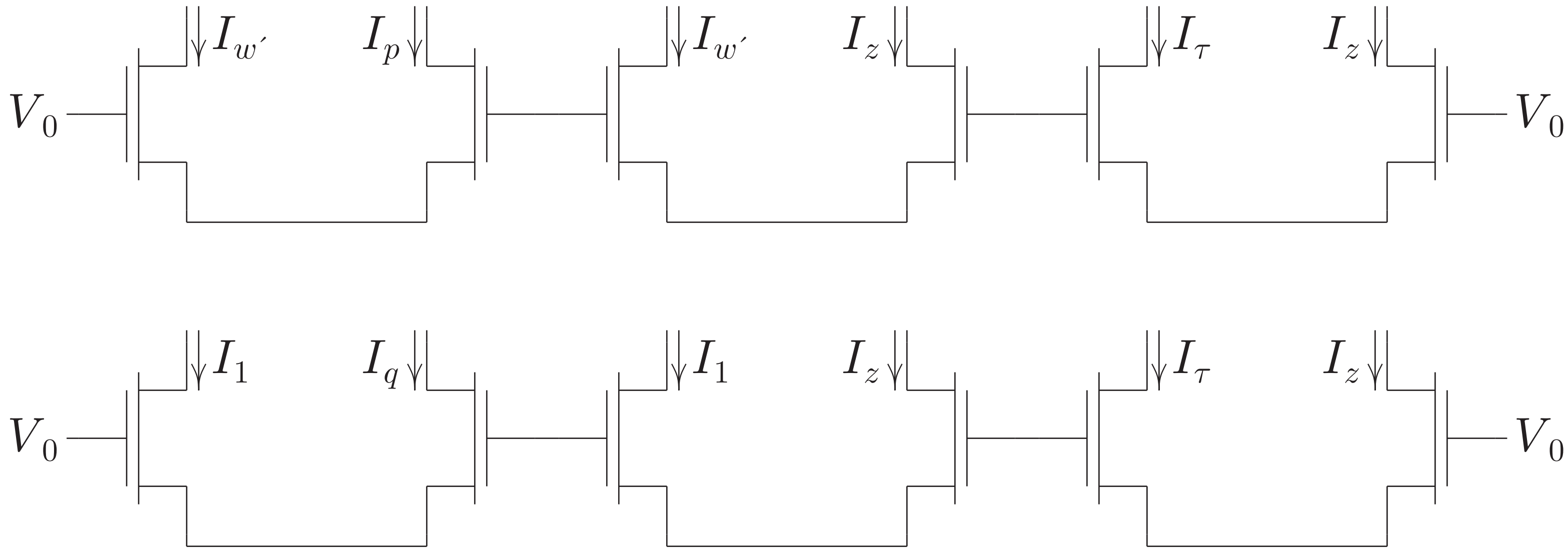

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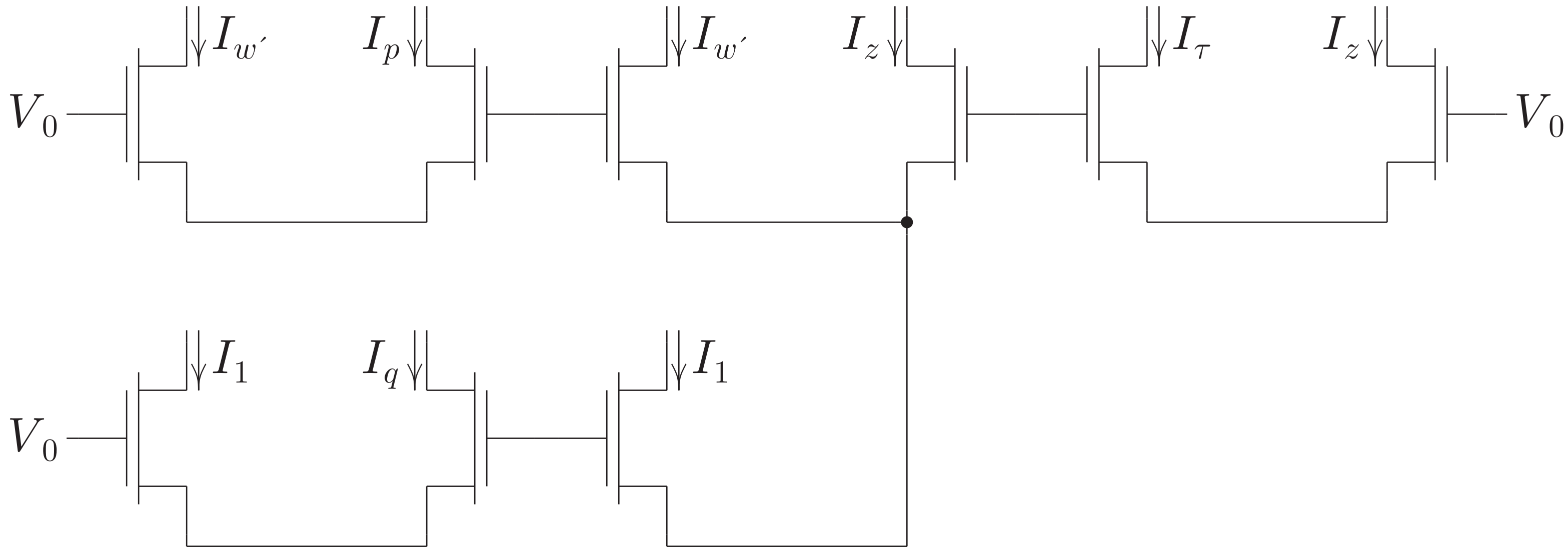

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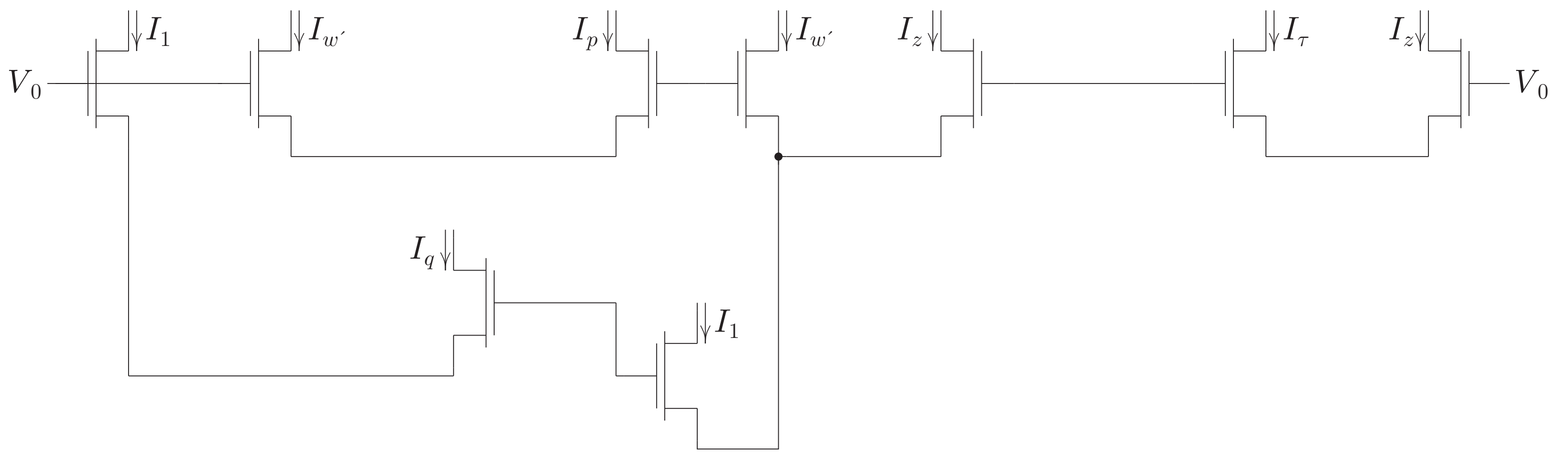

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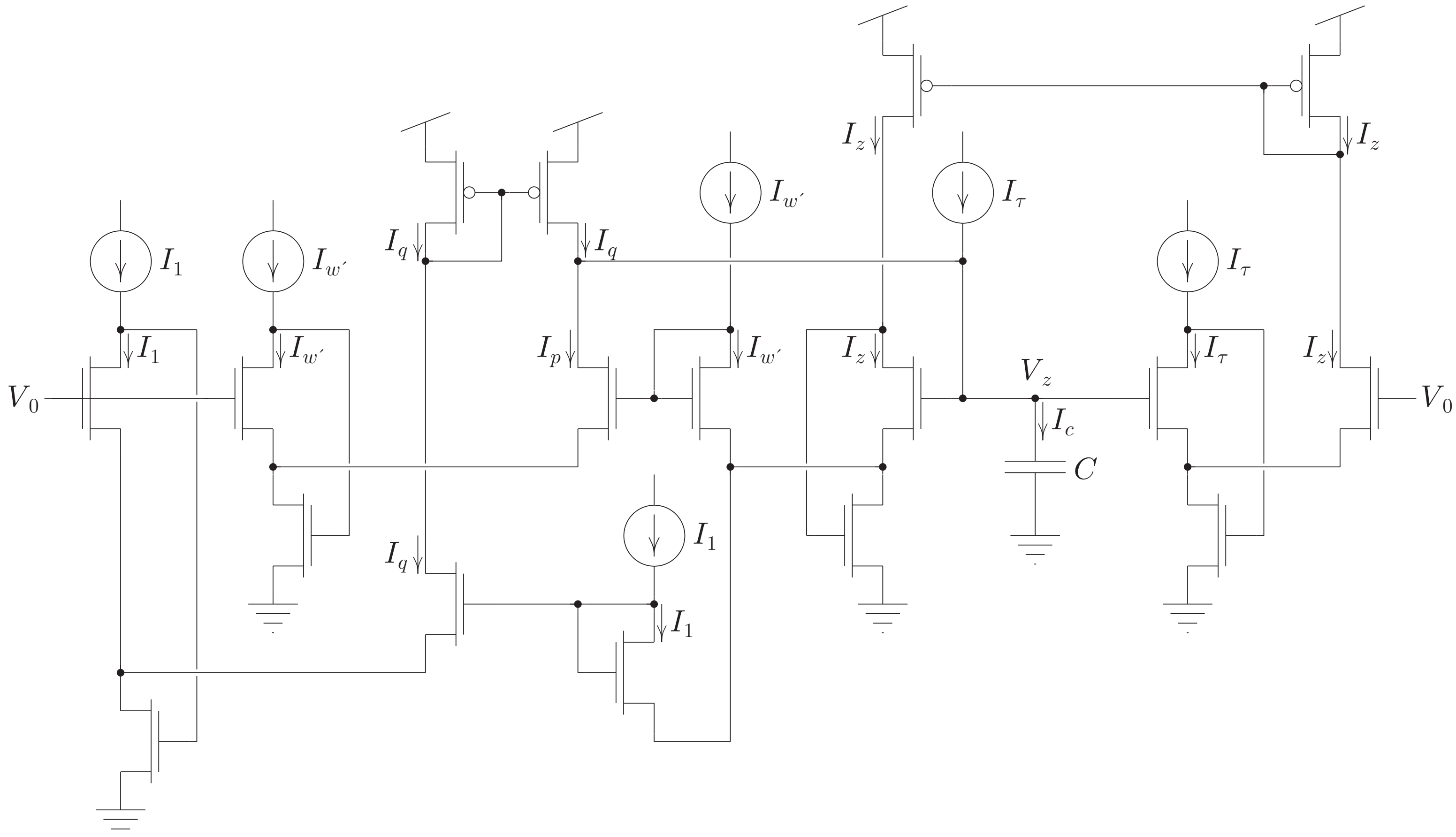

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