Static and Dynamic Translinear Circuits

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Translinear Circuits: What’s in a Name?

In 1975, Barrie Gilbert coined the term *translinear* to describe a class of circuits whose large-signal behavior hinges both on the precise exponential $I/V$ relationship of the bipolar transistor and on the intimate thermal contact and close matching of monolithically integrated devices.
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The word *translinear* refers to the exponential $I/V$ characteristic of the bipolar transistor—it* trans*conductance is linear in its collector current:

$$I_C = I_s e^{V_{BE}/U_T} \implies g_m = \frac{\partial I_C}{\partial V_B} = \frac{I_s e^{V_{BE}/U_T}}{I_C} \cdot \frac{1}{U_T} = \frac{I_C}{U_T}.$$
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Gilbert also meant the word *translinear* to refer to circuit analysis and design principles that bridge the gap between the familiar territory of linear circuits and the uncharted domain of nonlinear circuits.
Gummel Plot of a Forward-Active Bipolar Transistor

\[ I_C = I_s e^{V_B / U_T} \]

\[ I_B = \frac{I_C}{\beta_F} \]

\[ I_s = 105 \text{ fA} \]

\[ U_T = 25.6 \text{ mV} \]

\[ \beta_F = 679 \]
Translinearity of the Forward-Active Bipolar Transistor

\[ g_m = \frac{\partial I_C}{\partial V_B} = \frac{I_C}{U_T} \]

\[ \delta I_C \approx g_m \delta V_B \]
The Translinear Principle

Consider a closed loop of base-emitter junctions of four closely matched *n*-channel bipolar transistors biased in the forward-active region and operating at the same temperature. Kirchhoff’s voltage law (KVL) implies that

\[
V_1 + V_2 = V_3 + V_4
\]

\[
U_T \log \frac{I_1}{I_s} + U_T \log \frac{I_2}{I_s} = U_T \log \frac{I_3}{I_s} + U_T \log \frac{I_4}{I_s}
\]

\[
\log \frac{I_1 I_2}{I_s^2} = \log \frac{I_3 I_4}{I_s^2}
\]

\[
I_1 I_2 = I_3 I_4.
\]

This result is a particular case of Gilbert’s translinear principle (TLP): The product of the clockwise currents is equal to the product of the counterclockwise currents.
Why Translinear Circuits?
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- Translinear circuits are *universal*. **Conjecture:** In principle, we can realize any system whose description we can write down as a nonlinear ODE with time as its independent variable as a dynamic translinear circuit.

- Translinear circuits are *synthesizable* via highly structured methods. They should be very amenable to the development both of CAD tools and of reconfigurable FPAA architectures for rapid prototyping and deployment of translinear analog signal processing systems.

- Translinear circuits are fundamentally *large-signal* circuits. Linear dynamic translinear circuits are linear because of device nonlinearities rather than in spite of them.

- Translinear circuits are *tunable* electronically over a wide dynamic range of parameters (e.g., gains, corner frequencies, quality factors).

- Translinear circuits are *robust*. Carefully designed translinear circuits are temperature insensitive and do not depend on device or technology parameters.
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Simple EKV Model of the Saturated $n$MOS Transistor

We model the saturation current of an $n$MOS transistor by

$$I_{\text{sat}} = S I_s \log^2 \left( 1 + e^{(\kappa(V_G - V_{T0}) - V_S)/2U_T} \right)$$

$$\approx \begin{cases} 
S I_s \exp^{(\kappa(V_G - V_{T0}) - V_S)/U_T}, & \kappa (V_G - V_{T0}) - V_S < 0 \\
\frac{S I_s}{4U_T^2} (\kappa (V_G - V_{T0}) - V_S)^2, & \kappa (V_G - V_{T0}) - V_S > 0,
\end{cases}$$

where

$$U_T = \frac{kT}{q}, \quad S = \frac{W}{L}, \quad I_s = \frac{2\mu C_{\text{ox}} U_T^2}{\kappa}, \quad \text{and} \quad \kappa = \frac{C_{\text{ox}}}{C_{\text{ox}} + C_{\text{dep}}}.$$
Saturation Current of an $n$MOS Transistor

\[ I_{\text{sat}} = SI_s e^{\kappa(V_G - V_{T0})/U_T} \]

\[ I_{\text{sat}} = SI_s \log^2(1 + e^{\kappa(V_G - V_{T0})/2U_T}) \]

\[ I_{\text{sat}} = \frac{SI_s}{4U_T^2}(\kappa(V_G - V_{T0}))^2 \]

\[ V_{T0} = 0.647 \text{ V} \]

\[ \kappa = 0.715 \]

\[ U_T = 25.8 \text{ mV} \]
Translinearity of the Saturated $n$MOS Transistor

\[ g_m = \frac{\kappa}{U_T} I_{\text{sat}} \]

\[ g_m = \frac{\kappa}{U_T} \sqrt{S I_s I_{\text{sat}}} (1 - e^{-\sqrt{I_{\text{sat}}/SI_s}}) \]

\[ SI_s = 3.93 \mu A \]

\[ U_T = 25.8 \text{ mV} \]

\[ \kappa = 0.715 \]

\[ \delta I_{\text{sat}} \approx g_m \delta V_G \]
Weak-Inversion MOS Translinear Principle

When designing weak-inversion MOS translinear circuits, if we restrict ourselves to using translinear loops that alternate between clockwise and counterclockwise elements, we obtain Gilbert’s original TLP, with no dependence on the body effect (i.e., $\kappa$).

\[ TLP: I_1 I_2^\kappa = I_3^\kappa I_4 \]
\[ TLP: I_1 I_3 = I_2 I_4 \]

This restriction does not limit the class of systems that we can implement. However, designs based on alternating loops generally consume more current but operate on a lower power supply voltage than do designs based on stacked loops.
Static Translinear Circuits: Squaring/Reciprocal

\[ TLP \implies I_x^2 = I_y I_z \implies I_z = \frac{I_x^2}{I_y} \]
Static Translinear Circuits: Squaring/Reciprocal

\[ I_z = \frac{I_x^2}{I_y} \]
Static Translinear Circuits: Squaring/Reciprocal

\[ I_z = \frac{I_x^2}{I_y} \]
Static Translinear Circuit Synthesis: Pythagorator

Synthesize a two-dimensional vector-magnitude circuit implementing

$$r = \sqrt{x^2 + y^2}, \text{ where } x > 0 \text{ and } y > 0.$$
Static Translinear Circuit Synthesis: Pythagorator

Synthesize a two-dimensional vector-magnitude circuit implementing

\[ r = \sqrt{x^2 + y^2}, \quad \text{where} \quad x > 0 \quad \text{and} \quad y > 0. \]

We represent each signal as a ratio of a signal current to the unit current:

\[ x \equiv \frac{I_x}{I_1}, \quad y \equiv \frac{I_y}{I_1}, \quad \text{and} \quad r \equiv \frac{I_r}{I_1}. \]
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We substitute these into the original equation and rearrange to obtain

\[ \frac{I_r}{I_1} = \sqrt{\left(\frac{I_x}{I_1}\right)^2 + \left(\frac{I_y}{I_1}\right)^2} \]
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\[ \frac{I_r}{I_1} = \sqrt{\left(\frac{I_x}{I_1}\right)^2 + \left(\frac{I_y}{I_1}\right)^2} \quad \Rightarrow \quad I_r^2 = I_x^2 + I_y^2 \]
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\[ \frac{I_r}{I_1} = \sqrt{\left(\frac{I_x}{I_1}\right)^2 + \left(\frac{I_y}{I_1}\right)^2} \quad \implies \quad I_r^2 = I_x^2 + I_y^2 \quad \implies \quad I_r = \frac{I_x^2}{I_r} + \frac{I_y^2}{I_r}. \]
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We substitute these into the original equation and rearrange to obtain

\[ \frac{I_r}{I_1} = \sqrt{\left(\frac{I_x}{I_1}\right)^2 + \left(\frac{I_y}{I_1}\right)^2} \implies I_r^2 = I_x^2 + I_y^2 \implies I_r = \frac{I_x^2}{I_r} + \frac{I_y^2}{I_r} \]
Static Translinear Circuit Synthesis: Pythagorator

TLP: \( I_{r1}I_r = I_x^2 \)  
\( I_{r2}I_r = I_y^2 \)

KCL: \( I_r = I_{r1} + I_{r2} \)
**Static Translinear Circuit Synthesis: Pythagorator**

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$\quad I_{r2}I_r = I_y^2$

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Static Translinear Circuit Synthesis: Pythagorator

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$I_{r2}I_r = I_y^2$

KCL: $I_r = I_{r1} + I_{r2}$
Dynamic Translinear Circuit Synthesis: Output Structures

\[ I_n = I_\tau e^{(V_n - V_0)/U_T} \]
\[ \frac{\partial I_n}{\partial V_n} = \frac{I_n}{U_T} \]

\[ I_n = I_\tau e^{\kappa(V_0 - V_n)/U_T} \]
\[ \frac{\partial I_n}{\partial V_n} = -\frac{\kappa I_n}{U_T} \]
Dynamic Translinear Circuit Synthesis: First-Order LPF

Synthesize a first-order low-pass filter described by

\[ \tau \frac{dy}{dt} + y = x, \text{ where } x > 0. \]
Dynamic Translinear Circuit Synthesis: First-Order LPF

Synthesize a first-order low-pass filter described by

\[ \tau \frac{dy}{dt} + y = x, \quad \text{where} \quad x > 0. \]

We represent each signal as a ratio of a signal current to the unit current:

\[ x \equiv \frac{I_x}{I_1} \quad \text{and} \quad y \equiv \frac{I_y}{I_1}. \]
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\[ x \equiv \frac{I_x}{I_1} \quad \text{and} \quad y \equiv \frac{I_y}{I_1}. \]

Substituting these into the ODE, we obtain

\[ \tau \frac{d}{dt} \left( \frac{I_y}{I_1} \right) + \frac{I_y}{I_1} = \frac{I_x}{I_1}. \]
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Substituting these into the ODE, we obtain

\[ \tau \frac{d}{dt} \left( \frac{I_y}{I_1} \right) + \frac{I_y}{I_1} = \frac{I_x}{I_1} \quad \implies \quad \tau \frac{dI_y}{dt} + I_y = I_x. \]
Dynamic Translinear Circuit Synthesis: First-Order LPF

To implement the time derivative, we introduce a log-compressed voltage state variable, \( V_y \). Using the chain rule, we can express the preceding equation as

\[
\tau \frac{\partial I_y}{\partial V_y} \cdot \frac{dV_y}{dt} + I_y = I_x
\]
Dynamic Translinear Circuit Synthesis: First-Order LPF

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$$\tau \frac{\partial I_y}{\partial V_y} \cdot \frac{dV_y}{dt} + I_y = I_x \quad \Rightarrow \quad \tau \left( -\frac{\kappa}{U_T} I_y \right) \frac{dV_y}{dt} + I_y = I_x$$
Dynamic Translinear Circuit Synthesis: First-Order LPF

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\]

\[
\implies -\frac{\kappa \tau}{U_T} \cdot \frac{dV_y}{dt} + 1 = \frac{I_x}{I_y}
\]
Dynamic Translinear Circuit Synthesis: First-Order LPF

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\]

\[
\Rightarrow -\frac{\kappa \tau}{U_T} \cdot \frac{dV_y}{dt} + 1 = \frac{I_x}{I_y} \quad \Rightarrow \quad -\frac{\kappa \tau}{CU_T} \cdot C \frac{dV_y}{dt} + 1 = \frac{I_x}{I_y}
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Dynamic Translinear Circuit Synthesis: First-Order LPF

To implement the time derivative, we introduce a log-compressed voltage state variable, $V_y$. Using the chain rule, we can express the preceding equation as

$$\tau \frac{\partial I_y}{\partial V_y} \cdot \frac{dV_y}{dt} + I_y = I_x \quad \iff \quad \tau \left( -\frac{\kappa}{U_T} I_y \right) \frac{dV_y}{dt} + I_y = I_x$$

$$\iff -\frac{\kappa \tau}{U_T} \cdot \frac{dV_y}{dt} + 1 = \frac{I_x}{I_y} \quad \iff \quad -\frac{\kappa \tau}{C U_T} \cdot C \frac{dV_y}{dt} + 1 = \frac{I_x}{I_y}$$
Dynamic Translinear Circuit Synthesis: First-Order LPF

To implement the time derivative, we introduce a log-compressed voltage state variable, $V_y$. Using the chain rule, we can express the preceding equation as

$$\tau \frac{\partial I_y}{\partial V_y} \cdot \frac{dV_y}{dt} + I_y = I_x \quad \Rightarrow \quad \tau \left( -\frac{\kappa}{U_T} I_y \right) \frac{dV_y}{dt} + I_y = I_x$$

$$\Rightarrow -\frac{\kappa T}{U_T} \cdot \frac{dV_y}{dt} + 1 = \frac{I_x}{I_y} \quad \Rightarrow \quad -\frac{\kappa T}{CU_T} \cdot C \frac{dV_y}{dt} + 1 = \frac{I_x}{I_y}$$

$$\Rightarrow -\frac{I_c}{I_\tau} + 1 = \frac{I_x}{I_y}$$
Dynamic Translinear Circuit Synthesis: First-Order LPF

To implement the time derivative, we introduce a log-compressed voltage state variable, $V_y$. Using the chain rule, we can express the preceding equation as

$$\tau \frac{\partial I_y}{\partial V_y} \cdot \frac{dV_y}{dt} + I_y = I_x \quad \implies \quad \tau \left( -\frac{\kappa}{U_T} I_y \right) \frac{dV_y}{dt} + I_y = I_x$$

$$\implies \quad -\frac{\kappa \tau}{U_T} \cdot \frac{dV_y}{dt} + 1 = \frac{I_x}{I_y} \quad \implies \quad -\frac{\kappa \tau}{C U_T} \cdot C \frac{dV_y}{dt} + 1 = \frac{I_x}{I_y}$$

$$\implies \quad -\frac{I_c}{I_T} + 1 = \frac{I_x}{I_y} \quad \implies \quad I_T - I_c = \frac{I_T I_x}{I_y}.$$
Dynamic Translinear Circuit Synthesis: First-Order LPF

To implement the time derivative, we introduce a log-compressed voltage state variable, \( V_y \). Using the chain rule, we can express the preceding equation as

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\tau \frac{\partial I_y}{\partial V_y} \cdot \frac{dV_y}{dt} + I_y = I_x \quad \implies \quad \tau \left( -\frac{\kappa}{U_T} I_y \right) \frac{dV_y}{dt} + I_y = I_x
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\implies -\frac{I_c}{I_T} + 1 = \frac{I_x}{I_y} \quad \implies \quad I_T - I_c = \frac{I_T I_x}{I_y}.
\]
Dynamic Translinear Circuit Synthesis: First-Order LPF

TLP: \( I_p I_y = I_x I_\tau \)  \hspace{1cm} KCL: \( I_c + I_p = I_\tau \)
Dynamic Translinear Circuit Synthesis: First-Order LPF

TLP:  \( I_p I_y = I_x I_\tau \)  

KCL:  \( I_c + I_p = I_\tau \)
Dynamic Translinear Circuit Synthesis: First-Order LPF

TLP: \( I_p I_y = I_x I_\tau \)

KCL: \( I_c + I_p = I_\tau \)
Dynamic Translinear Circuit Synthesis: RMS-DC Converter

Synthesize an RMS-to-DC converter described by

\[ x = w^2, \quad \tau \frac{dy}{dt} + y = x, \quad \text{and} \quad z = \sqrt{y}. \]
Dynamic Translinear Circuit Synthesis: RMS-DC Converter

Synthesize an RMS-to-DC converter described by

\[ x = w^2, \quad \tau \frac{dy}{dt} + y = x, \quad \text{and} \quad z = \sqrt{y}. \]

We can eliminate \( x \) and \( y \) from the system description by substituting

\[ x = w^2, \quad y = z^2, \quad \text{and} \quad \frac{dy}{dt} = 2z \frac{dz}{dt} \]

into the linear ODE describing the low-pass filter, obtaining a first-order nonlinear ODE describing the system given by

\[ 2\tau z \frac{dz}{dt} + z^2 = w^2. \]
Dynamic Translinear Circuit Synthesis: RMS-DC Converter

\[ w_+ \equiv \frac{I_{w+}}{I_1} = \frac{1}{2} \left( 1 + e^{\kappa (V_w - V_0) / U_T} \right) \]

\[ w_- \equiv \frac{I_{w-}}{I_1} = \frac{1}{2} \left( 1 + e^{-\kappa (V_w - V_0) / U_T} \right) \]

\[ w \equiv \frac{I_w}{I_1} = w_+ - w_- \]
\[ = \sinh \frac{\kappa (V_w - V_0)}{U_T} \]

\[ w' \equiv \frac{I_{w'}}{I_1} = w_+ + w_- - 1 \]
\[ = \cosh \frac{\kappa (V_w - V_0)}{U_T} \]

\[ w^2 = (w')^2 - 1 \]
Dynamic Translinear Circuit Synthesis: RMS-DC Converter

The input signal, $w$, can be positive or negative. To remedy this situation, we adopt a sinh representation for $w$ and define an associated signal, $w'$, as just described. Substituting $w^2 = (w')^2 - 1$ into the nonlinear ODE, we obtain

$$2\tau z \frac{dz}{dt} + z^2 = (w')^2 - 1.$$
Dynamic Translinear Circuit Synthesis: RMS-DC Converter

The input signal, $w$, can be positive or negative. To remedy this situation, we adopt a sinh representation for $w$ and define an associated signal, $w'$, as just described. Substituting $w^2 = (w')^2 - 1$ into the nonlinear ODE, we obtain

$$2\tau z \frac{dz}{dt} + z^2 = (w')^2 - 1.$$ 

We represent each signal as a ratio of a signal current to the unit current:

$$w' \equiv \frac{I_{w'}}{I_1} \quad \text{and} \quad z \equiv \frac{I_z}{I_1}.$$
Dynamic Translinear Circuit Synthesis: RMS-DC Converter

The input signal, \( w \), can be positive or negative. To remedy this situation, we adopt a sinh representation for \( w \) and define an associated signal, \( w' \), as just described. Substituting \( w^2 = (w')^2 - 1 \) into the nonlinear ODE, we obtain

\[
2\tau z \frac{dz}{dt} + z^2 = (w')^2 - 1.
\]

We represent each signal as a ratio of a signal current to the unit current:

\[
w' \equiv \frac{I_w}{I_1} \quad \text{and} \quad z \equiv \frac{I_z}{I_1}.
\]

Substituting these into the nonlinear ODE, we obtain

\[
2\tau \frac{I_z}{I_1} \frac{d}{dt} \left( \frac{I_z}{I_1} \right) + \left( \frac{I_z}{I_1} \right)^2 = \left( \frac{I_{w'}}{I_1} \right)^2 - 1
\]
Dynamic Translinear Circuit Synthesis: RMS-DC Converter

The input signal, \( w \), can be positive or negative. To remedy this situation, we adopt a sinh representation for \( w \) and define an associated signal, \( w' \), as just described. Substituting \( w^2 = (w')^2 - 1 \) into the nonlinear ODE, we obtain

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We represent each signal as a ratio of a signal current to the unit current:

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w' \equiv \frac{I_w'}{I_1} \quad \text{and} \quad z \equiv \frac{I_z}{I_1}.
\]

Substituting these into the nonlinear ODE, we obtain

\[
2\tau \frac{I_z}{I_1} \cdot \frac{d}{dt} \left( \frac{I_z}{I_1} \right) + \left( \frac{I_z}{I_1} \right)^2 = \left( \frac{I_w'}{I_1} \right)^2 - 1 \quad \Rightarrow \quad 2\tau I_z \frac{dI_z}{dt} + I_z^2 = I_{w'}^2 - I_1^2.
\]
Dynamic Translinear Circuit Synthesis: RMS-DC Converter

To implement the time derivative, we introduce a log-compressed voltage state variable, $V_z$. Using the chain rule, we can express the preceding equation as

$$2\tau I_z \frac{\partial I_z}{\partial V_z} \cdot \frac{dV_z}{dt} + I_z^2 = I_{w'} - I_1^2$$
Dynamic Translinear Circuit Synthesis: RMS-DC Converter

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\[
2\tau I_z \frac{\partial I_z}{\partial V_z} \cdot \frac{dV_z}{dt} + I_z^2 = I_w^2 - I_1^2 \quad \Rightarrow \quad 2\tau I_z \left( -\frac{\kappa}{U_T} I_z \right) \frac{dV_z}{dt} + I_z^2 = I_w^2 - I_1^2
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Dynamic Translinear Circuit Synthesis: RMS-DC Converter

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\]

\[
\Rightarrow -\frac{2w\kappa}{U_T} \cdot \frac{dV_z}{dt} + 1 = \frac{I_{w'}}{I_z^2} - \frac{I_1^2}{I_z^2}
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Dynamic Translinear Circuit Synthesis: RMS-DC Converter

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\[
\Rightarrow -\frac{2w\kappa}{U_T} \cdot \frac{dV_z}{dt} + 1 = \frac{I_w'}{I_z^2} - \frac{I_1^2}{I_z^2} \quad \Rightarrow \quad -\frac{2\kappa}{CU_T} \cdot C \frac{dV_z}{dt} + 1 = \frac{I_w'}{I_z^2} - \frac{I_1^2}{I_z^2}
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Dynamic Translinear Circuit Synthesis: RMS-DC Converter

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$$\quad \Rightarrow \quad -\frac{2w\kappa}{U_T} \cdot \frac{dV_z}{dt} + 1 = \frac{I_{w'}^2}{I_z^2} - \frac{I_1^2}{I_z^2} \quad \Rightarrow \quad -\frac{2\kappa\tau}{C U_T} \cdot \frac{dV_z}{dt} + 1 = \frac{I_{w'}^2}{I_z^2} - \frac{I_1^2}{I_z^2}$$
Dynamic Translinear Circuit Synthesis: RMS-DC Converter

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$$2\tau I_z \frac{\partial I_z}{\partial V_z} \cdot \frac{dV_z}{dt} + I_z^2 = I_{w'}^2 - I_1^2 \quad \Longrightarrow \quad 2\tau I_z \left(-\frac{\kappa}{U_T}I_z\right) \frac{dV_z}{dt} + I_z^2 = I_{w'}^2 - I_1^2$$

$$\Longrightarrow -\frac{2w\kappa}{U_T} \cdot \frac{dV_z}{dt} + 1 = \frac{I_{w'}^2}{I_z^2} - \frac{I_1^2}{I_z^2} \quad \Longrightarrow \quad -\frac{2\kappa\tau}{C\frac{1}{I_T}} \cdot C \frac{dV_z}{dt} + 1 = \frac{I_{w'}^2}{I_z^2} - \frac{I_1^2}{I_z^2}$$

$$\Longrightarrow -\frac{I_c}{I_\tau} + 1 = \frac{I_{w'}^2}{I_z^2} - \frac{I_1^2}{I_z^2}$$
Dynamic Translinear Circuit Synthesis: RMS-DC Converter

To implement the time derivative, we introduce a log-compressed voltage state variable, \( V_z \). Using the chain rule, we can express the preceding equation as

\[
2\tau I_z \frac{\partial I_z}{\partial V_z} \cdot \frac{dV_z}{dt} + I_z^2 = I_w' - I_1^2 \quad \implies \quad 2\tau I_z \left(-\frac{\kappa}{U_T}I_z\right) \frac{dV_z}{dt} + I_z^2 = I_w' - I_1^2
\]

\[
\implies -\frac{2w\kappa}{U_T} \cdot \frac{dV_z}{dt} + 1 = \frac{I_w'}{I_z^2} - \frac{I_1^2}{I_z^2} \quad \implies \quad -\frac{2\kappa \tau}{CU_T} \cdot \frac{dV_z}{dt} + 1 = \frac{I_w'}{I_z^2} - \frac{I_1^2}{I_z^2}
\]

\[
\implies -\frac{I_c}{I_\tau} + 1 = \frac{I_w'}{I_z^2} - \frac{I_1^2}{I_z^2} \quad \implies \quad I_\tau - I_c = \frac{I_\tau I_w'}{I_z^2} - \frac{I_\tau I_1^2}{I_z^2}.
\]
Dynamic Translinear Circuit Synthesis: RMS-DC Converter

To implement the time derivative, we introduce a log-compressed voltage state variable, $V_z$. Using the chain rule, we can express the preceding equation as

$$2\tau I_z \frac{\partial I_z}{\partial V_z} \frac{dV_z}{dt} + I_z^2 = I_w' - I_1^2 \quad \implies \quad 2\tau I_z \left(-\frac{\kappa}{U_T} I_z\right) \frac{dV_z}{dt} + I_z^2 = I_w' - I_1^2$$

$$\implies \quad -\frac{2w\kappa}{U_T} \frac{dV_z}{dt} + 1 = \frac{I_w'}{I_z^2} - \frac{I_1^2}{I_z^2} \quad \implies \quad -\frac{2\kappa\tau}{CU_T} \frac{dV_z}{dt} + 1 = \frac{I_w'}{I_z^2} - \frac{I_1^2}{I_z^2}$$

$$\implies \quad -\frac{I_c}{I_\tau} + 1 = \frac{I_w'}{I_z^2} - \frac{I_1^2}{I_z^2} \quad \implies \quad I_\tau - I_c = \frac{I_\tau I_w'}{I_z^2} - \frac{I_\tau I_1^2}{I_z^2}.$$


Dynamic Translinear Circuit Synthesis: RMS-DC Converter

TLP: \[ I_p I_z^2 = I_T I_{w'}^2 \]
\[ I_q I_z^2 = I_T I_1^2 \]

KCL: \[ I_c + I_p = I_T + I_q \]
Dynamic Translinear Circuit Synthesis: RMS-DC Converter

TLP: $I_p I_z^2 = I_\tau I_{w'}^2$
$I_q I_z^2 = I_\tau I_1^2$

KCL: $I_c + I_p = I_\tau + I_q$

\[V_0\]

\[I_{w'} \quad I_p \quad I_{w'} \quad I_z \quad I_\tau \quad I_z \quad V_0\]
Dynamic Translinear Circuit Synthesis: RMS-DC Converter

TLP: \[ I_p I_z^2 = I_\tau I_w^2, \quad I_q I_z^2 = I_\tau I_1^2 \]

KCL: \[ I_c + I_p = I_\tau + I_q \]

\[ V_0 - I_w' - I_p - I_z - I_\tau - I_z - V_0 \]

\[ V_0 - I_1 - I_q - I_1 - I_z - I_\tau - I_z - V_0 \]
Dynamic Translinear Circuit Synthesis: RMS-DC Converter

TLP: \[ I_p I_z^2 = I_\tau I_w^2, \quad I_q I_z^2 = I_\tau I_1^2 \]

KCL: \[ I_c + I_p = I_\tau + I_q \]
Dynamic Translinear Circuit Synthesis: RMS-DC Converter

\[ TLP: \quad I_p I_z^2 = I_\tau I_{w'}^2 \quad KCL: \quad I_c + I_p = I_\tau + I_q \]

\[ I_q I_z^2 = I_\tau I_I^2 \]
Dynamic Translinear Circuit Synthesis: RMS-DC Converter

TLP: \( I_p I_z^2 = I_\tau I_w^2 \)
\( I_q I_z^2 = I_\tau I_1^2 \)

KCL: \( I_c + I_p = I_\tau + I_q \)
Dynamic Translinear Circuit Synthesis: RMS-DC Converter

TLP: \[ I_p I_z^2 = I_\tau I_{w'}^2, \]
\[ I_q I_z^2 = I_\tau I_1^2 \]

KCL: \[ I_c + I_p = I_\tau + I_q \]