# A Simple Class-AB Transconductor in CMOS 

Bradley A. Minch<br>Mixed Analog-Digital VLSI Circuits and Systems Lab<br>Franklin W. Olin College of Engineering<br>Needham, MA 02492-1200<br>bradley.minch@olin.edu

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## Simple EKV MOS Transistor Model

We model the channel current of an $n \mathrm{MOS}$ transistor as the difference between a forward current and a reverse current,

$$
I=I_{\mathrm{F}}-I_{\mathrm{R}},
$$

whose values are given by


$$
I_{\mathrm{F}(\mathrm{R})}=S I_{\mathrm{S}} \log ^{2}\left(1+e^{\left(\kappa\left(V_{\mathrm{G}}-V_{\mathrm{T} 0}\right)-V_{\mathrm{S}(\mathrm{D})}\right) / 2 U_{\mathrm{T}}}\right),
$$

where

$$
U_{\mathrm{T}}=\frac{k T}{q}, \quad S=\frac{W}{L}, \quad I_{\mathrm{s}}=\frac{2 \mu C_{\mathrm{ox}} U_{\mathrm{T}}^{2}}{\kappa}, \quad \text { and } \quad \kappa=\frac{C_{\mathrm{ox}}}{C_{\mathrm{ox}}+C_{\mathrm{dep}}} .
$$

Note that $S I_{\mathrm{s}}$ is approximately twice the saturation current at threshold. This simple model covers all regions of normal MOS transistor operation.

## Simple EKV MOS Transistor Model

The expressions for $I_{\mathrm{F}}$ and $I_{\mathrm{R}}$ reduce asymptotically to an exponential form in weak inversion and a quadratic form in strong inversion, given by

$$
I_{\mathrm{F}(\mathrm{R})} \approx\left\{\begin{array}{c}
S I_{\mathrm{s}} e^{\left(\kappa\left(V_{\mathrm{G}}-V_{\mathrm{T} 0}\right)-V_{\mathrm{S}(\mathrm{D})}\right) / U_{\mathrm{T}}}, \\
V_{\mathrm{G}}<V_{\mathrm{T} 0}+\frac{V_{\mathrm{S}(\mathrm{D})}}{\kappa} \\
\frac{S I_{\mathrm{S}}}{4 U_{\mathrm{T}}^{2}}\left(\kappa\left(V_{\mathrm{G}}-V_{\mathrm{T} 0}\right)-V_{\mathrm{S}(\mathrm{D})}\right)^{2}, \\
V_{\mathrm{G}}>V_{\mathrm{T} 0}+\frac{V_{\mathrm{S}(\mathrm{D})}}{\kappa} .
\end{array}\right.
$$



## Simple EKV MOS Transistor Model

The expressions for $I_{\mathrm{F}}$ and $I_{\mathrm{R}}$ are also explicitly invertible; the inverses are given by

$$
\begin{aligned}
\kappa\left(V_{\mathrm{G}}-V_{\mathrm{T} 0}\right)-V_{\mathrm{S}(\mathrm{D})} & =2 U_{\mathrm{T}} \log \left(e^{\sqrt{I_{\mathrm{F}(\mathrm{R})} / S I_{\mathrm{s}}}}-1\right) \\
& \approx \begin{cases}U_{\mathrm{T}} \log \frac{I_{\mathrm{F}(\mathrm{R})}}{S I_{\mathrm{s}}}, & I_{\mathrm{F}(\mathrm{R})} \ll S I_{\mathrm{s}} \\
2 U_{\mathrm{T}} \sqrt{\frac{I_{\mathrm{F}(\mathrm{R})}}{S I_{\mathrm{s}}}}, & I_{\mathrm{F}(\mathrm{R})} \gg S I_{\mathrm{s}}\end{cases}
\end{aligned}
$$

## Delbrück's Bump/Antibump Circuit

If the bias current, $I_{\mathrm{b}}$, is in weak inversion, then the bump current, $I_{3}$, is an even-symmetric, bell-shaped function of the differential-mode input voltage, $V_{\mathrm{dm}}=$ $V_{1}-V_{2}$, given by

$$
I_{3}=\frac{I_{\mathrm{b}}}{2} \operatorname{sech}^{2}\left(\frac{\kappa V_{\mathrm{dm}}}{2 U_{\mathrm{T}}}\right) .
$$

Note that the three output currents sum to a constant, $I_{\mathrm{b}}$, so the sum $I_{1}+I_{2}$ is just $I_{\mathrm{b}}$ less the bump current, which is the antibump current.

## A Variation of the Bump/Antibump Circuit

Now, suppose that we fix the bump current to be a constant, $I_{\mathrm{b}}$. If $I_{1}, I_{2}$, and $I_{\mathrm{b}}$ are in weak inversion, we have that

$$
\begin{aligned}
& I_{1}=S I_{\mathrm{s}} e^{\left(\kappa\left(V_{1}-V_{\mathrm{T} 0}\right)-V\right) / U_{\mathrm{T}}}, \\
& I_{2}=S I_{\mathrm{s}} e^{\left(\kappa\left(V_{2}-V_{\mathrm{T} 0}\right)-V\right) / U_{\mathrm{T}}},
\end{aligned}
$$

and

$$
I_{\mathrm{b}}=\frac{I_{1} I_{2}}{I_{1}+I_{2}}=I_{1} \| I_{2}
$$

## A Variation of the Bump/Antibump Circuit

Now, consider the quantity

$$
\begin{aligned}
I_{\mathrm{b}} & =I_{1} \| I_{2} \\
& =S I_{\mathrm{s}} e^{\left(\kappa\left(V_{1}-V_{\mathrm{T} 0}\right)-V\right) / U_{\mathrm{T}}} \| S I_{\mathrm{s}} e^{\left(\kappa\left(V_{2}-V_{\mathrm{T} 0}\right)-V\right) / U_{\mathrm{T}}} \\
& =S I_{\mathrm{s}} e^{-\kappa V_{\mathrm{T} 0} / U_{\mathrm{T}}} e^{-V / U_{\mathrm{T}}}\left(e^{\kappa V_{1} / U_{\mathrm{T}}} \| e^{\kappa V_{2} / U_{\mathrm{T}}}\right),
\end{aligned}
$$

which implies that

$$
S I_{\mathrm{s}} e^{-\kappa V_{\mathrm{T} 0} / U_{\mathrm{T}}} e^{-V / U_{\mathrm{T}}}=\frac{I_{\mathrm{b}}}{e^{\kappa V_{1} / U_{\mathrm{T}} \| e^{\kappa V_{2} / U_{\mathrm{T}}}} . . . . . .}
$$

## A Variation of the Bump/Antibump Circuit

By substituting this result back into the equations for $I_{1}$ and $I_{2}$, we find that

$$
I_{1}=\frac{I_{\mathrm{b}} e^{\kappa V_{1} / U_{\mathrm{T}}}}{e^{\kappa V_{1} / U_{\mathrm{T}}} \| e^{\kappa V_{2} / U_{\mathrm{T}}}}=I_{\mathrm{b}}\left(1+e^{\kappa\left(V_{1}-V_{2}\right) / U_{\mathrm{T}}}\right)=I_{\mathrm{b}}\left(1+e^{\kappa V_{\mathrm{dm}} / U_{\mathrm{T}}}\right)
$$

that

$$
I_{2}=\frac{I_{\mathrm{b}} e^{\kappa V_{2} / U_{\mathrm{T}}}}{e^{\kappa V_{1} / U_{\mathrm{T}}} \| e^{\kappa V_{2} / U_{\mathrm{T}}}}=I_{\mathrm{b}}\left(1+e^{-\kappa\left(V_{1}-V_{2}\right) / U_{\mathrm{T}}}\right)=I_{\mathrm{b}}\left(1+e^{-\kappa V_{\mathrm{dm}} / U_{\mathrm{T}}}\right)
$$

and that

$$
I_{1}-I_{2}=I_{\mathrm{b}}\left(e^{\kappa V_{\mathrm{dm}} / U_{\mathrm{T}}}-e^{-\kappa V_{\mathrm{dm}} / U_{\mathrm{T}}}\right)=2 I_{\mathrm{b}} \sinh \left(\frac{\kappa V_{\mathrm{dm}}}{U_{\mathrm{T}}}\right)
$$

Fixing the Bump Current at $I_{\mathrm{b}}$


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Bias Circuit for Folded-Cascode/Flipped-Follower


## Analytical Model of Output Current versus $V_{\mathrm{dm}}$

By using the simplified EKV model of the MOS transistor, we can show that

$$
I_{1}=I_{\mathrm{b}}+S I_{\mathrm{s}} \log ^{2}\left(1+e^{\kappa V_{\mathrm{dm}} / 2 U_{\mathrm{T}}}\left(e^{\sqrt{I_{\mathrm{b}} / S I_{\mathrm{s}}}}-1\right)\right)
$$

and that

$$
I_{2}=S I_{\mathrm{s}} \log ^{2}\left(1+e^{-\kappa V_{\mathrm{dm}} / 2 U_{\mathrm{T}}}\left(e^{\sqrt{I_{1} / S I_{\mathrm{s}}}}-1\right)\right)
$$

where all the symbols have their previously defined meanings.
Note that the model equation for $I_{2}$ is expressed in terms of $I_{1}$, which, in turn, is an explicit function of $V_{\mathrm{dm}}$. So, we can obtain explicit expressions for $I_{2}$ and $I_{1}-I_{2}$ in terms of $V_{\mathrm{dm}}$, but it is unclear that doing so is of much value, because doing so leads to very cumbersome expressions.

## Output Currents versus $V_{\mathrm{dm}}$



## Differential Output Current versus $V_{\mathrm{dm}}$



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## Differential-Mode Transconductance versus $V_{\mathrm{cm}}$



Enhanced-Slew-Rate Folded-Cascode Amplifier


Enhanced-Slew-Rate Folded-Cascode Amplifier Step Response


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