MOS Translinear Principle for All Inversion Levels

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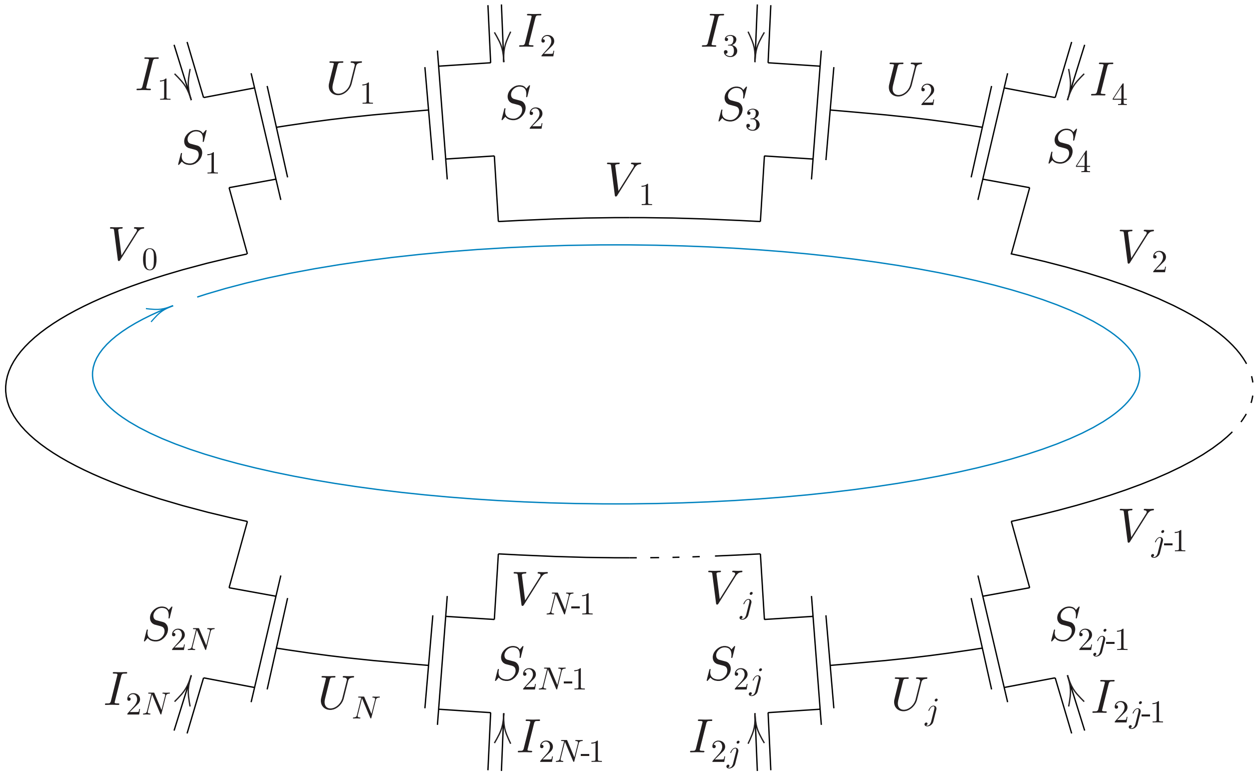
Translinear Principles for Alternating MOS Translinear Loops

Translinear Principle:

$$\prod_{j\in\mathrm{CW}} \frac{I_j}{S_j} = \prod_{j\in\mathrm{CCW}} \frac{I_j}{S_j}$$

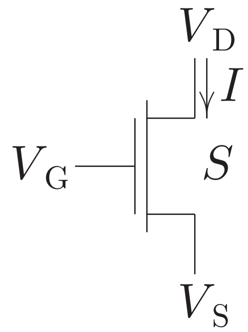
Voltage-Translinear Principle:

$$\sum_{j\in\mathrm{CW}} \sqrt{\frac{I_j}{S_j}} = \sum_{j\in\mathrm{CCW}} \sqrt{\frac{I_j}{S_j}}$$



Simple EKV MOS Transistor Model

We model the channel current of an nMOS transistor as the difference between a forward current and a reverse current,



$$I = I_F - I_R,$$

whose values are given by

$$I_{F(R)} = SI_s \log^2 \left(1 + e^{\left(\kappa(V_G - V_{T0}) - V_{S(D)}\right)/2U_T}\right),$$

where

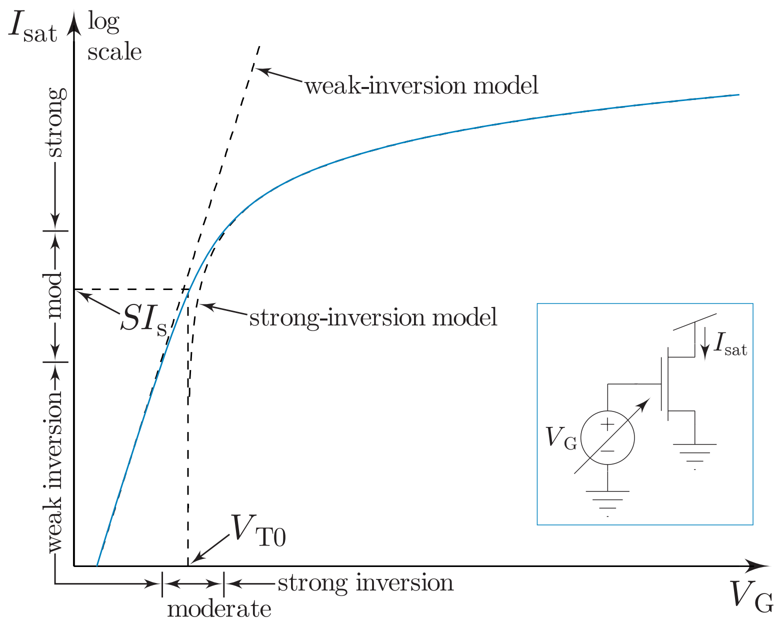
$$U_T = \frac{kT}{q}, \quad S = \frac{W}{L}, \quad I_s = \frac{2\mu C_{ox} U_T^2}{\kappa}, \quad \text{and} \quad \kappa = \frac{C_{ox}}{C_{ox} + C_{dep}}.$$

Note that SI_s is approximately twice the saturation current at threshold. This simple model covers all regions of normal MOS transistor operation.

Simple EKV MOS Transistor Model

The expressions for I_F and I_R reduce asymptotically to an exponential form in weak inversion and a quadratic form in strong inversion, given by

$$I_{\mathrm{F(R)}} \approx \begin{cases} SI_\mathrm{s} e^{\left(\kappa(V_\mathrm{G}-V_\mathrm{T0})-V_\mathrm{S(D)}\right)/U_\mathrm{T}}, \\ \qquad V_\mathrm{G} < V_\mathrm{T0} + \dfrac{V_\mathrm{S(D)}}{\kappa} \\ \dfrac{SI_\mathrm{s}}{4U_\mathrm{T}^2}\left(\kappa\left(V_\mathrm{G}-V_\mathrm{T0}\right)-V_\mathrm{S(D)}\right)^2, \\ \qquad V_\mathrm{G} > V_\mathrm{T0} + \dfrac{V_\mathrm{S(D)}}{\kappa}. \end{cases}$$



Simple EKV MOS Transistor Model

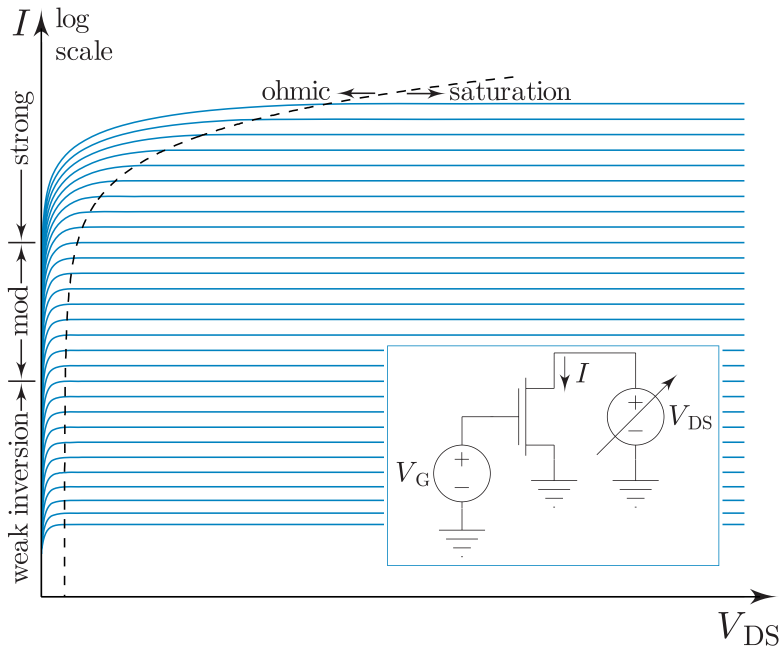
The expressions for I_F and I_R are also explicitly invertible; the inverses are given by

$$\begin{aligned}\kappa\left(V_\mathrm{G} - V_\mathrm{T0}\right) - V_\mathrm{S(D)} &= 2U_\mathrm{T}\log\left(e^{\sqrt{I_\mathrm{F(R)}/SI_\mathrm{s}}} - 1\right)\\ &\approx \begin{cases} U_\mathrm{T}\log\dfrac{I_\mathrm{F(R)}}{SI_\mathrm{s}}, & I_\mathrm{F(R)} \ll SI_\mathrm{s}\\ 2U_\mathrm{T}\sqrt{\dfrac{I_\mathrm{F(R)}}{SI_\mathrm{s}}}, & I_\mathrm{F(R)} \gg SI_\mathrm{s}.\end{cases}\end{aligned}$$

Simple EKV Model: The Onset of Saturation

Note that I_F depends only on V_G and V_S and that I_R depends only on V_G and V_D. If $I_F \gg I_R$, then $I \approx I_F$ and is nearly independent of V_D, which corresponds qualitatively to the saturation region of operation, so $I_{sat} \approx I_F$. We can define the onset of saturation operationally in terms of an arbitrary parameter, $A \gg 1$: We say that an MOS transistor is saturated if $I_F/I_R \geq A$. Using this notion, we can find an expression for V_{DSsat}, given by

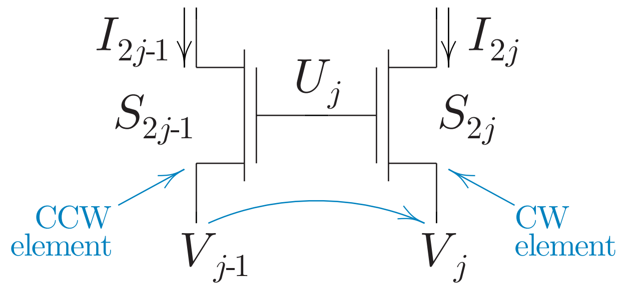
$$V_{DSsat} = 2U_T \log \frac{e^{\sqrt{I_{sat}/SI_s}} - 1}{e^{\sqrt{I_{sat}/ASI_s}} - 1} \approx \begin{cases} U_T \log A, & I_{sat} \ll SI_s \\ 2U_T \left(1 - \frac{1}{\sqrt{A}}\right) \sqrt{\frac{I_{sat}}{SI_s}}, & I_{sat} \gg SI_s \end{cases}$$



Counterclockwise/Clockwise Pairs

For the counterclockwise transistor, indexed by $2j-1$, we have that

$$\kappa\left(U_j - V_{\mathrm{T0}}\right) - V_{j-1} = 2U_{\mathrm{T}}\log\left(e^{\sqrt{I_{2j-1}/S_{2j-1}I_{\mathrm{s}}}} - 1\right),$$



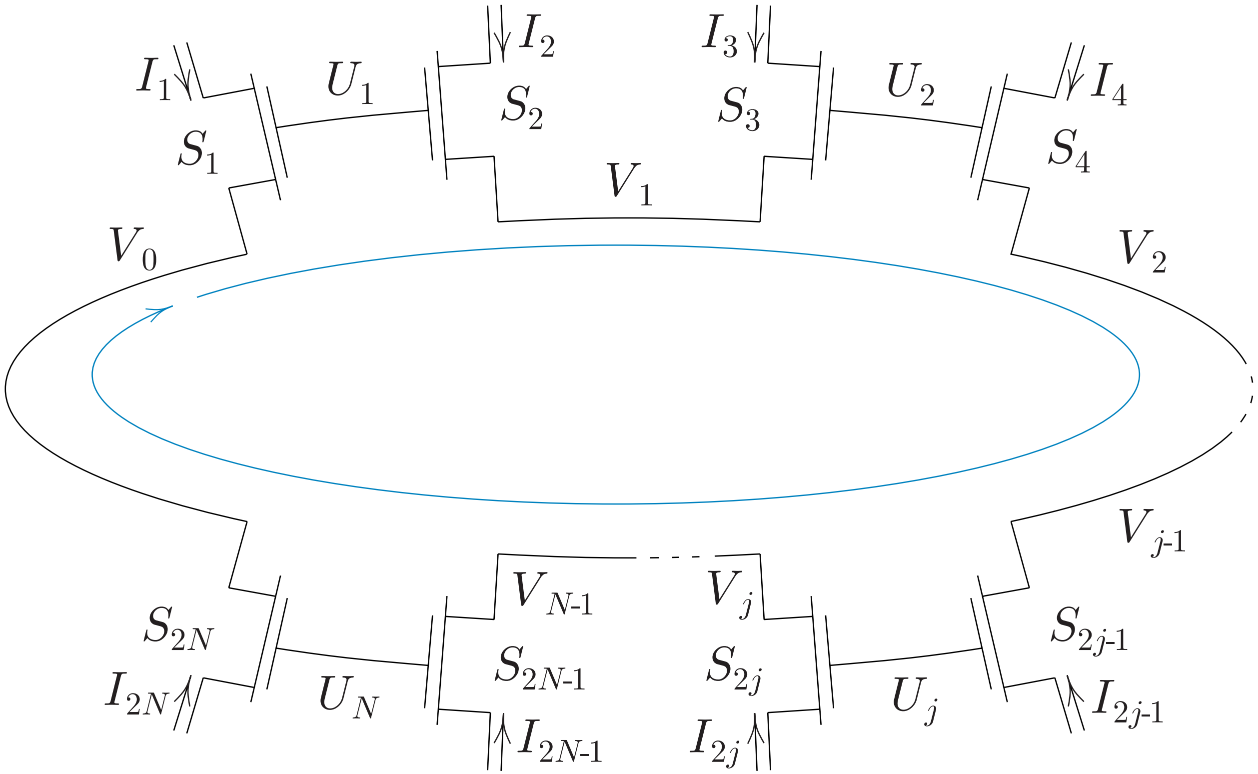
and for the clockwise transistor, indexed by $2j$, we have that

$$\kappa\left(U_j - V_{\mathrm{T0}}\right) - V_j = 2U_{\mathrm{T}}\log\left(e^{\sqrt{I_{2j}/S_{2j}I_{\mathrm{s}}}} - 1\right).$$

By subtracting these equations, we obtain

$$V_{j-1} - V_j = 2U_{\mathrm{T}}\log\frac{e^{\sqrt{I_{2j}/S_{2j}I_{\mathrm{s}}}} - 1}{e^{\sqrt{I_{2j-1}/S_{2j-1}I_{\mathrm{s}}}} - 1}.$$

Alternating MOS Translinear Loop



Generalized MOS Translinear Principle

Kirchhoff's voltage law (KVL) around the loop implies that

$$
\begin{aligned}
(V_0 - V_1) + \cdots + (V_{j-1} - V_j) + \\
\cdots + (V_{N-1} - V_0) &= 0 \\
\sum_{j=1}^{N} 2U_{\mathrm{T}} \log \frac{e^{\sqrt{I_{2j}/S_{2j}I_{\mathrm{s}}}} - 1}{e^{\sqrt{I_{2j-1}/S_{2j-1}I_{\mathrm{s}}}} - 1} &= 0 \\
2U_{\mathrm{T}} \log \prod_{j=1}^{N} \frac{e^{\sqrt{I_{2j}/S_{2j}I_{\mathrm{s}}}} - 1}{e^{\sqrt{I_{2j-1}/S_{2j-1}I_{\mathrm{s}}}} - 1} &= 0 \\
\log \prod_{j=1}^{N} \frac{e^{\sqrt{I_{2j}/S_{2j}I_{\mathrm{s}}}} - 1}{e^{\sqrt{I_{2j-1}/S_{2j-1}I_{\mathrm{s}}}} - 1} &= 0.
\end{aligned}
$$

Generalized MOS Translinear Principle

By exponentiating both sides of this equation, we have that

$$
\begin{aligned}
\prod_{j=1}^{N} \frac{e^{\sqrt{I_{2j}/S_{2j}I_{\mathrm{s}}}} - 1}{e^{\sqrt{I_{2j-1}/S_{2j-1}I_{\mathrm{s}}}} - 1} &= 1 \\
\prod_{j=1}^{N} \left(e^{\sqrt{I_{2j}/S_{2j}I_{\mathrm{s}}}} - 1\right) &= \prod_{j=1}^{N} \left(e^{\sqrt{I_{2j-1}/S_{2j-1}I_{\mathrm{s}}}} - 1\right) \\
\prod_{j \in \mathrm{CW}} \left(e^{\sqrt{I_{j}/S_{j}I_{\mathrm{s}}}} - 1\right) &= \prod_{j \in \mathrm{CCW}} \left(e^{\sqrt{I_{j}/S_{j}I_{\mathrm{s}}}} - 1\right),
\end{aligned}
$$

which is a generalized TLP for alternating loops of saturated MOS transistors.

Generalized MOS TLP: Weak Inversion Limit

If $I_j \ll S_j I_\mathrm{s}$ (i.e., weak inversion operation) for all j,

$$\prod_{j\in\mathrm{CW}} \left(e^{\sqrt{I_j/S_j I_\mathrm{s}}} - 1\right) = \prod_{j\in\mathrm{CCW}} \left(e^{\sqrt{I_j/S_j I_\mathrm{s}}} - 1\right)$$

$$\prod_{j\in\mathrm{CW}} \left(1 + \sqrt{\frac{I_j}{S_j I_\mathrm{s}}} - 1\right) = \prod_{j\in\mathrm{CCW}} \left(1 + \sqrt{\frac{I_j}{S_j I_\mathrm{s}}} - 1\right)$$

$$\prod_{j\in\mathrm{CW}} \sqrt{\frac{I_j}{S_j I_\mathrm{s}}} = \prod_{j\in\mathrm{CCW}} \sqrt{\frac{I_j}{S_j I_\mathrm{s}}}$$

$$\prod_{j\in\mathrm{CW}} \sqrt{\frac{I_j}{S_j}} = \prod_{j\in\mathrm{CCW}} \sqrt{\frac{I_j}{S_j}}$$

$$\prod_{j\in\mathrm{CW}} \frac{I_j}{S_j} = \prod_{j\in\mathrm{CCW}} \frac{I_j}{S_j}$$

Generalized MOS TLP: Strong Inversion Limit

If $I_j \gg S_j I_\mathrm{s}$ (i.e., strong inversion operation) for all j,

$$\prod_{j\in\mathrm{CW}} \left(e^{\sqrt{I_j/S_j I_\mathrm{s}}} - 1\right) = \prod_{j\in\mathrm{CCW}} \left(e^{\sqrt{I_j/S_j I_\mathrm{s}}} - 1\right)$$

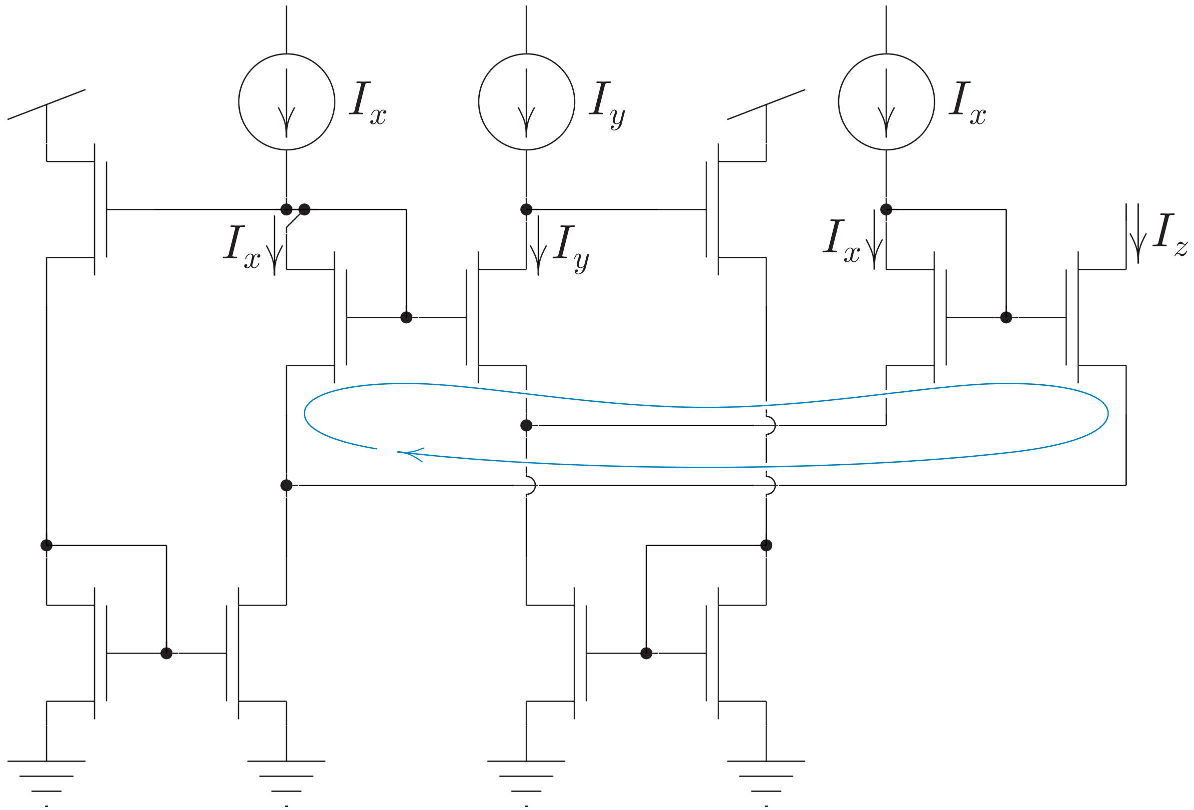
$$\prod_{j\in\mathrm{CW}} e^{\sqrt{I_j/S_j I_\mathrm{s}}} = \prod_{j\in\mathrm{CCW}} e^{\sqrt{I_j/S_j I_\mathrm{s}}}$$

$$\exp\left(\sum_{j\in\mathrm{CW}} \sqrt{\frac{I_j}{S_j I_\mathrm{s}}}\right) = \exp\left(\sum_{j\in\mathrm{CCW}} \sqrt{\frac{I_j}{S_j I_\mathrm{s}}}\right)$$

$$\sum_{j\in\mathrm{CW}} \sqrt{\frac{I_j}{S_j I_\mathrm{s}}} = \sum_{j\in\mathrm{CCW}} \sqrt{\frac{I_j}{S_j I_\mathrm{s}}}$$

$$\sum_{j\in\mathrm{CW}} \sqrt{\frac{I_j}{S_j}} = \sum_{j\in\mathrm{CCW}} \sqrt{\frac{I_j}{S_j}}$$

Simple MOS Translinear Circuit



Simple MOS Translinear Circuit

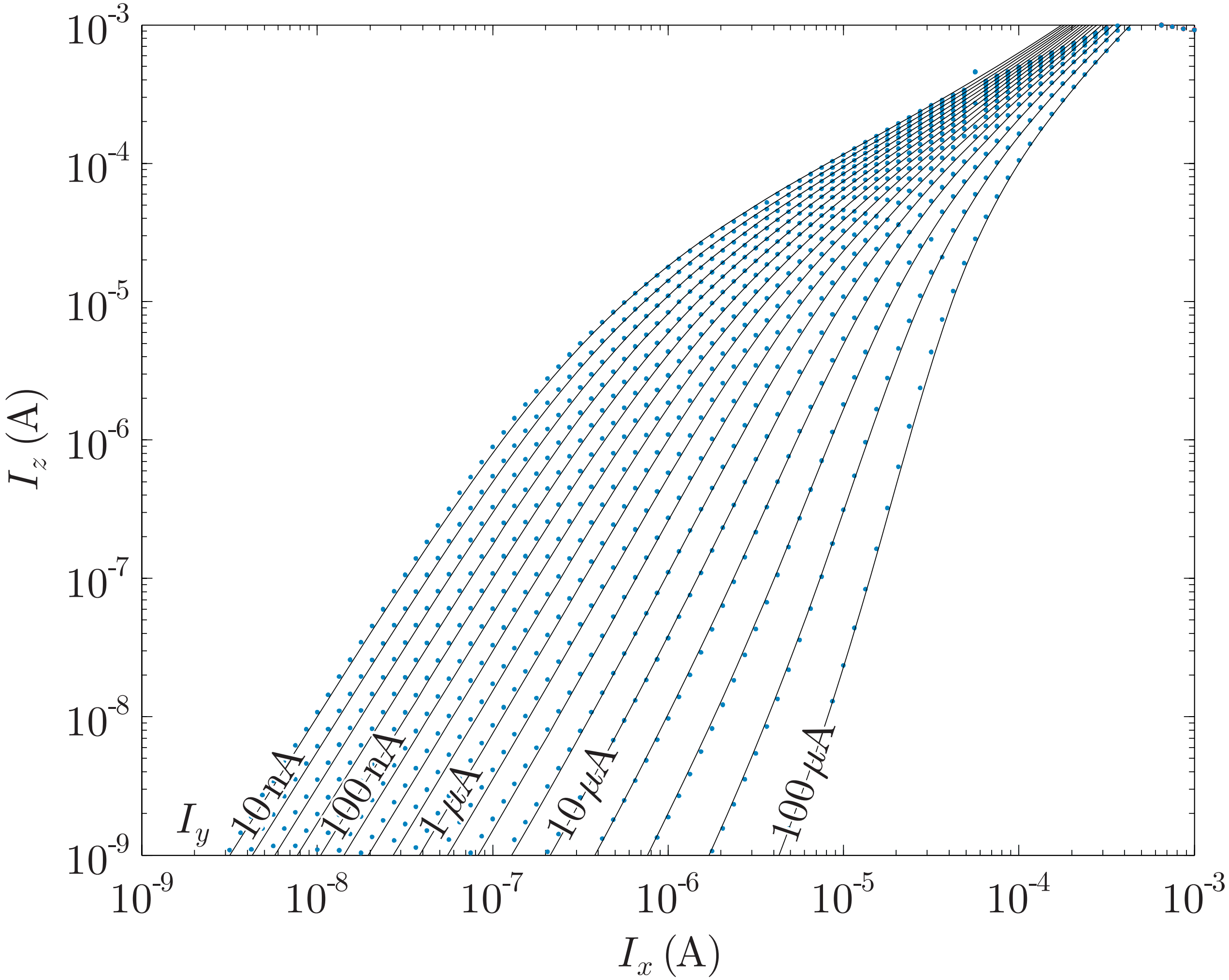
By applying the generalized TLP around the loop indicated, we have that

$$\left(e^{\sqrt{I_y/SI_\mathrm{s}}}-1\right)\left(e^{\sqrt{I_z/SI_\mathrm{s}}}-1\right)=\left(e^{\sqrt{I_x/SI_\mathrm{s}}}-1\right)^2,$$

which we can solve for I_z, thereby obtaining

$$\begin{aligned} I_z &= SI_\mathrm{s}\log^2\left(1+\frac{\left(e^{\sqrt{I_x/SI_\mathrm{s}}}-1\right)^2}{e^{\sqrt{I_y/SI_\mathrm{s}}}-1}\right) \\ &\approx \begin{cases} \dfrac{I_x^2}{I_y}, & I_x, I_y, I_z \ll SI_\mathrm{s} \\ 4I_x+I_y-4\sqrt{I_xI_y}, & I_x, I_y, I_z \gg SI_\mathrm{s}. \end{cases} \end{aligned}$$

Simple MOS Translinear Circuit



Simple MOS Translinear Circuit